6.034 Recitation #8
May 4, 2007

1 FOL Proof - Rags $\vee$ Riches?

(a) Unification

Find a most general unifier for each of the following pairs of sentences. *Bob* and *Alice* are constants.

S_1	S_2	**Unifier**
$H(Bob)$	$H(x)$	x/Bob
$Eq(f(Bob), Alice)$	$Eq(x, y)$	$x/f(Bob)$, $y/Alice$
$P(f(x), Bob)$	$P(Alice, x)$	**none**
$Eq(f(f(Bob)), f(Bob))$	$Eq(f(x), x)$	$x/f(Bob)$
$P(y, Bob)$	$P(f(Bob), x)$	x/Bob, $y/f(Bob)$

(b) Proof

Say we have the following predicates:

H(x)	x is an heir.
M(x)	x is male.
P(y,x)	y is in the Trump family and is the parent of x.
Eq(x,y)	x and y are equal.

Convert the following English sentence into first-order logic clausal form.

1. A person is an heir if and only if he or she is male, and has a parent from the Trump family who is an heir.

   Solution:

   $$\forall x . H(x) \leftrightarrow M(x) \wedge (\exists y . P(y, x) \wedge H(y))$$

   $$\Rightarrow (\neg H(x_1) \vee M(x_1)) \wedge (\neg H(x_2) \vee P(f(x_2), x2)) \wedge \\ (\neg H(x_3) \vee H(f(x_3))) \wedge (\neg M(x_4) \vee \neg P(y_4, x_4) \vee \neg H(y_4) \vee H(x_4))$$

2. A person only has one parent from the Trump family.

   Solution:

   $$\forall x.\ \forall y.\ \forall z.\ (p(y, x) \wedge P(z, x)) \rightarrow Eq(y, z)$$

   $$\Rightarrow \neg P(y_1, x_1) \vee \neg P(z_1, x_1) \vee Eq(y_1, z_1)$$

3. If two people are Eq and one of them is an heir, the other is an heir. (That is, allow substitutions for predicate H)

   Solution:

   $$\forall x . \forall y . (H(x) \wedge Eq(x, y)) \rightarrow H(y)$$

   $$\Rightarrow \neg H(x_1) \vee \neg Eq(x_1, y_1) \vee H(y_1)$$

4. Eq is symmetric.

   Solution:

$$\forall x.\forall y.Eq(x, y) \rightarrow Eq(y, x)$$

$$\Rightarrow \neg Eq(x_1, y_1) \vee Eq(y_1, x_1)$$

On the next pages are a framework for conducting two proofs about Bob and Alice. A few specifics about them have been stated along with axioms from the given knowledge base. All axioms that we converted to clausal form have already been stated. You need only provide the proofs. Be sure to note any unification substitutions you make.

Prove that Alice is not an heir.

step	reason	result
1.	KB 4	$\neg Eq(x_1, y_1) \vee Eq(y_1, x_1)$
2.	KB 1	$\neg H(x_2) \vee M(x_2)$
3.	KB 1	$\neg H(x_3) \vee H(f(x_3))$
4.	KB 1	$\neg H(x_4) \vee P(f(x_4), x_4)$
5.	KB 1	$H(x_5) \vee \neg M(x_5) \vee \neg P(y_5, x_5) \vee \neg H(y_5)$
6.	KB 2	$\neg P(y_6, x_6) \vee \neg P(z_6, x_6) \vee Eq(y_6, z_6)$
7.	KB 3	$\neg H(x_7) \vee \neg Eq(x_7, y_7) \vee H(y_7)$
8.	KB	$P(Alice, Bob)$
9.	KB	$\neg M(Alice)$
10.	**neg. concl.**	$H(Alice)$
11.	**2,10 {x_2/Alice}**	**M(Alice)**
12.	**9,11**	**false**

Assume that Alice is not an heir. Prove that Bob is not an heir.

step	reason	result
1.	KB 4	$\neg Eq(x_1, y_1) \vee Eq(y_1, x_1)$
2.	KB 1	$\neg H(x_2) \vee M(x_2)$
3.	KB 1	$\neg H(x_3) \vee H(f(x_3))$
4.	KB 1	$\neg H(x_4) \vee P(f(x_4), x_4)$
5.	KB 1	$H(x_5) \vee \neg M(x_5) \vee \neg P(y_5, x_5) \vee \neg H(y_5)$
6.	KB 2	$\neg P(y_6, x_6) \vee \neg P(z_6, x_6) \vee Eq(y_6, z_6)$
7.	KB 3	$\neg H(x_7) \vee \neg Eq(x_7, y_7) \vee H(y_7)$
8.	KB	$P(Alice, Bob)$
9.	KB	$\neg H(Alice)$
10.	**neg. concl.**	$H(Bob)$
11.	**10,4 {x_4/Bob}**	$P(f(Bob), Bob)$
12.	**11,6 {x_6/Bob,y_6/f(Bob)}**	$\neg P(z_6, Bob) \vee Eq(f(Bob), z_6)$
13.	**12,8 {z_6/Alice}**	$Eq(f(Bob), Alice)$
14.	**13,7 {x_7/f(Bob),y_7/Alice}**	$\neg H(f(Bob)) \vee H(Alice)$
15.	**14,9**	$\neg H(f(Bob))$
16.	**15,3 {x_3/Bob}**	$\neg H(Bob)$
17.	**16,10**	**false**