Supervised Learning

- Given data (training set)

$$D = \left\{ \left\langle x^1, y^1 \right\rangle, \left\langle x^2, y^2 \right\rangle, \ldots . \left\langle x^m, y^m \right\rangle \right\}$$

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$\left\langle x_1^1, x_2^1, \ldots, x_n^1 \right\rangle$

input

Supervised Learning

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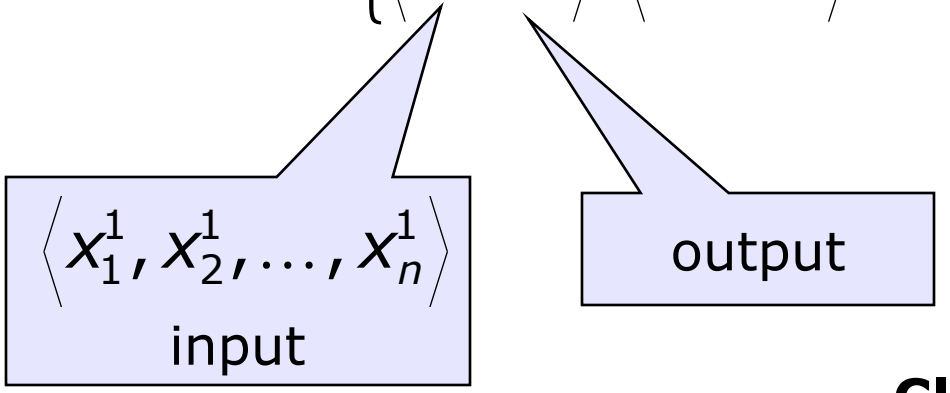

Classification: discrete Y

Regression: continuous Y

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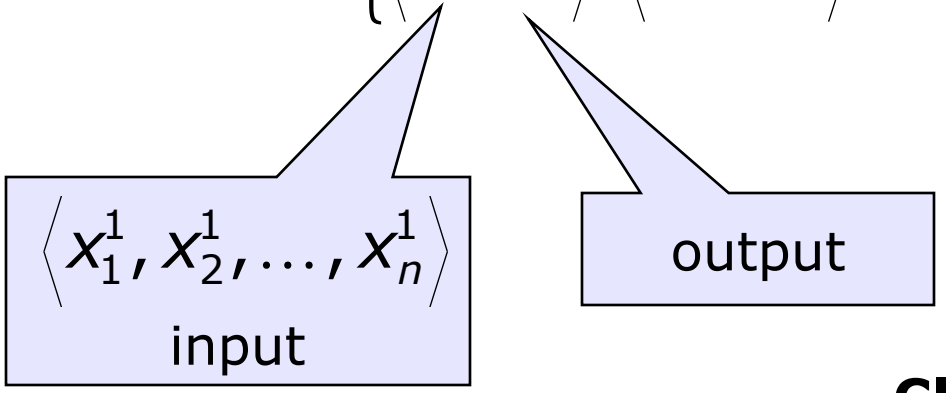

Classification: discrete Y

Regression: continuous Y

- Goal: find a hypothesis h in hypothesis class H that does a good job of mapping x to y

Best Hypothesis

Hypothesis should

- do a good job of describing the data
- not be too complex

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  - ideally: $h(x^i) = y^i$
  - number of errors: $E(h,D)$

- not be too complex

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  - measure: C(h)

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William of Ockham

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  - measure: C(h)

Non sunt multiplicanda entia praeter necessitatem

William of Ockham

Minimize $E(h,D) + \alpha C(h)$

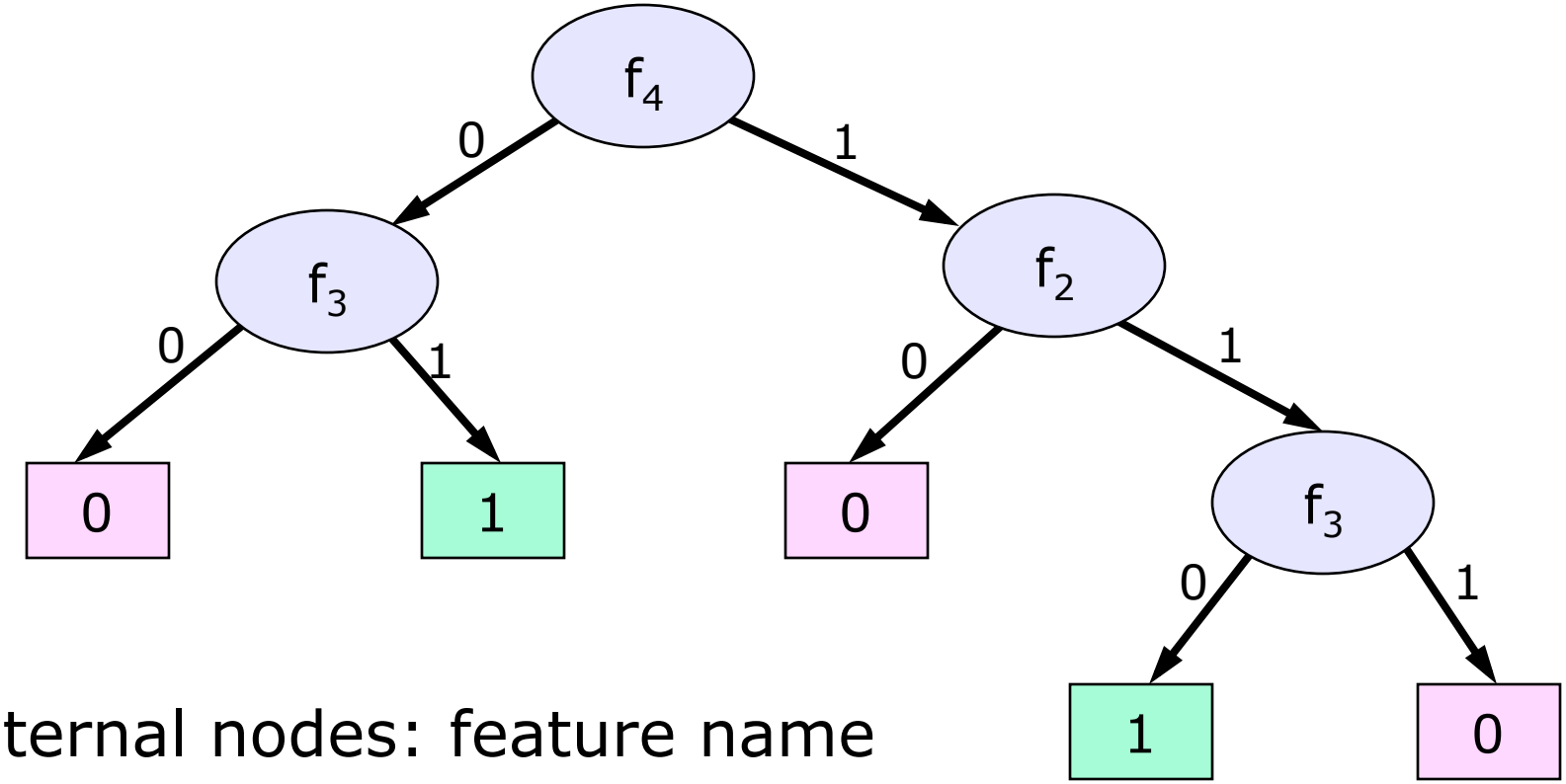
trade-off

Congressional Voting

0. handicapped-infants
1. water-project-cost-sharing
2. adoption-of-the-budget-resolution
3. physician-fee-freeze
4. el-salvador-aid
5. religious-groups-in-schools
6. anti-satellite-test-ban
7. aid-to-nicaraguan-contras
8. mx-missile
9. immigration
10. synfuels-corporation-cutback
11. education-spending
12. superfund-right-to-sue
13. crime
14. duty-free-exports
15. export-administration-act-south-africa

