6.042 Final Review: Topics

These are the main topics we have covered in 6.042 this semester. In studying for the final exam, you may find it useful to use this handout as a guide.

1 Proofs

- propositions, axioms, deductions
- predicates and quantifiers
- proof by contradiction
- proof by contrapositive

2 Induction

- proof by induction
- strong induction
- state machines and invariants
- the 8 puzzle (and friends)

3 Number theory

- divisibility
- Division Theorem
- GCD and linear combinations
- Pulverizer Algorithm
- modular arithmetic
- multiplicative inverses
- Fermat’s Theorem
- relative primality and Euler’s theorem

4 Graph theory

- graphs: simple, weighted, directed, trees
- graph colorings
- connectivity
• matchings, perfect matchings
• bipartite graphs
• stable marriage problem, the mating algorithm
• induction on graphs (build up error)
• minimum weight spanning trees, MST algorithm
• paths, cycles, walks, tours
• Hamiltonian paths, Euler circuits
• tournament graphs
• PageRank

5 Communication Networks

• definitions of latency, diameter, switch size, switch count, congestion
• network architectures: binary tree, 2D array, butterfly, Beneš

6 Relations

• basic definitions: reflexive, symmetric, antisymmetric, transitive
• equivalency relations and classes
• partial orders, total order, topological sorts, Hasse diagrams
• task scheduling theorems, chains, antichains, Dilworth’s Theorem

7 Sums and asymptotics

• geometric sums, finite and infinite
• solving sums through differentiation and integration
• sums of powers (e.g. $\sum i$, $\sum i^2$)
• integration bounds for sums
• harmonic numbers
• solving products through logarithms

8 Recurrences

• guess-and-check
• plug-and-chug
• Akra-Bazzi theorem
• linear recurrences
9  **Asymptotic Notation**

- asymptotic notation
- induction involving asymptotics

10  **Counting**

- functions, bijections
- sum rule
- product rule
- the pigeonhole principle
- generalized product rule
- bookkeeper formula
- permutations
- division rule
- inclusion-exclusion principle
- Stirling’s approximation
- counting bit sequences
- counting poker hands
- combinatorial proofs

11  **Basic Probability**

- Monty Hall problem
- tree diagrams
- sample points/outcomes, events
- product rule for intersection of events
- inclusion-exclusion for unions of events
- independence (pairwise and mutual)
- conditional probability, Bayes rule
- union bound
- law of total probability
- birthday paradox

12  **Random Variables**

- random variables, indicator random variables
- probability distributions: Bernoulli, binomial, uniform and their expectation, variance
- independence of random variables
- expectation, conditional expectation, expectation of functions
- expectation of sums (distributive for all random variables)
• expectation of products (distributive for independent random variables)
• expectation of quotients (in general, not distributive)
• law of total expectation
• mean time to failure
• variance, different definitions
• variance of sums (distributive for independent random variables)
• polling, approximating the binomial distribution
• expectation bounds: Markov, Chebyshev, Chernoff, Murphy’s
• random walks: gambler’s ruin, random walks on graphs