Interpretations and Entailment 1

1. Fill in the first three columns of the table with all possible interpretations in the domain $\{A, B, C\}$:

A	В	C	S1	S2

Now, consider the following two sentences, S1 and S2:

- $\begin{array}{ll} S1 & (A \lor B) \to (\neg B \land (C \lor A)) \\ S2 & (B \leftrightarrow C) \to A \end{array}$

In the column labeled "S1," place a mark next to each interpretation in which S1 holds. Do the same for S2 in the column labeled "S2." Does S1 entail S2?

2. Here is a sentence in propositional logic:

 $(A \to (B \lor (C \land D))) \leftrightarrow (B \lor C)$

Does it hold given the interpretation $i = \{A = t, B = f, C = t, D = f\}$? If so, give an interpretation in which it does not hold. If not, give an interpretation in which it does hold.

Does $(A \lor B) \land (\neg A \lor C)$ entail $(B \lor C)$?

3. Show that $(A \lor B) \land (\neg A \lor C) \to (B \lor C)$ using truth tables.

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Image: Second	A	B	C	$A \lor B$	$\neg A \lor C$	$(A \lor B) \land (\neg A \lor C)$	$B \lor C$		
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2 Writing FOL

Assume that you can use the following predicates in a universe of all baseball players:

- 1. Yankees(x) x plays for the Yankees
- 2. $\operatorname{RedSox}(x)$ x plays for the Red Sox
- 3. Better(x,y) player x is better than player y
- 4. Loves(x,y) player x loves player y
- 5. Cursed(x) player x is cursed

Now convert the following English sentences to FOL statements:

- 1. Every Red Sox player has no love for any Yankee player.
- 2. There is not a single Red Sox player who is not cursed.
- 3. If a baseball player is cursed, he cannot love anyone.
- 4. All Yankee players have the same skill level (no player is better than another).
- 5. Not a single Red Sox player is better than any Yankee.

3 Interpretations

1. Determine whether each of the following sentences holds or fails given the interpretation from the lecture slides.



- 1. $\forall x.Above(x, Fred)$
- 2. $\forall x.Above(x, Fred) \rightarrow Square(x)$
- 3. $\exists x. \forall y. Circle(y) \rightarrow Above(y, x)$

2. List a universe and interpretation that makes the first two sentences true and the third sentence false. This can be done with a universe of size 2.

- 1. $\forall x.H(x) \rightarrow G(x)$
- 2. $\forall x.F(x) \to G(x)$
- 3. $\exists x.F(x) \land H(x)$

3. List a universe and interpretation that makes the first sentence true and the second sentence false. This can be done with a universe of size 3.

- 1. $\forall x. \exists y. F(x, y)$
- 2. $\exists y. \forall x. F(x, y)$