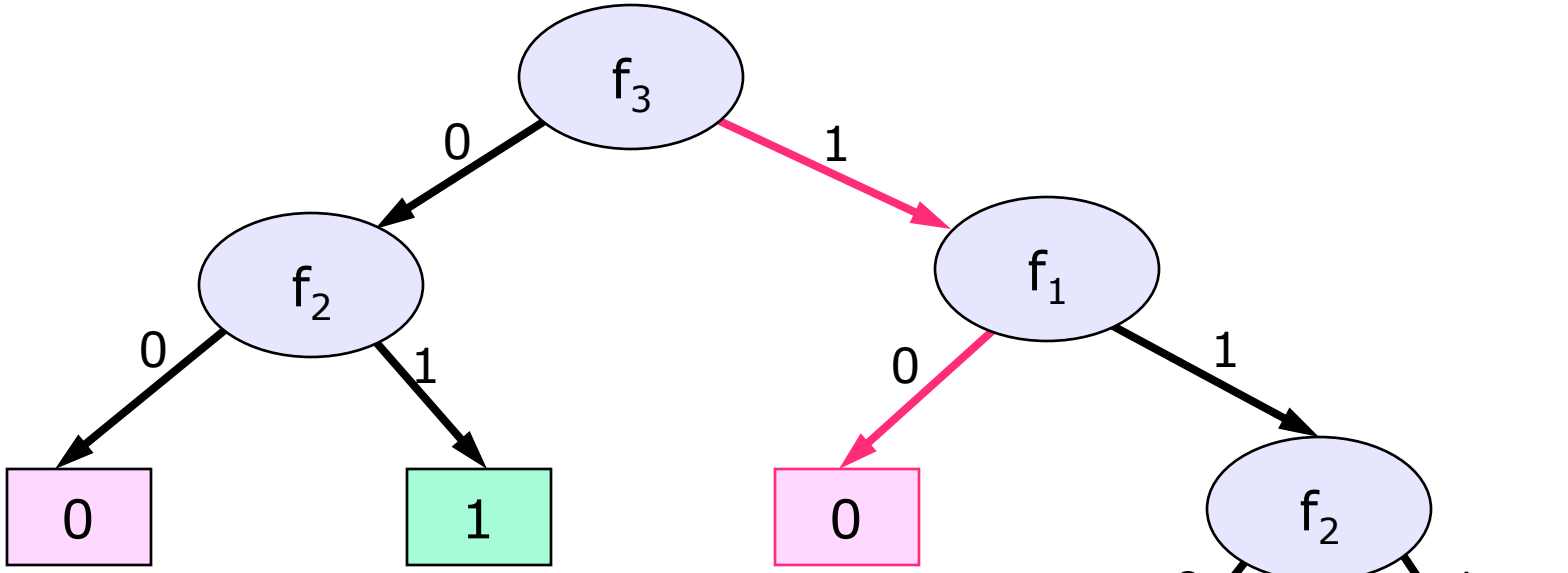
232 data points

Decision Trees: Hypothesis Class



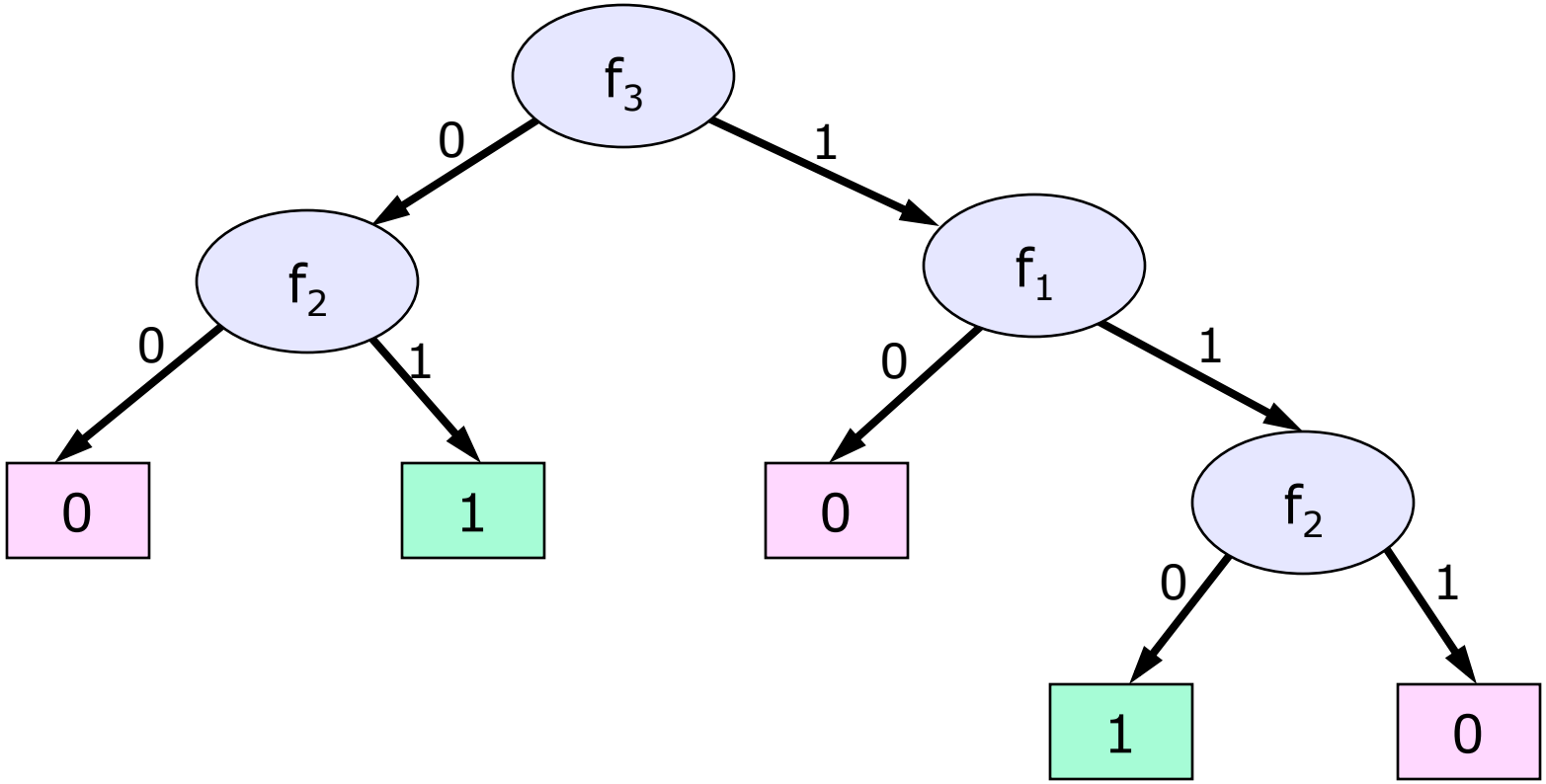
- Internal nodes: feature name
- One child for each value of the feature
- Leaf nodes: output

Hypothesis Class



$h(\langle 0,1,1,0 \rangle) = 0$

Hypothesis Class



$$h = \left(\neg f_3 \wedge f_2\right) \vee \left(f_3 \wedge f_1 \wedge \neg f_2\right)$$

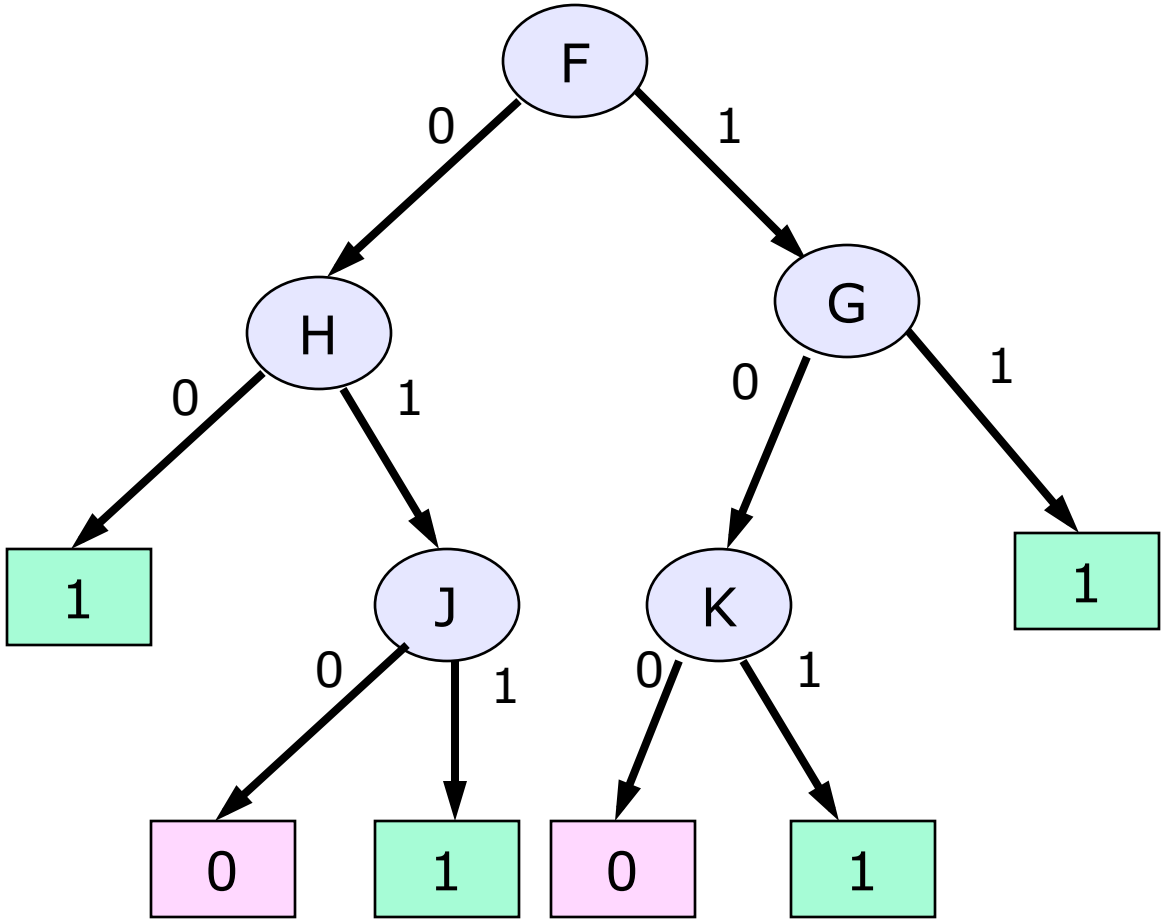
Tree Bias

- Both decision trees and DNF with negation can represent any Boolean function. So why bother with trees?
- Because we have a nice algorithm for growing trees that is consistent with a bias for simple trees (few nodes)

Tree Bias

- Both decision trees and DNF with negation can represent any Boolean function. So why bother with trees?
- Because we have a nice algorithm for growing trees that is consistent with a bias for simple trees (few nodes)
- Too hard to find the smallest good tree, so we'll be greedy again
- Have to watch out for overfitting

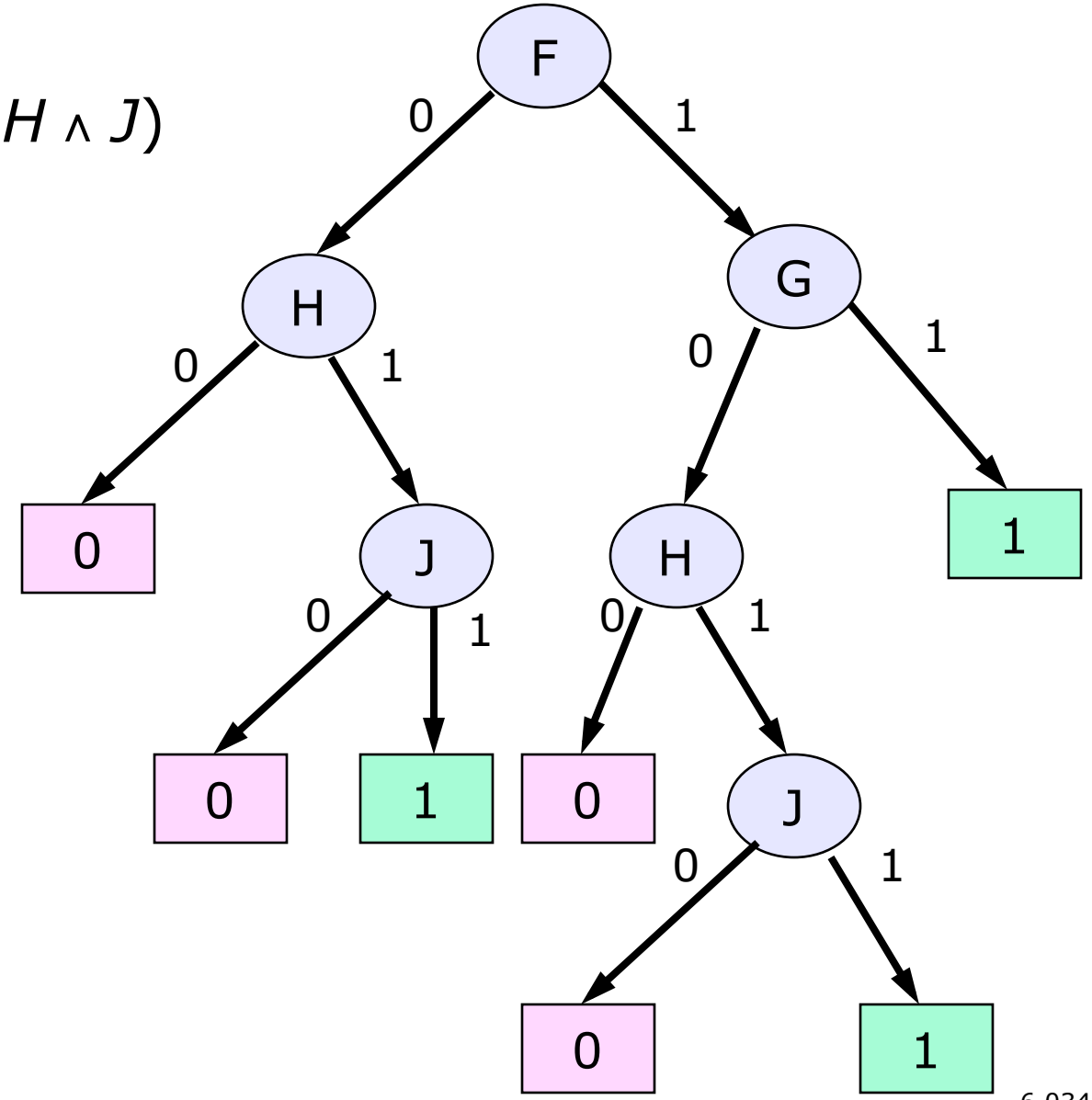
Trees vs DNF



$$(\neg F \wedge \neg H) \vee (\neg F \wedge H \wedge J) \vee (F \wedge \neg G \wedge K) \vee (F \wedge G)$$

