

1 Lists and Trees

- (a) You are given two lists:
- ```
(define x (list 1 2 3))
(define y (list 4 5 6))
```

What would the following evaluate to:

```
(append x y)
(cons x y)
(list x y)
```

- (b) Draw the tree structure corresponding to the following list.

```
(1 ((2 3) 4) (5 6 7) (8 (9 10)))
```

- (c) Draw the box-and-pointer representation of the above list.

## 2 sum-leaves

Write a Scheme procedure `sum-leaves` that takes one argument (a tree with integer leaves) and returns the sum of the leaves.

### 3 Match

The code on the following pages implements a simple list matcher. The matcher is given two lists as input. The first list (the pattern) contains variables, and the second list (the data) does not. The procedure returns a list of variable bindings that would make the pattern match the data, if possible. Simple variables are denoted by symbols whose first character is a question mark, e.g. `?x`.

Evaluate the following expressions:

```
(match '(a ?x c) '(a b c)) =>
```

```
(match '(a (?x c) d) '(a (b c) d)) =>
```

```
(match '(a (?x c) ?y) '(a (b c) c)) =>
```

```
(match '(a (b c) d) '(a (b c) d)) =>
```

```
(match '(a b c d) '(a (b c) c)) =>
```

```
(match '(a ?x c ?x e) '(a b c b e)) =>
```

```
(match '(a ?x c ?x e) '(a b c d e)) =>
```

```
(match '(a ?x c ?y e) '(a b c d e)) =>
```

```
(match '(a ?x d) '(a b c d)) =>
```

```
;;; DATA STRUCTURES
```

```
;; Variables are indicated by ?<var>, e.g., ?x.
;; Binding ?<var>=<value> is represented by (?<var> . <value>)
;; in an association, e.g., ((?x . a), (?y . b)).
;; A pattern is a list of symbols and variables.
;; A datum is a list of symbols.
```

```
(define (match p d) (do-match p d *no-bindings*))
```

```
;; Arguments: Pattern (p), datum (d), bindings.
;; Returns: A list of bindings or #f
```

```
(define (do-match p d bindings)
 (cond ((eq? p d) bindings)
 ;; is p down to one variable?
 ((variable? p) (match-variable p d bindings))
 ((segment-variable? p)
 ;; A stand-alone segment variable (match '*x ...) is not well
 ;; defined. The more general case is caught by match-pair.
 fail)
 ((pair? p) (match-pair p d bindings))
 (else *fail*)))
```

```
;; Arguments: Variable (var), datum (d), bindings.
;; Returns: A list of bindings or #f
```

```
(define (match-variable var d bindings)
 (let ((binding (get-binding var bindings)))
 ;; Is the pattern variable on the list of bindings:
 (if binding
 ;; If it is, substitute its value and try again:
 (do-match (binding-val binding) d bindings)
 ;; Otherwise, add new binding:
 (extend-bindings var d bindings))))
```

```

;; Arguments: Pattern list (p), datum (d), bindings.
;; Returns: A list of bindings or #f

(define (match-pair p d bindings)
 (if (segment-variable? (first p))
 ;; to be implemented in Problem Set 1
 (match-segment-variable p d bindings)
 (if (pair? d) ; p is a pair, so d better be
 (let ((result (do-match (first p) (first d) bindings)))
 ;; See if the FIRST parts match producing new bindings:
 (if (eq? result *fail*)
 ;; If they do not match, fail.
 fail
 ;; If they do match, try the REST parts using the
 ;; resulting bindings:
 (do-match (rest p) (rest d) result)
))
 fail)))

;;; VARIABLE ABSTRACTION

;;; Is x a variable (a symbol beginning with '?')?
(define (variable? x)
 (and (symbol? x) (equal? (string-ref (symbol->string x) 0) #\?)))

;;; Is x a segment variable (a symbol beginning with '*')?
(define (segment-variable? x)
 (and (symbol? x) (equal? (string-ref (symbol->string x) 0) #*)))

;;; BINDINGS ABSTRACTION

;;; Indicates unification/match failure
(define *fail* #f)

;;; Indicates unification/match success, with no variables.
(define *no-bindings* '((#f)))

