6.034 Recitation 3 - Constraint Satisfaction
April 6, 2007

1. Sudoku-BOT

You are involved as a consultant for a startup that plans to build robots that can automatically solve the Sudoku puzzles in the daily paper. An example of such a puzzle is visible in Figure 1.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 8 | | | | 7 | | |
| | | | 3 | | | 2 | | |
| | 7 | | | | | | | |
| | | | | 7 | 1 | | | |
| 6 | | | | | | | 4 | |
| 3 | | | | | | | | |
| 4 | | | 5 | | | | | 3 |
| | 2 | | | 8 | | | | |
| | | | | | | | 6 | |

Figure 1: A Sudoku puzzle.

The rules of the game are that each unfilled box must be filled in with a digit within a particular range (in this case 0..9), such that no two digits appear in the same *row*, *column*, or *region*. Thus, in a solved sudoku puzzle, each row, column and region will have a complete set of digits {0..9}. You plan to apply your newly acquired knowledge about Constraint Propagation algorithms to make your robot able to solve Sudoku puzzles with exceeding speed and grace.

a. What do variables represent in your CSP algorithm?

b. What are the domains of the variables represented by your CSP algorithm?

c. What are the constraints among the variables of your algorithm? What order are they?

| | | | |
|---|---|---|---|
| 2 | | | |
| | 3 | | |
| | | 0 | |
| 0 | | 1 | |

Figure 2: Mini-sudoku. Values of squares constrained to range {0,1,2,3}

d. Draw a constraint hypergraph for the mini-sudoku puzzle in Figure 2, then solve the puzzle using pure arc-consistency (if possible)

e. For a general Sudoku puzzle of N by N squares (where regions are $\sqrt{N}$ by $\sqrt{N}$ ) , how many constraint arcs are there?

f. For a general Sudoku puzzle of N by N squares using forward checking and most constrained variable reordering, what determines the depth of the search tree? What determines the branching factor at each level?