Trees vs DNF

$(F \wedge G) \vee (H \wedge J)$



Algorithm

- Developed in parallel in AI by Quinlan and in statistics by Breiman, Friedman, Olsen and Stone

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BuildTree (Data)

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BuildTree (Data)

if all elements of Data have the same y value, then

MakeLeafNode(y)

Algorithm

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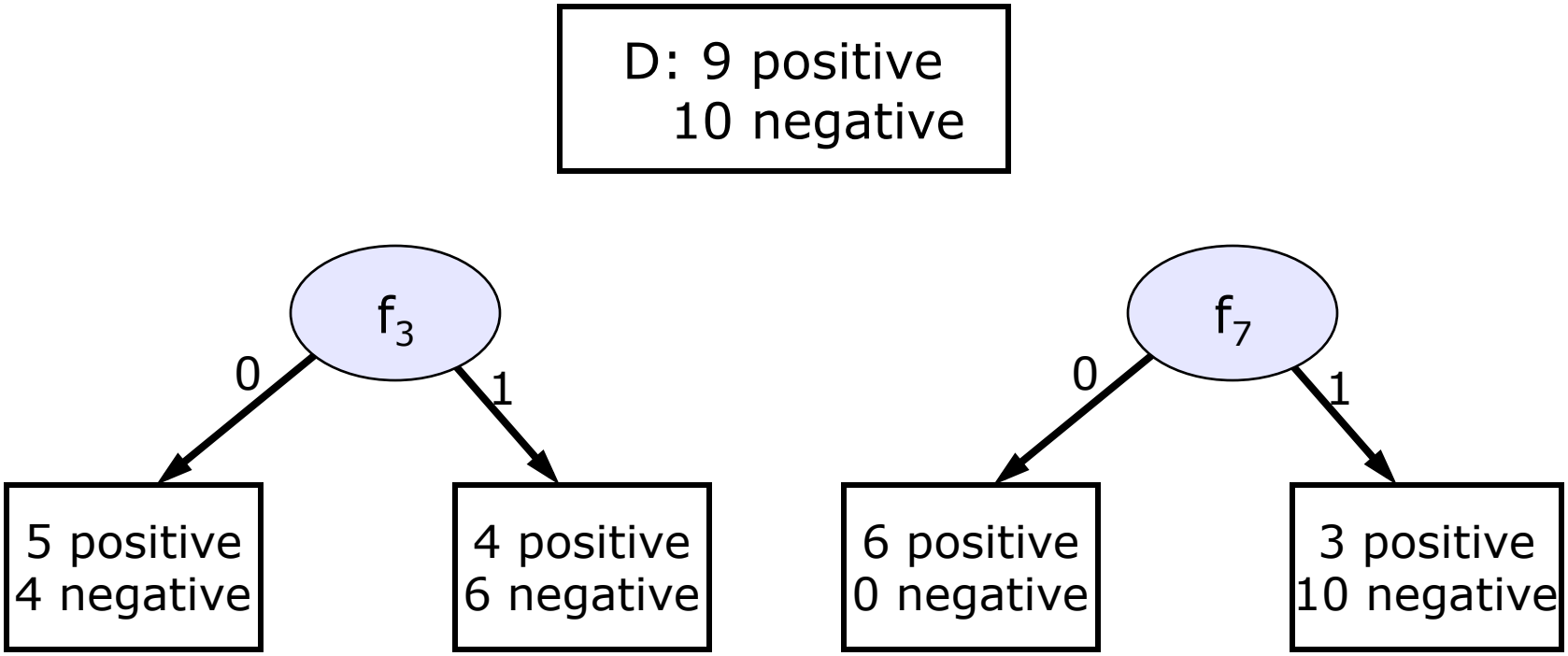
            BuildTree(SelectFalse(Data, feature)),
            BuildTree(SelectTrue(Data, feature)))
```

Let's Split

D: 9 positive
10 negative

Let's Split


D: 9 positive
10 negative
f_3
0
1
5 positive
4 negative
4 positive
6 negative
f_7
0
1
6 positive
0 negative
3 positive
10 negative

Entropy

p : proportion of positive examples in a data set

$$H = -p \log_2 p - (1 - p) \log_2 (1 - p)$$

Entropy

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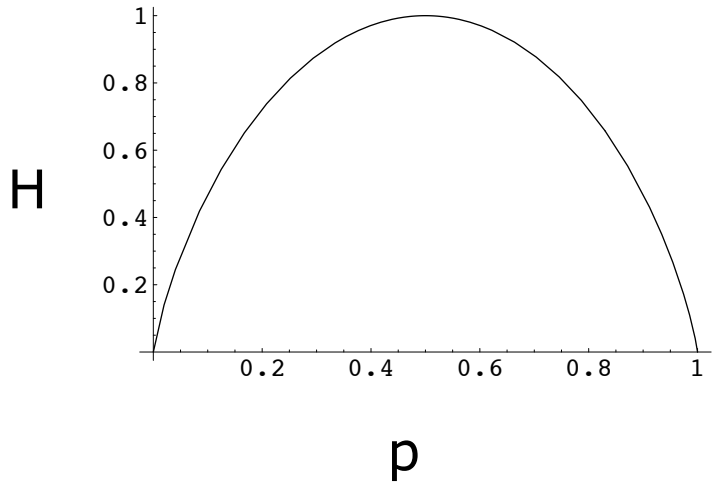
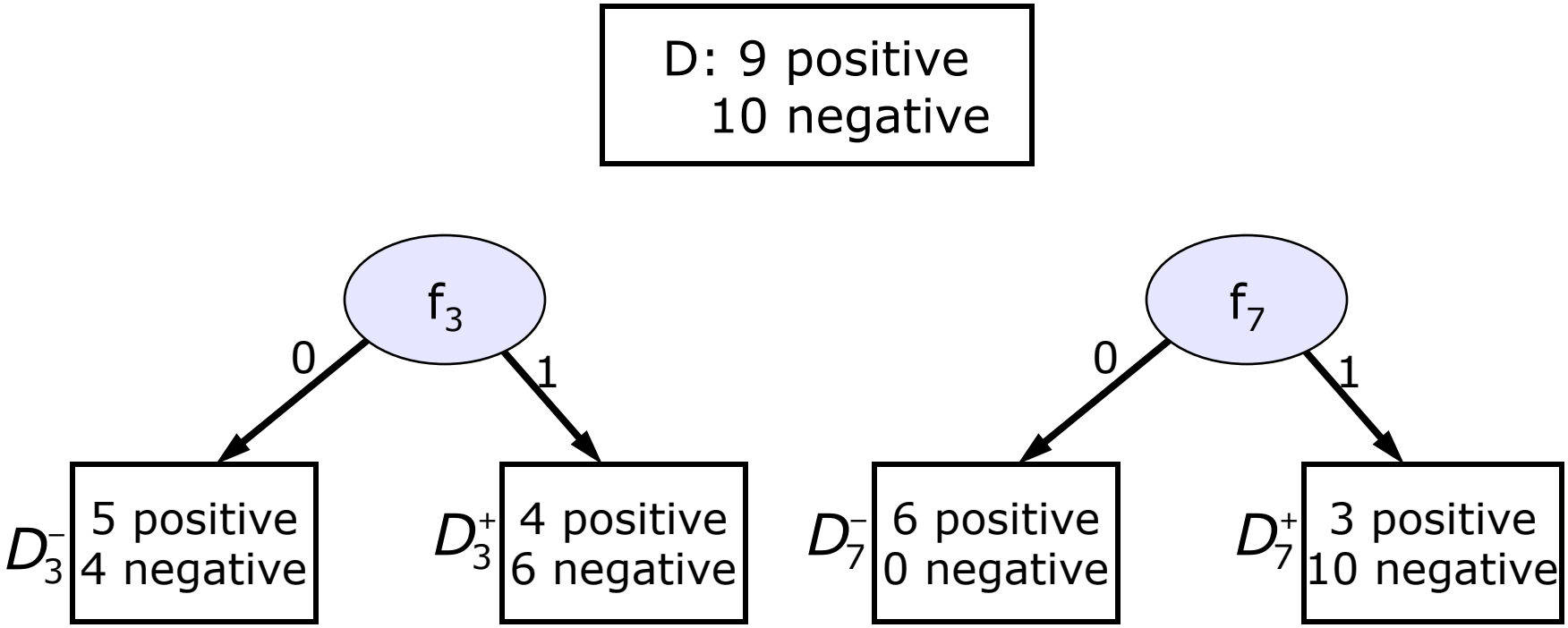
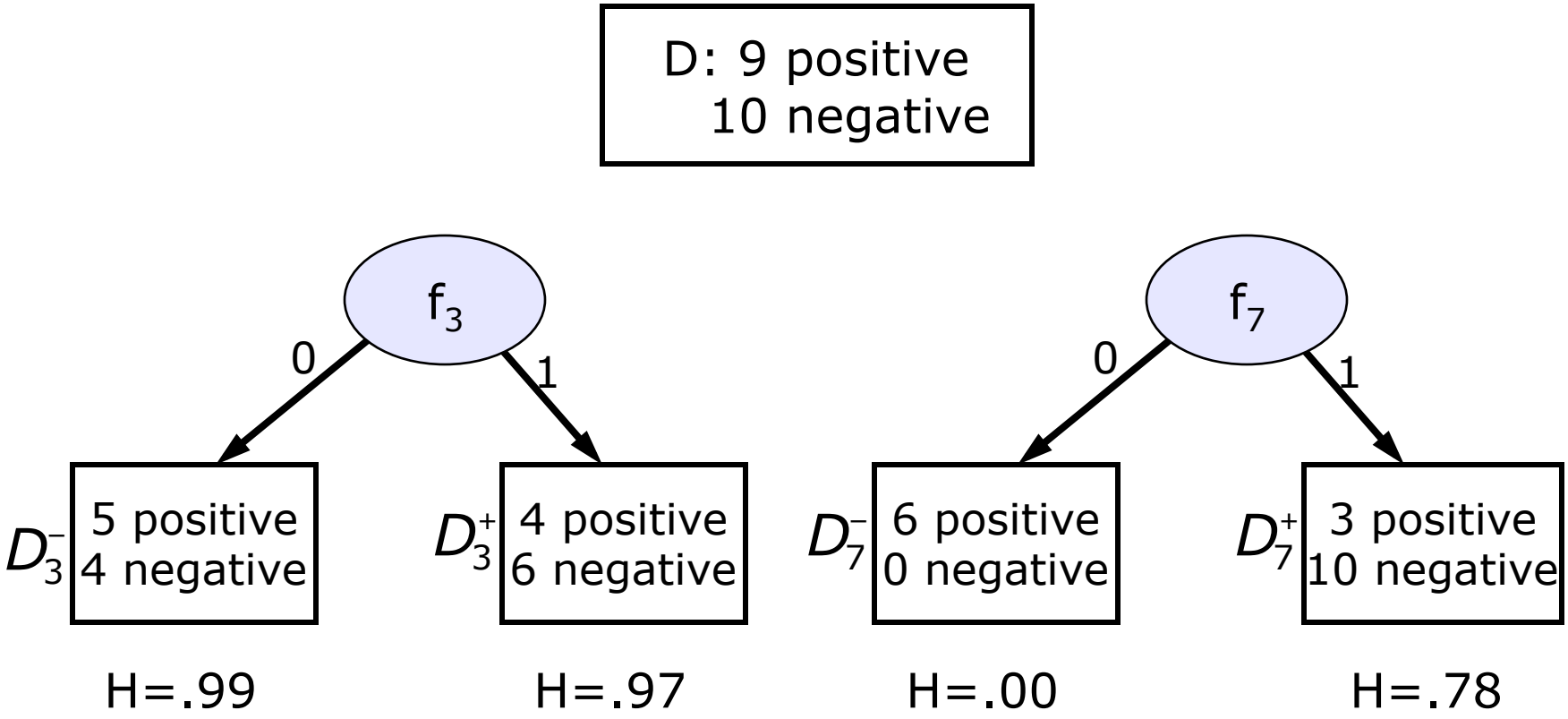


$$0 \log_2 0 = 0$$

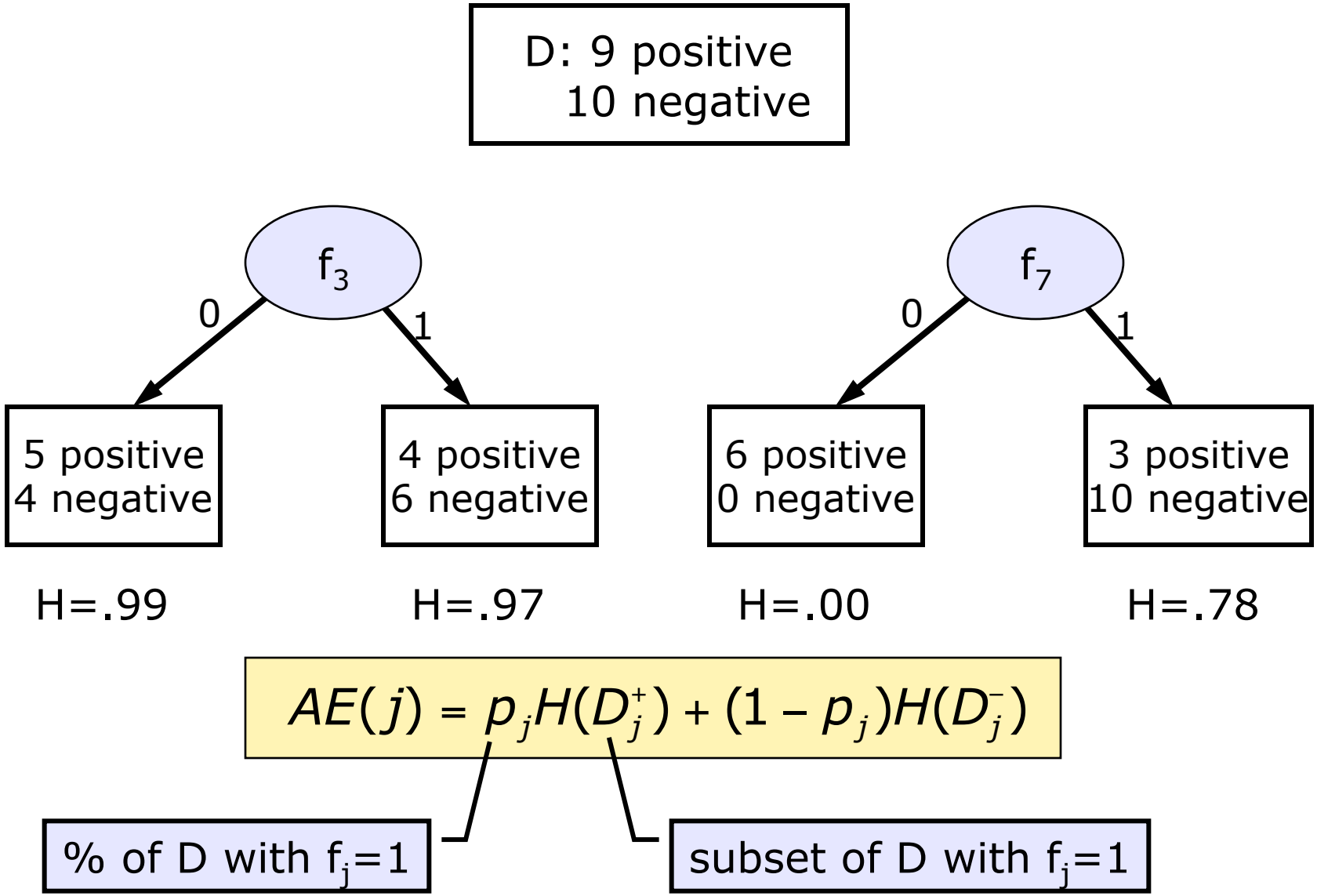
Let's Split


D: 9 positive
10 negative
f_3
0
1
D_3^-
5 positive
4 negative
D_3^+
4 positive
6 negative
f_7
0
1
D_7^-
6 positive
0 negative
D_7^+
3 positive
10 negative

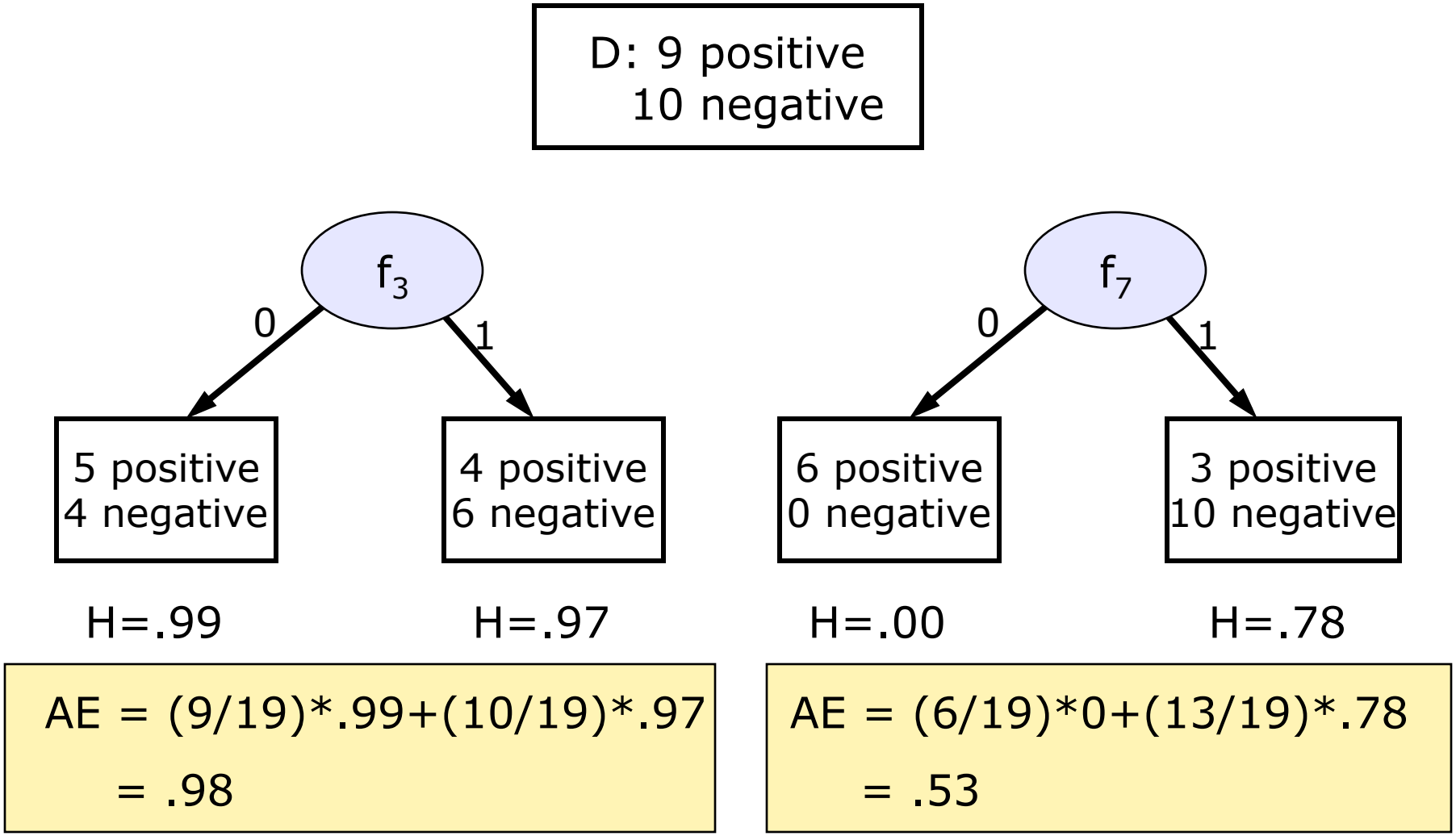
Let's Split


D: 9 positive
10 negative
f_3
0
1
D_3^-
5 positive
4 negative
D_3^+
4 positive
6 negative
H=.99
H=.97
f_7
0
1
D_7^-
6 positive
0 negative
D_7^+
3 positive
10 negative
H=.00
H=.78

Let's Split


D: 9 positive
10 negative
f3
0
1
f7
0
1
5 positive
4 negative
4 positive
6 negative
6 positive
0 negative
3 positive
10 negative
H=.99
H=.97
H=.00
H=.78
$AE(j) = p_j H(D_j^+) + (1 - p_j) H(D_j^-)$
% of D with f_j=1
subset of D with f_j=1

Let's Split


D: 9 positive
10 negative
f_3
0
1
5 positive
4 negative
4 positive
6 negative
H=.99
H=.97
AE = (9/19)*.99+(10/19)*.97
= .98
f_7
0
1
6 positive
0 negative
3 positive
10 negative
H=.00
H=.78
AE = (6/19)*0+(13/19)*.78
= .53

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BuildTree (Data)
    if all elements of Data have the same y value, then
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    else
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        MakeInternalNode(feature,
            BuildTree(SelectFalse(Data, feature)),
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```
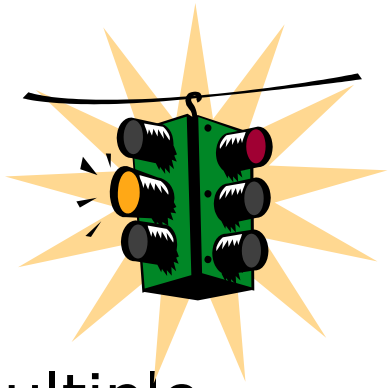
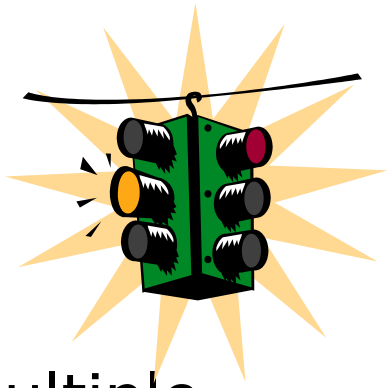

- Best feature minimizes average entropy of data in the children

Stopping



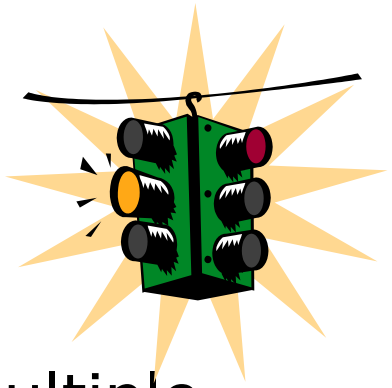
- Stop recursion if data contains only multiple instances of the same x with different y values

