1 Interpretations and Entailment

1. Fill in the first three columns of the table with all possible interpretations in the domain $\{A, B, C\}$:

Solution:

A	B	C	$S1$	$S2$
t	t	t		x
t	t	f		x
t	f	t	x	x
t	f	f	x	x
f	t	t		
f	t	f		x
f	f	t	x	x
f	f	f	x	

Now, consider the following two sentences, $S1$ and $S2$:

$S1 \quad (A \vee B) \rightarrow (\neg B \wedge (C \vee A))$
$S2 \quad (B \leftrightarrow C) \rightarrow A$

In the column labeled "S1," place a mark next to each interpretation in which $S1$ holds. Do the same for $S2$ in the column labeled "S2." Does $S1$ entail $S2$?

Solution: No.

2. Here is a sentence in propositional logic:

$(A \rightarrow (B \vee (C \wedge D))) \leftrightarrow (B \vee C)$

Does it hold given the interpretation $i = \{A = t, B = f, C = t, D = f\}$? If so, give an interpretation in which it does not hold. If not, give an interpretation in which it does hold.

Solution: No, it does not hold. It reduces to $(t \rightarrow f) \leftrightarrow t$ or $f \leftrightarrow t$ which is false.

A	B	C	D	the-big-long-sentence
t	t	t	t	t
t	t	t	f	t
t	t	f	t	t
t	t	f	f	t
t	f	t	t	t
t	f	t	f	f
t	f	f	t	f
t	f	f	f	f
f	t	t	t	t
f	t	t	f	t
f	t	f	t	t
f	t	f	f	t
f	f	t	t	t
f	f	t	f	t
f	f	f	t	t
f	f	f	f	t

Does $(A \vee B) \wedge (\neg A \vee C)$ entail $(B \vee C)$?

Solution: Yes (see truth table below)

A	B	C	$(A \vee B) \wedge (\neg A \vee C)$	$(B \vee C)$
t	t	t	x	x
t	t	f		x
t	f	t	x	x
t	f	f		
f	t	t	x	x
f	t	f	x	x
f	f	t		x
f	f	f		

3. Show that $(A \vee B) \wedge (\neg A \vee C) \rightarrow (B \vee C)$ using truth tables.

Solution:

A	B	C	$A \vee B$	$\neg A \vee C$	$(A \vee B) \wedge (\neg A \vee C)$	$B \vee C$
t	t	t	t	t	t	t
t	t	f	t	f	f	t
t	f	t	t	t	t	t
t	f	f	t	f	f	f
f	t	t	t	t	t	t
f	t	f	t	t	t	t
f	f	t	f	t	f	t
f	f	f	f	t	f	f

To see that $(A \vee B) \wedge (\neg A \vee C) \rightarrow (B \vee C)$, note that it is true in all interpretations of {A,B,C}.

2 Writing FOL

Assume that you can use the following predicates in a universe of all baseball players:

1. Yankees(x) - x plays for the Yankees
2. RedSox(x) - x plays for the Red Sox
3. Better(x,y) - player x is better than player y
4. Loves(x,y) - player x loves player y
5. Cursed(x) - player x is cursed

Now convert the following English sentences to FOL statements:

1. Every Red Sox player has no love for any Yankee player.

   Solution:

   (a) $\forall x.\forall y.(\text{RedSox}(x) \wedge \text{Yankee}(y)) \rightarrow \neg\text{Loves}(x, y)$
   (b) $\neg\exists x.\exists y.\text{RedSox}(x) \wedge \text{Yankee}(y) \wedge \text{Loves}(x, y)$

2. There is not a single Red Sox player who is not cursed.

   Solution:

   (a) $\forall x.\text{RedSox}(x) \rightarrow \text{Cursed}(x)$
   (b) $\neg\exists x.\text{RedSox}(x) \wedge \neg\text{Cursed}(x)$

3. If a baseball player is cursed, he cannot love anyone.

   Solution:

   (a) $\forall x.\forall y.\text{Cursed}(x) \rightarrow \neg\text{Loves}(x, y)$
   (b) $\forall x.\text{Cursed}(x) \rightarrow \neg\exists y.\text{Loves}(x, y)$

4. All Yankee players have the same skill level (no player is better than another).

   Solution:

   (a) $\forall x.\forall y.(\text{Yankee}(x) \wedge \text{Yankee}(y)) \rightarrow \neg\text{Better}(x, y)$

   (b) $\neg\exists x.\exists y.\text{Yankee}(x) \wedge \text{Yankee}(y) \wedge \text{Better}(x, y)$

5. Not a single Red Sox player is better than any Yankee.

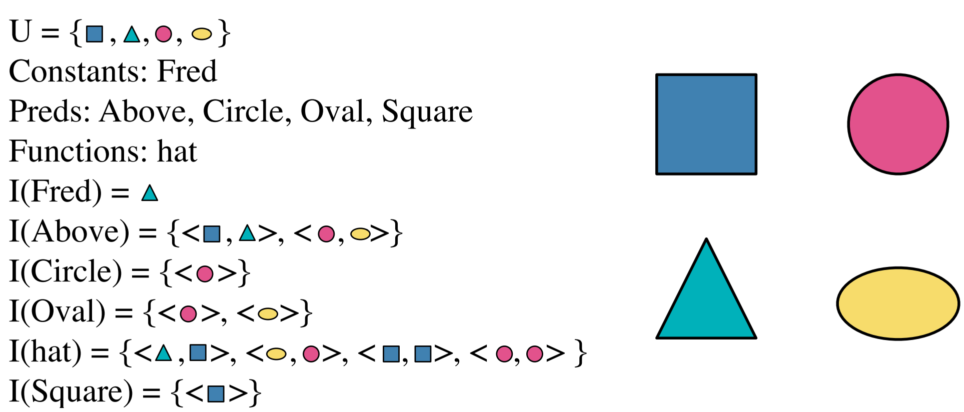
   Solution:

   (a) $\forall x.\forall y.(\text{RedSox}(x) \wedge \text{Yankee}(y)) \rightarrow \neg\text{Better}(x, y)$

   (b) $\neg\exists x.\exists y.\text{RedSox}(x) \wedge \text{Yankee}(y) \wedge \text{Better}(x, y)$

3 Interpretations

1. Determine whether each of the following sentences holds or fails given the interpretation from the lecture slides.



1. $\forall x.Above(x, Fred)$

   Solution: False

2. $\forall x.Above(x, Fred) \rightarrow Square(x)$

   Solution: True

3. $\exists x.\forall y.Circle(y) \rightarrow Above(y, x)$

   Solution: True

2. List a universe and interpretation that makes the first two sentences true and the third sentence false. This can be done with a universe of size 2.

1. $\forall x.H(x) \rightarrow G(x)$
2. $\forall x.F(x) \rightarrow G(x)$
3. $\exists x.F(x) \wedge H(x)$

Solution:

U = {A, B}
I(F) = {A}
I(G) = {A, B}
I(H) = {B}

Another possible interpretation is for F, G, and H to be empty.

3. List a universe and interpretation that makes the first sentence true and the second sentence false. This can be done with a universe of size 3.

1. $\forall x.\exists y.F(x, y)$
2. $\exists y.\forall x.F(x, y)$

Solution:

U = {A, B, C}
I(F) = {(A, B), (B, C), (C, A)}