Staff Solutions to Mini-Quiz 1, afternoon

Problem 1 (10 points).
Prove that $\sqrt[3]{4}$ is irrational.

Solution. Proof. Assume for the sake of contradiction that $\sqrt[3]{4}$ is rational. Under this assumption, there exist integers $a$ and $b$ such that

$$\sqrt[3]{4} = \frac{a}{b},$$

where $a$ and $b$ have no common factor. Now we prove that $a$ and $b$ are both even, that is, they have 2 as a common factor, a contradiction.

$$\sqrt[3]{4} = \frac{a}{b}, \quad \text{(by assumption)}$$
$$4 = \frac{a^3}{b^3}, \quad \text{(cubing both sides)}$$
$$4b^3 = a^3.\quad \text{(dividing by 4)}$$

The lefthand side of the last equation is even, so $a^3$ is even, which implies that $a$ is even. In particular, $a = 2c$ for some integer $c$. Thus,

$$4b^3 = (2c)^3 = 8c^3,\quad \text{(dividing by 4)}$$
$$b^3 = 2c^3.$$