Stopping



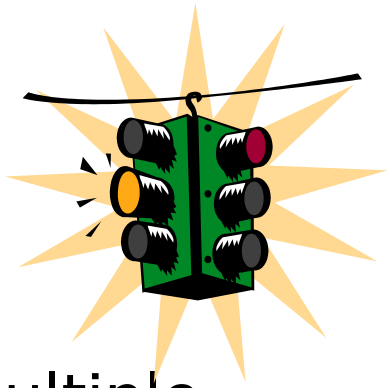
- Stop recursion if data contains only multiple instances of the same x with different y values
  - Make leaf node with output equal to the y value that occurs in the majority of the cases in the data

Stopping



- Stop recursion if data contains only multiple instances of the same x with different y values
  - Make leaf node with output equal to the y value that occurs in the majority of the cases in the data
- Consider stopping to avoid overfitting when:

Stopping



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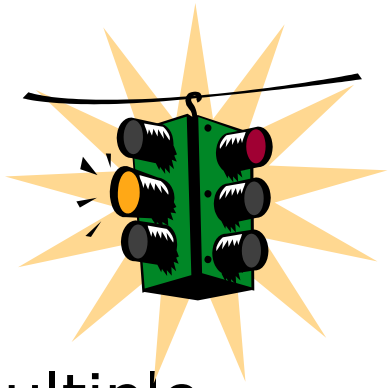
- Consider stopping to avoid overfitting when:
  - entropy of a data set is below some threshold

Stopping



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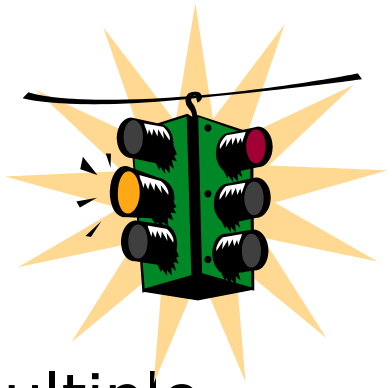
Stopping



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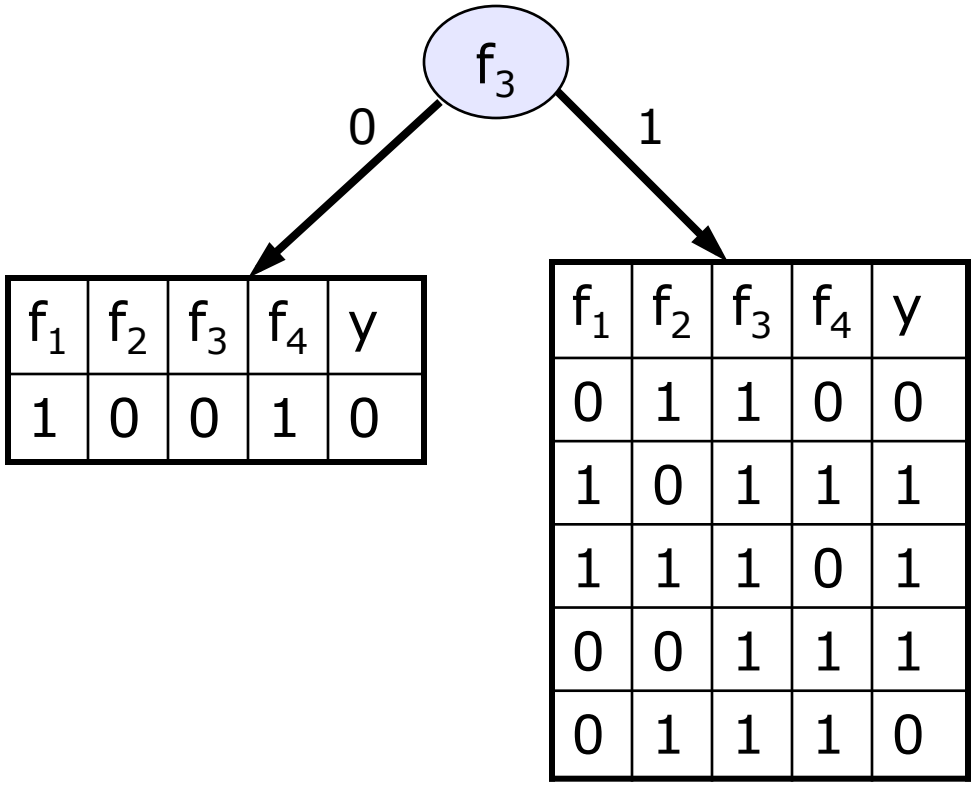
  - Make leaf node with output equal to the y value that occurs in the majority of the cases in the data
- Consider stopping to avoid overfitting when:
  - entropy of a data set is below some threshold
  - number elements in a data set is below threshold
  - best next split doesn't decrease average entropy (but this can get us into trouble)

Simulation

f_1	f_2	f_3	f_4	y
0	1	1	0	0
1	0	1	1	1
1	1	1	0	1
0	0	1	1	1
1	0	0	1	0
0	1	1	1	0

- H(D) = .92
- AE_1=.92, AE_2=.92, AE_3=.81, AE_4=1

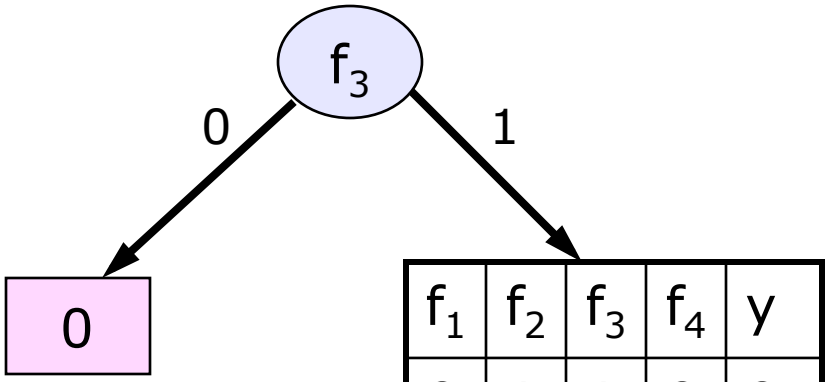
Simulation



f_1	f_2	f_3	f_4	y
1	0	0	1	0

f_1	f_2	f_3	f_4	y
0	1	1	0	0
1	0	1	1	1
1	1	1	0	1
0	0	1	1	1
0	1	1	1	0

Simulation



f_1	f_2	f_3	f_4	y
0	1	1	0	0
1	0	1	1	1
1	1	1	0	1
0	0	1	1	1
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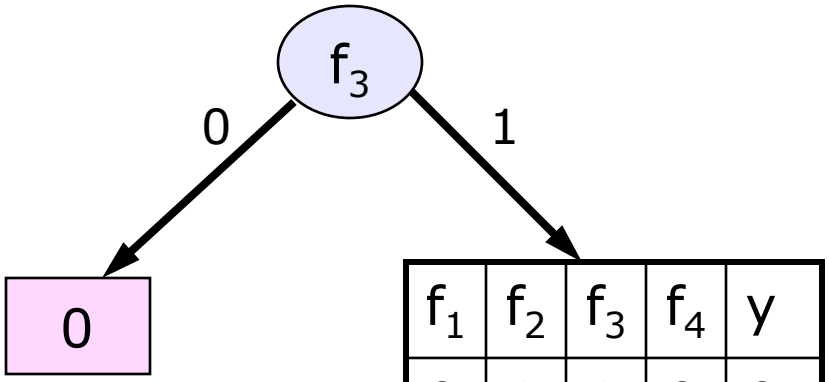
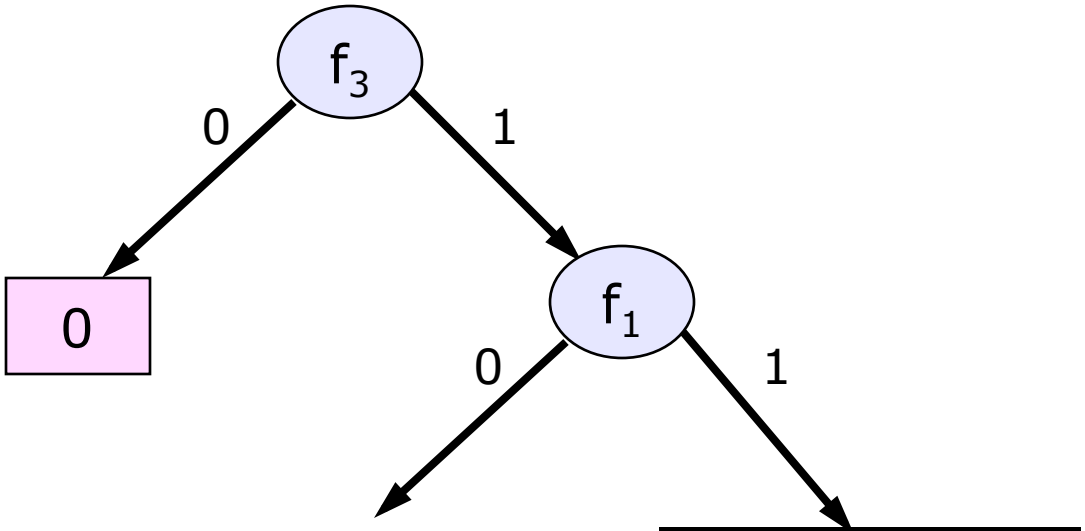
Simulation



f_1	f_2	f_3	f_4	y
0	1	1	0	0
1	0	1	1	1
1	1	1	0	1
0	0	1	1	1
0	1	1	1	0

- AE_1=.55, AE_2=.55, AE_4=.95

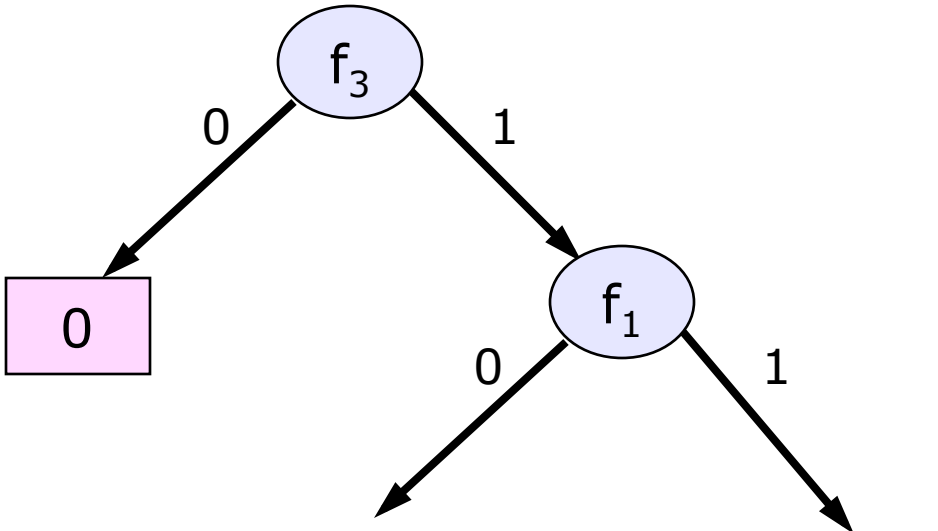
Simulation



f_1	f_2	f_3	f_4	y
0	1	1	0	0
0	0	1	1	1
0	1	1	1	0

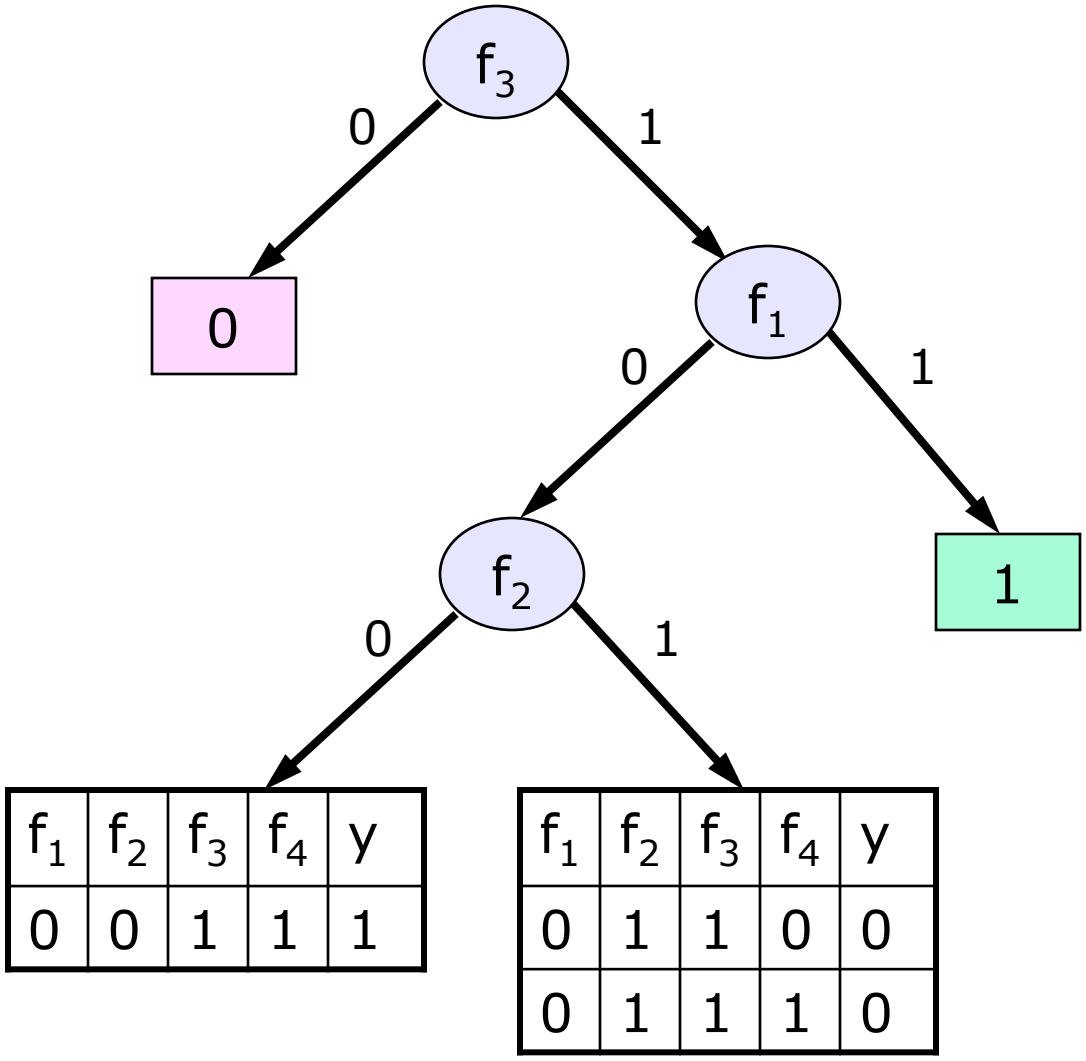
f_1	f_2	f_3	f_4	y
1	0	1	1	1
1	1	1	0	1

Simulation



f_1	f_2	f_3	f_4	y
0	1	1	0	0
0	0	1	1	1
0	1	1	1	0

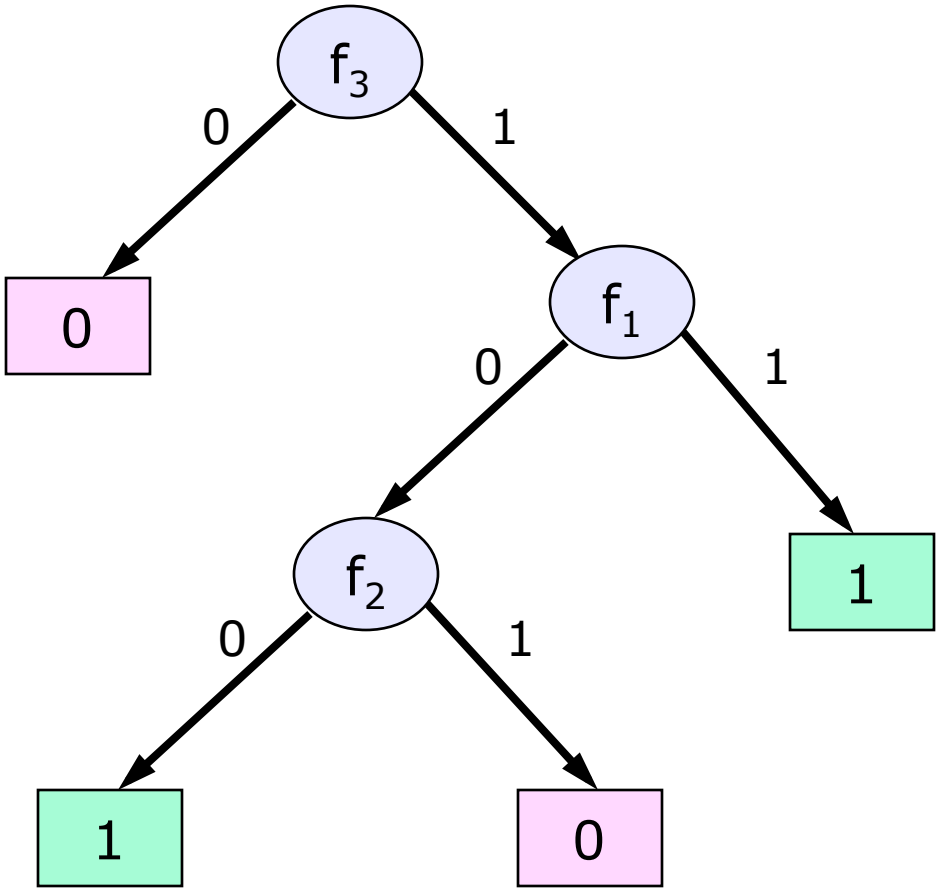
Simulation



f_1	f_2	f_3	f_4	y
0	0	1	1	1

f_1	f_2	f_3	f_4	y
0	1	1	0	0
0	1	1	1	0

Simulation



Exclusive OR

$$\left(A \wedge \neg B\right) \vee \left(\neg A \wedge B\right)$$

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1	0	1	0	0
1	1	1	0	1
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- H(D) = .92
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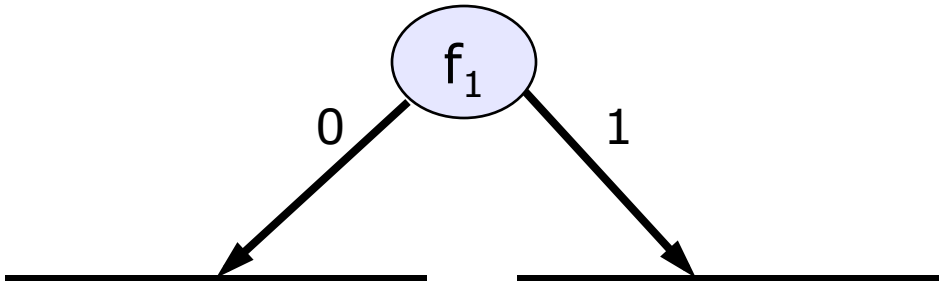
