1.

14.11 This question exercises many aspects of the student's understanding of Bayesian networks and uncertainty.

a. A suitable network is shown in Figure S14.2. The key aspects are: the failure nodes are parents of the sensor nodes, and the temperature node is a parent of both the gauge and the gauge failure node. It is exactly this kind of correlation that makes it difficult for humans to understand what is happening in complex systems with unreliable sensors.

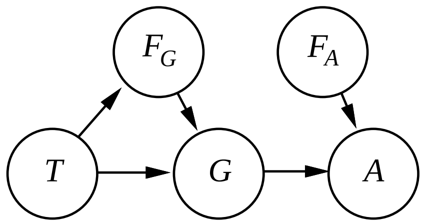

Figure S14.2 A Bayesian network for the nuclear alarm problem.

b. No matter which way the student draws the network, it should not be a polytree because of the fact that the temperature influences the gauge in two ways.

c. The CPT for G is shown below. Students should pay careful attention to the semantics of F_G, which is true when the gauge is *faulty*, i.e., *not* working.

	$T = Normal$		$T = High$	
	F_G	$\neg F_G$	F_G	$\neg F_G$
$G = Normal$	y	x	$1-y$	$1-x$
$G = High$	$1-y$	$1-x$	y	x

d. The CPT for A is as follows:

	$G = Normal$		$G = High$	
	F_A	$\neg F_A$	F_A	$\neg F_A$
A	0	0	0	1
$\neg A$	1	1	1	0

e. This part actually asks the student to do something usually done by Bayesian network algorithms. The great thing is that doing the calculation without a Bayesian network makes it easy to see the nature of the calculations that the algorithms are systematizing. It illustrates the magnitude of the achievement involved in creating complete and correct algorithms.

Abbreviating $T = High$ and $G = High$ by T and G, the probability of interest here is $P(T|A, \neg F_G, \neg F_A)$. Because the alarm's behavior is deterministic, we can reason that if the alarm is working and sounds, G must be $High$. Because F_A and A are d-separated from T, we need only calculate $P(T|\neg F_G, G)$.

There are several ways to go about doing this. The "opportunistic" way is to notice that the CPT entries give us $P(G|T, \neg F_G)$, which suggests using the generalized Bayes' Rule to switch G and T with $\neg F_G$ as background:

$$P(T|\neg F_G, G) \propto P(G|T, \neg F_G)P(T|\neg F_G)$$

We then use Bayes' Rule again on the last term:

$$P(T|\neg F_G, G) \propto P(G|T, \neg F_G)P(\neg F_G|T)P(T)$$

A similar relationship holds for $\neg T$:

$$P(\neg T|\neg F_G, G) \propto P(G|\neg T, \neg F_G)P(\neg F_G|\neg T)P(\neg T)$$

Normalizing, we obtain

$$P(T|\neg F_G, G) = \frac{P(G|T,\neg F_G)P(\neg F_G|T)P(T)}{P(G|T,\neg F_G)P(\neg F_G|T)P(T)+P(G|\neg T,\neg F_G)P(\neg F_G|\neg T)P(\neg T)}$$

The "systematic" way to do it is to revert to joint entries (noticing that the subgraph of T, G, and F_G is completely connected so no loss of efficiency is entailed). We have

$$P(T|\neg F_G, G) = \frac{P(T, \neg F_G, G)}{P(G, \neg F_G)} = \frac{P(T, \neg F_G, G)}{P(T, G, \neg F_G) + P(T, G, \neg F_G)}$$

Now we use the chain rule formula (Equation 15.1 on page 439) to rewrite the joint entries as CPT entries:

$$P(T|\neg F_G, G) = \frac{P(T)P(\neg F_G|T)P(G|T,\neg F_G)}{P(T)P(\neg F_G|T)P(G|T,\neg F_G)+P(\neg T)P(\neg F_G|\neg T)P(G|\neg T,\neg F_G)}$$

which of course is the same as the expression arrived at above. Letting $P(T) = p$, $P(F_G|T) = g$, and $P(F_G|\neg T) = h$, we get

$$P(T|\neg F_G, G) = \frac{p(1-g)(1-x)}{p(1-g)(1-x) + (1-p)(1-h)x}$$

2.

(a) Sum the sizes of the conditional probability tables of $P(A), P(B|A), P(C|B), P(D|B), P(E|B), P(F|C, D, E, G), P(G|A)$: $1 + 2 + 2 + 2 + 2 + 16 + 2 = 27$.

(b) No.

(c) No.

(d) No.

(e) Irrelevant variables: E, F, G.

(f) $P(d|c) = P(c, d) \times P(c)$

$P(c, d) = \sum_b P(c|b)P(d|b)P(b) = \sum_b P(c|b)P(d|b) \sum_a P(b|a)P(a)$

$P(c) = \sum_b P(c|b) \sum_a P(b|a)P(a)$

(g) Factors created: $F_1(B, G), F_2(C, D, E, G), F_3(C, D, F, G), F_4(C, D, G), F_5(C, D)$.

(h) No.