```

```

;;; Find a (variable . value) pair in a binding list.
(define (get-binding var bindings)
 (assoc var bindings))

;;; Get the variable part of a single binding.
(define (binding-var binding)
 (and binding (car binding)))

;;; Get the value part of a single binding.
(define (binding-val binding)
 (and binding (cdr binding)))

(define make-binding cons)

;;; Add a (var . value) pair to a binding list.
(define (extend-bindings var val bindings)
 (cons (make-binding var val)
 ;;; Once we add a "real" binding,
 ;;; we can get rid of the dummy *no-bindings*
 (if (eq? bindings *no-bindings*)
 '()
 bindings)))

;;;; HELPER FUNCTION

;;; Add elt to the end of lst.
(define (add-to-end elt lst)
 (append lst (list elt)))

(define rest cdr)

```

## 4 Scheme Quick Reference

Scheme specification: <http://www.schemers.org/Documents/Standards/R5RS/>

Notation:

|                                      |                                                               |
|--------------------------------------|---------------------------------------------------------------|
| <code>&lt;object&gt;</code>          | any Scheme data object.                                       |
| <code>&lt;object&gt;*</code>         | zero or more objects                                          |
| <code>&lt;object&gt;+</code>         | one or more objects                                           |
| <code>[&lt;object&gt;]</code>        | optional object                                               |
| <code>( &lt;whatever&gt; )...</code> | Zero or more occurrences of <code>( &lt;whatever&gt; )</code> |

Built-in Commands Signatures:

```
(lambda <name> <exp>+)
(lambda (<name>*) <exp>+)
(and <exp>*)
(or <exp>*)
(if <test-exp> <if-true> [<if-false>])
(cond (<test> <exp>*)... [(else <exp>+)])
(case <key-exp> ((<datum>+) <exp>*)... [(else <exp>+)])
(define (<name> <name>*) <exp>+)
(define <name> <exp>)
(let [<name>] ((<vname> <value-exp>)...) <exp>+)
(let* ((<vname> <value-exp>)...) <exp>+)
(letrec ((<vname> <value-exp>)...) <exp>+)
(begin <expression>+)
(do ((<var> <init> <step>)...) (<test> <exp>*) <exp>*)
```

```
(not <object>)
(boolean? <object>)
```

```

(eq? <obj1> <obj2>)
(eqv? <obj1> <obj2>)
(equal? <obj1> <obj2>)

(pair? <object>)
(cons <obj1> <obj2>)
(car <pair>)
(cdr <pair>)
(set-car! <pair> <object>)
(set-cdr! <pair> <object>)
(caar <list>) (cadr <list>) (cdar <list>) (cddr <list>)
(caaar <list>) (caadr <list>) (cadar <list>) (caddr <list>)
(cdaar <list>) (cdadr <list>) (cddar <list>) (cdddr <list>)
(caaaar <list>) (caaaadr <list>) (caadar <list>) (caaddr <list>)
(cadaar <list>) (cadadr <list>) (caddar <list>) (cadddr <list>)
(cdaaar <list>) (cdaadr <list>) (cdadar <list>) (cdaddr <list>)
(cddaar <list>) (cddadr <list>) (cdddar <list>) (cddddr <list>)
(null? <object>)
(list? <object>)
(list <object>*)
(length <list>)
(append <list>+)
(reverse <list>)
(list-ref <list> <index>)

(memq <object> <list>)
(memv <object> <list>)
(member <object> <list>)

(assq <object> <alist>)
(assv <object> <alist>)
(assoc <object> <alist>)

(symbol? <object>) (symbol->string <symbol>) (string->symbol <string>)

(number? <object>)
(complex? <object>)
(real? <object>)
(rational? <object>)