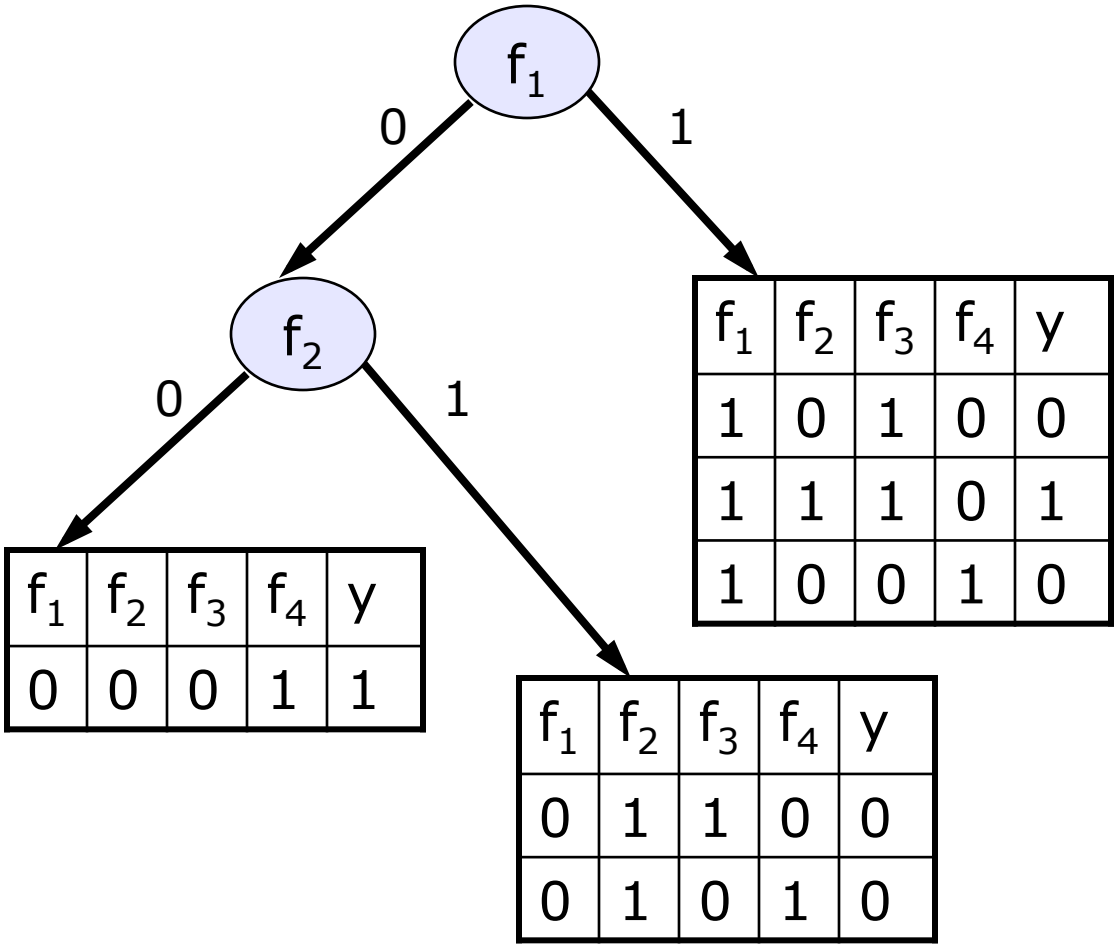
Exclusive OR



f_1	f_2	f_3	f_4	y
0	1	1	0	0
0	0	0	1	1
0	1	0	1	0

f_1	f_2	f_3	f_4	y
1	0	1	0	0
1	1	1	0	1
1	0	0	1	0

Exclusive OR

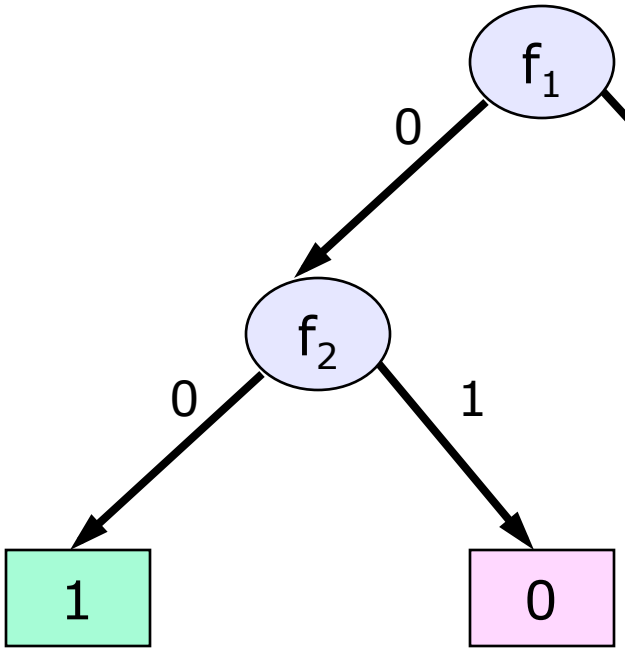


f_1	f_2	f_3	f_4	y
0	0	0	1	1

f_1	f_2	f_3	f_4	y
1	0	1	0	0
1	1	1	0	1
1	0	0	1	0

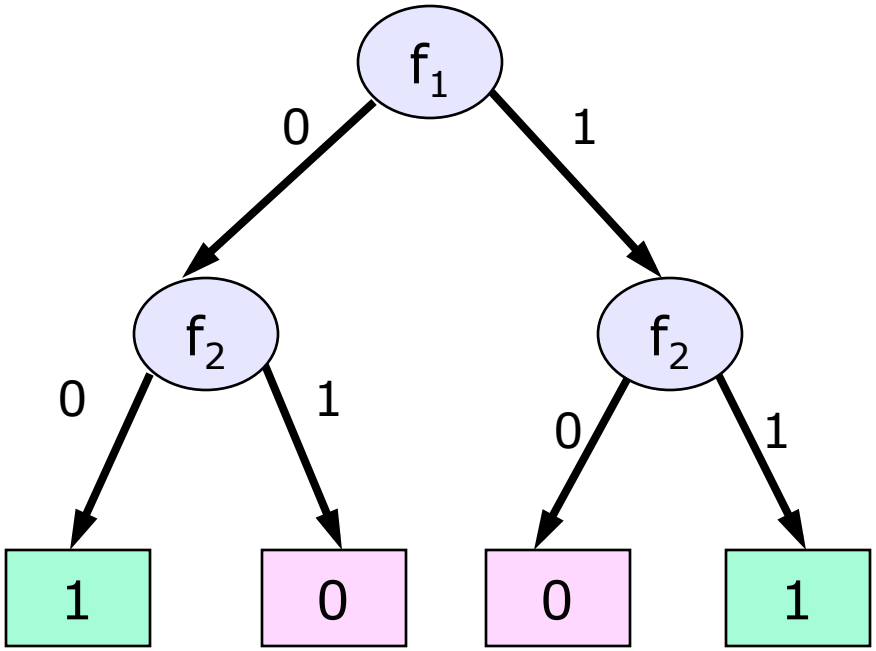
f_1	f_2	f_3	f_4	y
0	1	1	0	0
0	1	0	1	0

Exclusive OR



f_1	f_2	f_3	f_4	y
1	0	1	0	0
1	1	1	0	1
1	0	0	1	0

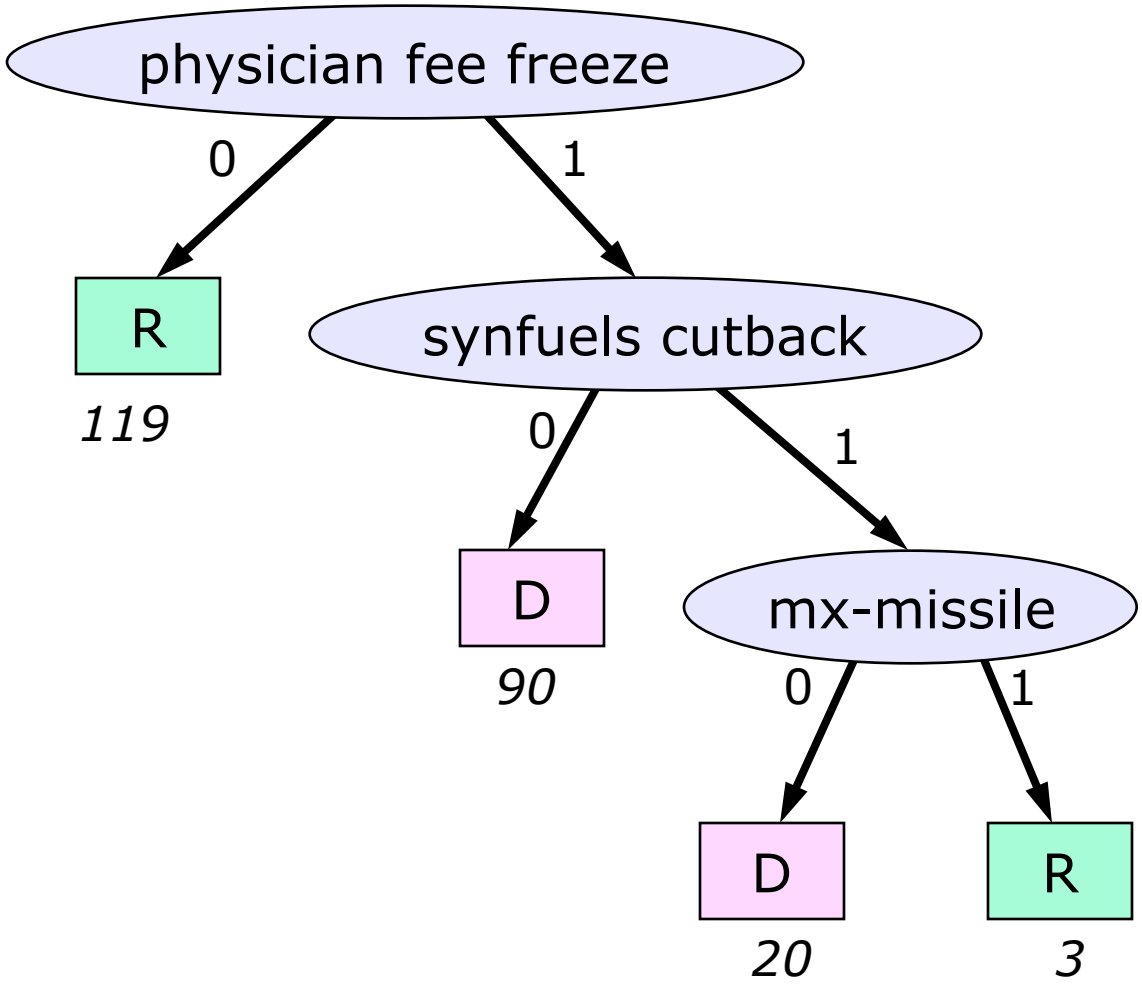
Exclusive OR



Pruning

- Best way to avoid overfitting and not get tricked by short-term lack of progress
  - Grow tree as far as possible
    - leaves are uniform or contain a single X
  - Prune the tree until it performs well on held-out data
  - Amount of pruning is like epsilon in the DNF algorithm

Congressional Voting



min leaf size = 20

Data Mining

- Making useful predictions in (usually corporate) applications
- Decision trees very popular because
  - easy to implement
  - efficient (even on huge data sets)
  - easy for humans to understand resulting hypotheses

Naïve Bayes

- Founded on Bayes' rule for probabilistic inference
- Update probability of hypotheses based on evidence
- Choose hypothesis with the maximum probability after the evidence has been incorporated

Rev. Thomas Bayes

- Algorithm is particularly useful for domains with **lots** of features

Example

f_1	f_2	f_3	f_4	y
0	1	1	0	1
0	0	1	1	1
1	0	1	0	1
0	0	1	1	1
0	0	0	0	1
1	0	0	1	0
1	1	0	1	0
1	0	0	0	0
1	1	0	1	0
1	0	1	1	0

- $R_1(1,1)=1/5$: fraction of all **positive** examples that have feature 1 **on**
- $R_1(0,1)=4/5$: fraction of all **positive** examples that have feature 1 **off**

Example

f_1	f_2	f_3	f_4	y
0	1	1	0	1
0	0	1	1	1
1	0	1	0	1
0	0	1	1	1
0	0	0	0	1
1	0	0	1	0
1	1	0	1	0
1	0	0	0	0
1	1	0	1	0
1	0	1	1	0

- $R_1(1,1)=1/5$: fraction of all **positive** examples that have feature 1 **on**
- $R_1(0,1)=4/5$: fraction of all **positive** examples that have feature 1 **off**
- $R_1(1,0)=5/5$: fraction of all **negative** examples that have feature 1 **on**
- $R_1(0,0)=0/5$: fraction of all **negative** examples that have feature 1 **off**

Example

f_1	f_2	f_3	f_4	y
0	1	1	0	1
0	0	1	1	1
1	0	1	0	1
0	0	1	1	1
0	0	0	0	1
1	0	0	1	0
1	1	0	1	0
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1	1	0	1	0
1	0	1	1	0

$R_1(1,1)=1/5$ $R_1(0,1)=4/5$
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$R_2(1,1)=1/5$ $R_2(0,1)=4/5$
$R_2(1,0)=2/5$ $R_2(0,0)=3/5$

$R_3(1,1)=4/5$ $R_3(0,1)=1/5$
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$R_4(1,1)=2/5$ $R_4(0,1)=3/5$
$R_4(1,0)=4/5$ $R_4(0,0)=1/5$

Prediction

$R_1(1,1)=1/5$ $R_1(0,1)=4/5$
$R_1(1,0)=5/5$ $R_1(0,0)=0/5$
