6.034 Midterm Quiz, Spring 2012

March 5, 2012

Name:

1 (25pts)	
2 (25pts)	
3 (25pts)	
4 (25pts)	
Total	

1 Propositional Entailment (25 points)

Circle the numbers of ALL the formulas for which the indicated entailment holds.

Please make sure that we can figure out which ones you mean to circle, if there's any possibility of confusion because of crossing out, write out clearly a list of the ones you meant to circle.

1. $(p \to q) \wedge (\neg p) \models \neg q$
2. $(p \vee q) \wedge (\neg q) \models p$
3. $((p \vee q) \to r) \wedge (p \to (\neg q)) \wedge r \models q$
4. $(p \to q) \wedge (p \leftrightarrow r) \wedge (r) \models q$
5. $(p \leftrightarrow q) \wedge (q) \models p$
6. $(p \to q) \wedge (\neg q) \models \neg p$
7. $(p \vee \neg q) \wedge (\neg q) \models \neg p$
8. $((p \wedge q) \to r) \wedge (\neg r) \models (\neg p \vee \neg q)$
9. $(p \to q) \wedge (q) \models p$

2 Propositional Proof Methods (25 points)

Here are some simple propositional statements:

- if Man then Mortal
- if Mortal then Boring
- not (Man and Boring)

From these statements we want to prove "not Man".
You can use the following for your formulas, A for Man, O for Mortal, B for Boring.

1. Carry out a resolution refutation proof. Label your clauses with numbers and indicate which clauses are being used at each step.

2. Explain briefly how you can use an algorithm for checking propositional satisfiability as an alternative way of proving "not Man" from the input statements. Write down in detail the propositional sentence involved in this approach to finding a proof.

3. Indicate whether you should use DPLL or WalkSAT for this application. Explain briefly why.

4. What is the size of the complete DPLL search space for this proof? Explain.

5. Would DPLL need to examine every state in the search space for this problem? Explain briefly, based on the operation of the algorithm, why or why not.

6. The Minesweeper game agent does two "proofs" per square: calling DPLL to find models for:

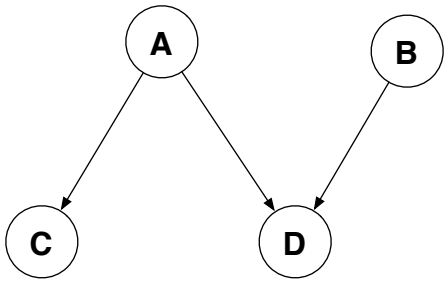
   (a) $Board \wedge X_{i,j}$, and

   (b) $Board \wedge \neg X_{i,j}$

   where $Board$ is the formula encoding the board. There are four possible outcomes to these two calls: (Model, Model), (Model, None), (None, Model) and (None, None). Explain the meaning of each of these outcomes and what it means in the game.

3 Bayes Nets (25 points)

Here is a Bayesian Net involving four variables.



A has two values (a_1, a_2), B has three values (b_1, b_2, b_3), C has two values (c_1 and c_2) and D has two values (d_1 and d_2).

1. What is the number of (non-redundant) probability values that need to be specified at each node of this network? What is the total number for the whole network?

2. Suppose you find out that $D = d_1$, write a formula for the probability distribution over C, given this, that is $P(C|D = d_1)$. The formula should only use expressions whose values can be read off from the CPTs in the network.

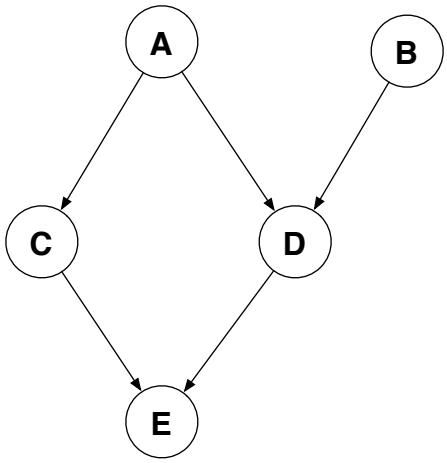
3. If we knew nothing else (ignore previous question), could learning that $C = c_1$ affect the probability of $B = b_2$? Explain.

4. If we knew $A = a_1$ (and nothing else), could learning that $C = c_1$ affect the probability of $B = b_2$? Explain.

5. If we knew $D = d_1$ (and nothing else), could learning that $C = c_1$ affect the probability of $B = b_2$? Explain.

4 Bayes Net Inference (25 points)

Here is a Bayesian Net involving five variables.



Assume each variable X has two values, x_1 and x_2, e.g. a variable A has values a_1 and a_2. In your answers below, specify a factor by writing down the variables involved in the factor, if evidence is involved, use the value of the evidence instead of the variable name, e.g. $f_1(A, E)$ or $f_3(E, C, d_1)$.

1. Show how $P(B)$ is computed via the Variable Elimination algorithm using the variable order: A, E, C, D. Show the sequence of new factors created by the algorithm; show which factors are being multiplied and summed over to create the new one. Identify each new factor created by a unique name, such as $f_5(A, E)$.

2. What is the largest factor created during the computation and how big is it (in terms of assignments)? Give the size before the call to `sumOut`.

3. How big is the biggest factor if we used the variable order D, C, A, E? Hint: You don't need to do the full VE process to answer this.

4. If you wanted to compute $P(B|C = c_1)$ instead, how would the VE computation you showed in part 1 change? Be specific.

5. If you are doing likelihood weighting to compute $P(E = e_1|C = c_1, A = a_2)$, what is a formula for the weight that you have to assign to the sample $(a_2, b_1, c_1, d_2, e_1)$? Write the formula in terms of the network CPTs.

6. Assume we are using a very large Bayesian Net that represents the connections between hundreds of diseases and thousands of symptoms. The network has intermediate nodes (between diseases and symptoms) that represent the states of internal organs, e.g. kidneys and lungs. A given symptom may ultimately be caused by multiple diseases.

   We want to use sampling to estimate conditional probabilities. For both sampling algorithms we worked with (rejection sampling and likelihood weighting) mention strengths (if any) and weaknesses (if any) you see with that algorithm for the particular problem. Which one would you recommend and (briefly) why?

   (a) $P(symptom_i|disease_j)$

      i. Rejection sampling

      ii. Likelihood weighting

   (b) $P(disease_j|symptom_i)$

      i. Rejection sampling

      ii. Likelihood weighting