```

```

(integer? <object>)
(exact? <number>) (inexact? <number>)
(= <number>+)
(< <number>+) (> <number>+)
(<= <number>+) (>= <number>+)
(zero? <number>)
(positive? <number>) (negative? <number>)
(odd? <number>) (even? <number>)
(max <number>+) (min <number>+)
(+ <number>+)
(* <number>+)
(- <number>+)
(/ <number>+)
(abs <number>)
(quotient <num1> <num2>) (remainder <num1> <num2>)
(modulo <num1> <num2>)
(gcd <number>*) (lcm <number>*)
(numerator <rational>) (denominator <rational>)
(floor <number>) (ceiling <number>)
(truncate <number>) (round <number>)
(rationalize <num1> <num2>)
(exp <number>) (log <number>)
(sin <number>) (cos <number>) (tan <number>)
(asin <number>) (acos <number>) (atan <number> [<number>])
(sqrt <number>)
(expt <num1> <num2>)
(make-rectangular <num1> <num2>) (make-polar <num1> <num2>)
(real-part <number>) (imag-part <number>)
(magnitude <number>) (angle <number>)
(exact->inexact <number>) (inexact->exact <number>)
(number->string <number>) (string->number <string>)

(char? <object>)
(char=? <char1> <char2>) (char-ci=? <char1> <char2>)
(char<? <char1> <char2>) (char-ci<? <char1> <char2>)
(char>? <char1> <char2>) (char-ci>? <char1> <char2>)
(char<=? <char1> <char2>) (char-ci<=? <char1> <char2>)
(char>=? <char1> <char2>) (char-ci>=? <char1> <char2>)
(char-alphabetic? <character>)
(char-numeric? <character>)

```

```

(char-whitespace? <character>)
(char-upper-case? <character>) (char-lower-case? <character>)
(char->integer <character>) (integer->char <integer>)
(char-upcase <character>) (char-downcase <character>)

(string? <object>)
(make-string <length> [<character>])
(string <character>+)
(string-length <string>)
(string-ref <string> <index>)
(string-set! <string> <index> <character>)
(string=? <string1> <string2>) (string-ci=? <string1> <string2>)
(string<? <string1> <string2>) (string-ci<? <string1> <string2>)
(string>? <string1> <string2>) (string-ci>? <string1> <string2>)
(string<=? <string1> <string2>) (string-ci<=? <string1> <string2>)
(string>=? <string1> <string2>) (string-ci>=? <string1> <string2>)
(substring <string> <start-index> <end-index>)
(string-append <string>+)

(vector? <object>)
(make-vector <length> [<object>])
(vector <object>*)
(vector-length <vector>)
(vector-ref <vector> <index>)
(vector-set! <vector> <index> <object>)

(procedure? <object>)
(apply <procedure> <arg>* <arg-list>)
(map <procedure> <list>+)
(for-each <procedure> <list>+)
(call-with-current-continuation <one-argument-procedure>)

(call-with-input-file <string> <procedure>)
(call-with-output-file <string> <procedure>)
(input-port? <object>) (output-port? <object>)
(current-input-port) (current-output-port)
(open-input-file <string>) (open-output-file <string>)
(close-input-port <input-port>) (close-output-port <output-port>)
(eof-object? <object>)
(read [<input-port>])

```

```
(read-char [<input-port>])
(peek-char [<input-port>])
(write <object> [<output-port>])
(display <object> [<output-port>])
(newline [<output-port>])
(write-char <character> [<output-port>])
```

```
(list-tail <list> <index>)
(string->list <string>)
(list->string <list-of-characters>)
(string-copy <string>)
(string-fill! <string> <character>)
(vector->list <vector>)
(list->vector <list>)
(vector-fill! <vector> <object>)
(delay <expression>)
(force <promise>)
(with-input-from-file <string> <thunk>)
(with-output-to-file <string> <thunk>)
(char-ready? [<input-port>])
(load <string>)
(transcript-on <string>)
(transcript-off)
```