$R_2(1,1)=1/5$ $R_2(0,1)=4/5$
$R_2(1,0)=2/5$ $R_2(0,0)=3/5$
$R_3(1,1)=4/5$ $R_3(0,1)=1/5$
$R_3(1,0)=1/5$ $R_3(0,0)=4/5$
$R_4(1,1)=2/5$ $R_4(0,1)=3/5$
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Prediction

$R_1(1,1)=1/5$ $R_1(0,1)=4/5$
$R_1(1,0)=5/5$ $R_1(0,0)=0/5$
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$R_2(1,0)=2/5$ $R_2(0,0)=3/5$
$R_3(1,1)=4/5$ $R_3(0,1)=1/5$
$R_3(1,0)=1/5$ $R_3(0,0)=4/5$
$R_4(1,1)=2/5$ $R_4(0,1)=3/5$
$R_4(1,0)=4/5$ $R_4(0,0)=1/5$

- New x = <0,0,1,1>
- $S(1) = R_1(0,1)*R_2(0,1)*R_3(1,1)*R_4(1,1) = .205$

Prediction

$R_1(1,1)=1/5$ $R_1(0,1)=4/5$
$R_1(1,0)=5/5$ $R_1(0,0)=0/5$
$R_2(1,1)=1/5$ $R_2(0,1)=4/5$
$R_2(1,0)=2/5$ $R_2(0,0)=3/5$
$R_3(1,1)=4/5$ $R_3(0,1)=1/5$
$R_3(1,0)=1/5$ $R_3(0,0)=4/5$
$R_4(1,1)=2/5$ $R_4(0,1)=3/5$
$R_4(1,0)=4/5$ $R_4(0,0)=1/5$

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- $S(1) = R_1(0,1)*R_2(0,1)*R_3(1,1)*R_4(1,1) = .205$
- $S(0) = R_1(0,0)*R_2(0,0)*R_3(1,0)*R_4(1,0) = 0$

Prediction

$R_1(1,1)=1/5$ $R_1(0,1)=4/5$
$R_1(1,0)=5/5$ $R_1(0,0)=0/5$
$R_2(1,1)=1/5$ $R_2(0,1)=4/5$
$R_2(1,0)=2/5$ $R_2(0,0)=3/5$
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$R_4(1,1)=2/5$ $R_4(0,1)=3/5$
$R_4(1,0)=4/5$ $R_4(0,0)=1/5$

- New x = <0,0,1,1>
- $S(1) = R_1(0,1)*R_2(0,1)*R_3(1,1)*R_4(1,1) = .205$
- $S(0) = R_1(0,0)*R_2(0,0)*R_3(1,0)*R_4(1,0) = 0$
- S(1) > S(0), so predict class 1

Learning Algorithm

- Estimate from the data, for all j:

$$R_j(1,1) = \frac{\#(x_j^i = 1 \wedge y^i = 1)}{\#(y^i = 1)}$$

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$$R_j(0,1) = 1 - R_j(1,1)$$

$$R_j(1,0) = \frac{\#(x_j^i = 1 \wedge y^i = 0)}{\#(y^i = 0)}$$

$$R_j(0,0) = 1 - R_j(1,0)$$

Prediction Algorithm

- Given a new x,

$$S(1) = \prod_j \begin{cases} R_j(1,1) & \text{if } x_j = 1 \\ R_j(0,1) & \text{otherwise} \end{cases}$$

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$$S(0) = \prod_j \begin{cases} R_j(1,0) & \text{if } x_j = 1 \\ R_j(0,0) & \text{otherwise} \end{cases}$$

- Output 1 if $S(1) > S(0)$

Prediction Algorithm

- Given a new x,

$$\log S(1) = \sum_j \begin{cases} \log R_j(1,1) & \text{if } x_j = 1 \\ \log R_j(0,1) & \text{otherwise} \end{cases}$$

$$\log S(0) = \sum_j \begin{cases} \log R_j(1,0) & \text{if } x_j = 1 \\ \log R_j(0,0) & \text{otherwise} \end{cases}$$

- Output 1 if $\log S(1) > \log S(0)$

Better to add logs than to multiply small probabilities

Laplace Correction

- Avoid getting 0 or 1 as an answer:

$$R_j(1,1) = \frac{\#(x_j^i = 1 \wedge y^i = 1) + \mathbf{1}}{\#(y^i = 1) + \mathbf{2}}$$

$$R_j(0,1) = 1 - R_j(1,1)$$

$$R_j(1,0) = \frac{\#(x_j^i = 1 \wedge y^i = 0) + \mathbf{1}}{\#(y^i = 0) + \mathbf{2}}$$

