

1. The sample space of a fair coin flip is  $\{H, T\}$ .

The sample space of a sequence of three fair coin flips is all  $2^3$  possible sequences of outcomes:  $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ .

The sample space of a sequence of five fair coin flips in which at least four flips are heads is  $\{HHHHH, HHHHT, HHHTH, HHTHH, HTHHH, THHHH\}$ .

The probability of heads doesn't matter (unless it's 0 or 1).

2.  $P(\text{sum is } 8) = P(2, 6) + P(6, 2) + P(3, 5) + P(5, 3) + P(4, 4) = \frac{5}{36}$

$$P(\text{sum is } 8 | \text{first is } 3) = P(5) = \frac{1}{6}$$

3. 
$$P(\text{disease} | \text{positive}) = \frac{P(\text{positive} | \text{disease})P(\text{disease})}{P(\text{positive})}$$

$$= \frac{P(\text{positive} | \text{disease})P(\text{disease})}{P(\text{positive} | \text{disease})P(\text{disease}) + P(\text{positive} | \text{no disease})P(\text{no disease})}$$

$$= \frac{0.95 \times 0.001}{(0.95 \times 0.001) + (0.05 \times 0.999)} \approx 0.0187$$

4.  $P(\neg T, \neg C) = 1 - (0.05 + 0.05 + 0.1) = 0.8$

$$P(T) = 0.05 + 0.05 = 0.1$$

$$P(C) = 0.05 + 0.1 = 0.15$$

They are not independent.

$$P(T | C) = \frac{0.05}{0.05 + 0.1} = \frac{1}{3} \neq 0.1 = P(T).$$

5.  $P(A \cap B) = P(\text{first roll is } 3) = \frac{1}{6}$

$$P(A \cap C) = P(\text{first roll is } 3 \text{ and second roll is a } 6) = \frac{1}{36}$$

$$P(B \cap C) = P(\text{rolls are } (3,6), (4,5), \text{ or } (5,4)) = \frac{3}{36} = \frac{1}{12}$$

$$P(A \cap B \cap C) = P(\text{first roll is } 3 \text{ and second roll is a } 6) = \frac{1}{36}$$

6. 
$$P(\text{biased} | \text{all 10 heads}) = \frac{P(\text{all 10 heads} | \text{biased})P(\text{biased})}{P(\text{all 10 heads})}$$

$$= \frac{P(\text{all 10 heads} | \text{biased})P(\text{biased})}{P(\text{all 10 heads} | \text{biased})P(\text{biased}) + P(\text{all 10 heads} | \text{fair})P(\text{fair})}$$

$$= \frac{1 \times 0.001}{(1 \times 0.001) + (0.5^{10} \times 0.999)} \approx 0.506$$