MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.01—Introduction to EECS I
Fall Semester, 2007

NanoQuiz Week 4 (R 27 Sep)

Name: Solutions **Athena userid:** ________ @mit.edu

1. Given $y(n) = -\frac{3}{2}y(n-1) - \frac{5}{8}y(n-2)$, does $y(n)$ go to zero as n goes to infinity, regardless of initial conditions? You must justify your answer.

<u>Examine Nat Freqs</u>

$$z^2 + \frac{3}{2}z + \frac{5}{8} = 0 \qquad z_{1,2} = -\frac{3}{4} \pm \sqrt{\frac{9}{16} - \frac{10}{16}} = -\frac{3}{4} \pm j\frac{1}{4}$$

$$|z_2| = |z_1| = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \sqrt{\frac{10}{16}} < 1 \qquad |z_1| \; \& \; |z_2| < 1$$

$$\Rightarrow y(n) = c_1 z_1^n + c_2 z_2^n \to 0 \text{ as } n \to \infty$$

2. Suppose $y(n) = Ky(n-1) + y(n-2)$. For what range of values of K is it possible for $|y(n)|$ to grow without bound as n goes to infinity? You must justify your answer.

<u>Examine Natural Freqs</u>

$$z^2 - Kz - 1 = 0 \qquad z_{1,2} = \frac{K}{2} \pm \sqrt{\frac{K^2}{4} + 1}$$

$K=0$	$K>0$	$K<0$
$z_1 = +1$	$\left\lvert \frac{K}{2} + \sqrt{\frac{K^2}{4}+1} \right\rvert$	$\left\lvert \frac{K}{2} - \sqrt{\frac{K^2}{4}+1} \right\rvert$
$z_2 = -1$	> 1	> 1
$\lvert z_1\rvert = \lvert z_2\rvert = 1$		

For all values of K, except $K=0$, the magnitude of <u>at least one</u> natural frequency is > 1. Therefore $y(n) \to \infty$ as $n \to \infty$ {Given approp initial conditions}