$$R_j(0,0) = 1 - R_j(1,0)$$

Example with Correction

f_1	f_2	f_3	f_4	y
0	1	1	0	1
0	0	1	1	1
1	0	1	0	1
0	0	1	1	1
0	0	0	0	1
1	0	0	1	0
1	1	0	1	0
1	0	0	0	0
1	1	0	1	0
1	0	1	1	0

- $R_1(1,1)=2/7$ $R_1(0,1)=5/7$
- $R_1(1,0)=6/7$ **$R_1(0,0)=1/7$**
- $R_2(1,1)=2/7$ $R_2(0,1)=5/7$
- $R_2(1,0)=3/7$ $R_2(0,0)=4/7$
- $R_3(1,1)=5/7$ $R_3(0,1)=2/7$
- $R_3(1,0)=2/7$ $R_3(0,0)=5/7$
- $R_4(1,1)=3/7$ $R_4(0,1)=4/7$
- $R_4(1,0)=5/7$ $R_4(0,0)=2/7$

Prediction with Correction

- $R_1(1,1)=2/7$ $R_1(0,1)=5/7$
- $R_1(1,0)=6/7$ $R_1(0,0)=1/7$
- $R_2(1,1)=2/7$ $R_2(0,1)=5/7$
- $R_2(1,0)=3/7$ $R_2(0,0)=4/7$
- $R_3(1,1)=5/7$ $R_3(0,1)=2/7$
- $R_3(1,0)=2/7$ $R_3(0,0)=5/7$
- $R_4(1,1)=3/7$ $R_4(0,1)=4/7$
- $R_4(1,0)=5/7$ $R_4(0,0)=2/7$

- New x = <0,0,1,1>
- $S(1) = R_1(0,1)*R_2(0,1)*R_3(1,1)*R_4(1,1) = .156$
- $S(0) = R_1(0,0)*R_2(0,0)*R_3(1,0)*R_4(1,0) = .017$
- S(1) > S(0), so predict class 1

Hypothesis Space

- Output 1 if

$$\prod_j \alpha_j x_j + (1 - \alpha_j)(1 - x_j) > \prod_j \beta_j x_j + (1 - \beta_j)(1 - x_j)$$

- Depends on parameters $\alpha_1 \ldots \alpha_n, \beta_1 \ldots \beta_n$ (which we set to be the R_j values)

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- Our method of computing parameters doesn't minimize training set error, but it's fast!

Hypothesis Space

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- Depends on parameters $\alpha_1 \ldots \alpha_n, \beta_1 \ldots \beta_n$ (which we set to be the R_j values)

- Our method of computing parameters doesn't minimize training set error, but it's fast!

- Weight of feature j's "vote" in favor of output 1:

$$\log \frac{\alpha_j}{1-\alpha_j} - \log \frac{\beta_j}{1-\beta_j}$$

Exclusive Or

f_1	f_2	f_3	f_4	y
0	1	1	0	0
1	0	1	0	0
1	0	0	1	0
0	1	0	1	0
1	1	1	0	1
0	0	0	1	1

- $R_1(1,1)=2/4$ $R_1(0,1)=2/4$
- $R_1(1,0)=3/6$ $R_1(0,0)=3/6$
- $R_2(1,1)=2/4$ $R_2(0,1)=2/4$
- $R_2(1,0)=3/6$ $R_2(0,0)=3/6$
- $R_3(1,1)=2/4$ $R_3(0,1)=2/4$
- $R_3(1,0)=3/6$ $R_3(0,0)=3/6$
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Exclusive Or

f_1	f_2	f_3	f_4	y
0	1	1	0	0
1	0	1	0	0
1	0	0	1	0
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- $R_1(1,1)=2/4$ $R_1(0,1)=2/4$
- $R_1(1,0)=3/6$ $R_1(0,0)=3/6$
- $R_2(1,1)=2/4$ $R_2(0,1)=2/4$
- $R_2(1,0)=3/6$ $R_2(0,0)=3/6$
- $R_3(1,1)=2/4$ $R_3(0,1)=2/4$
- $R_3(1,0)=3/6$ $R_3(0,0)=3/6$
- $R_4(1,1)=2/4$ $R_4(0,1)=2/4$
- $R_4(1,0)=3/6$ $R_4(0,0)=3/6$

- For any new x
- S(1) = .5 * .5 * .5 * .5 = .0625
- S(0) = .5 * .5 * .5 * .5 = .0625
- We're indifferent between classes

Congressional Voting

- Accuracy on the congressional voting domain is about 0.91
- Somewhat worse than decision trees (0.95)
- Decision trees can express more complex hypotheses over combinations of attributes
- Domain is small enough so that speed is not an issue
- So, prefer trees or DNF in this domain

Congressional Voting

-6.82	physician-fee-freeze
-4.20	el-salvador-aid
-4.20	crime
-3.56	education-spending
3.36	adoption-of-the-budget-resolution
3.25	aid-to-nicaraguan-contras
3.07	mx-missile
-2.51	superfund-right-to-sue
2.40	duty-free-exports
2.14	anti-satellite-test-ban
-2.07	religious-groups-in-schools
2.01	export-administration-act-south-africa
1.66	synfuels-corporation-cutback
1.63	handicapped-infants
-0.17	immigration
-0.08	water-project-cost-sharing

republican
democrat

Probabilistic Inference

Probabilistic Inference

- Think of features and output as random variables
- Learn $\Pr(Y = 1 | f_1, \ldots, f_n)$
- Given new example, compute probability it has value 1
- Generate answer 1 if that value is > 0.5, else 0

- Concentrate on estimating this distribution from data

$$\Pr(Y = 1 | f_1, \ldots, f_n)$$

Bayes' Rule

- Generically:

$$\Pr(A \mid B) = \Pr(B \mid A)\frac{\Pr(A)}{\Pr(B)}$$

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$$\Pr(A \mid B) = \Pr(B \mid A)\frac{\Pr(A)}{\Pr(B)}$$

- Specifically:

$$\Pr(Y = 1 \mid f_1 \ldots f_n) = \Pr(f_1 \ldots f_n \mid Y = 1)\frac{\Pr(Y = 1)}{\Pr(f_1 \ldots f_n)}$$

Bayes' Rule

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$$\Pr(A \mid B) = \Pr(B \mid A)\frac{\Pr(A)}{\Pr(B)}$$

- Specifically:

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independent of Y

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- Specifically:

$$\Pr(Y = 1 \mid f_1 \ldots f_n) = \Pr(f_1 \ldots f_n \mid Y = 1)\frac{\Pr(Y = 1)}{\Pr(f_1 \ldots f_n)}$$

independent of Y

prior

- Concentrate on:

$$\Pr(f_1 \ldots f_n \mid Y = 1)$$

Why is Bayes Naïve?

- Make a big independence assumption

$$\Pr(f_1 \ldots f_n \mid Y = 1) = \prod_j \Pr(f_j \mid Y = 1)$$

Learning Algorithm

- Estimate from the data, for all j:

$$R(f_j = 1 \mid Y = 1) = \frac{\#(x_j^i = 1 \wedge y^i = 1)}{\#(y^i = 1)}$$

$$R(f_j = 0 \mid Y = 1) = 1 - R(f_j = 1 \mid Y = 1)$$

$$R(f_j = 1 \mid Y = 0) = \frac{\#(x_j^i = 1 \wedge y^i = 0)}{\#(y^i = 0)}$$

$$R(f_j = 0 \mid Y = 0) = 1 - R(f_j = 1 \mid Y = 0)$$

Prediction Algorithm

- Given a new x,

$$S(x_1 \ldots x_n \mid Y = 1) = \prod_j \begin{cases} R(f_j = 1 \mid Y = 1) & \text{if } x_j = 1 \\ R(f_j = 0 \mid Y = 1) & \text{otherwise} \end{cases}$$

$$S(x_1 \ldots x_n \mid Y = 0) = \prod_j \begin{cases} R(f_j = 1 \mid Y = 0) & \text{if } x_j = 1 \\ R(f_j = 0 \mid Y = 0) & \text{otherwise} \end{cases}$$

- Output 1 if

$$S(x_1 \ldots x_n \mid Y = 1) > S(x_1 \ldots x_n \mid Y = 0)$$