10/23/07 6.01

Today - Circuit Design

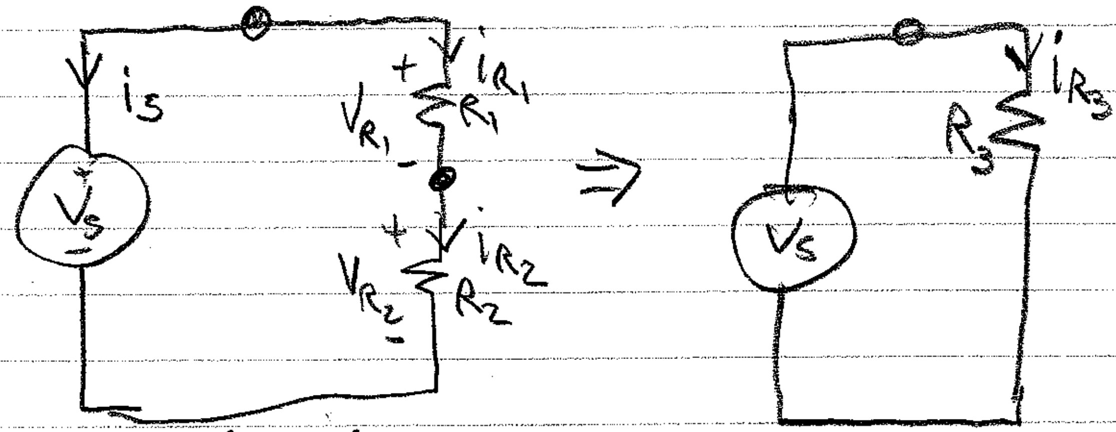
1) Common Patterns
   a) Series, Parallel, Dividers

2) Linearity and Superposition

3) Thevenin Equivalents

Series & Parallel Combos

Series

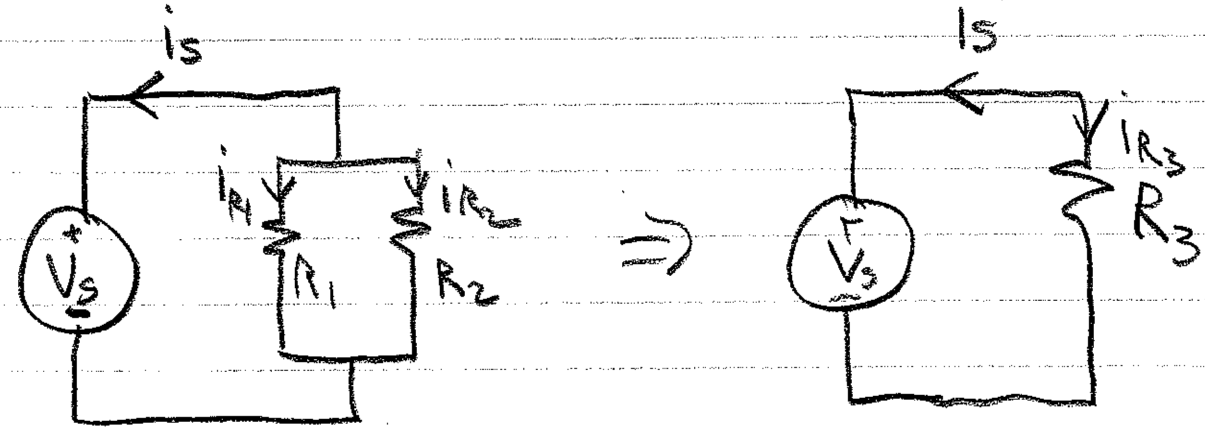


$$i_s + i_{R_1} = 0$$
$$-i_{R_1} + i_{R_2} = 0$$

Resistances add in series

$$V_s = R_1 i_{R_1} + R_2 i_{R_2} = \underbrace{(R_1 + R_2)}_{R_3} i_{R_1}$$

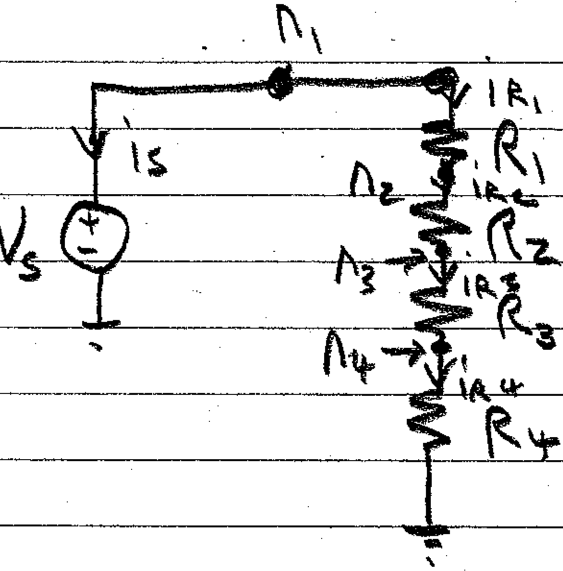
Parallel



Conductances add in parallel

$$i_s + i_{R_1} + i_{R_2} = 0$$
$$\frac{V_s}{R_1} + \frac{V_s}{R_2} + i_s = 0$$
$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_s + i_s = 0 \qquad \frac{1}{R_3} V_s + i_s = 0$$

Resistors In Series



Crnts Appear Twice

$$i_s + i_{R_1} = 0$$

$$-i_{R_1} + i_{R_2} = 0$$

$$-i_{R_2} + i_{R_3} = 0$$

$$-i_{R_3} + i_{R_4} = 0$$

At Gnd

$$-i_s - i_{R_4} = 0$$

$$V_{n_1} - V_{n_2} - i_{R_1} R_1 = 0$$

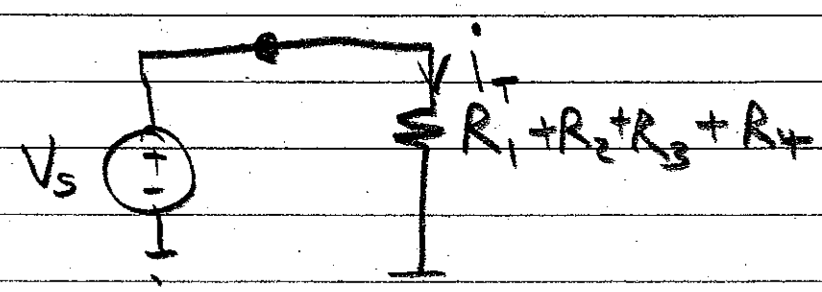
$$V_{n_2} - V_{n_3} - i_{R_2} R_2 = 0$$

$$V_{n_3} - V_{n_4} - i_3 R_3 = 0$$

$$V_{n_4} - 0 - i_4 R_4 = 0$$

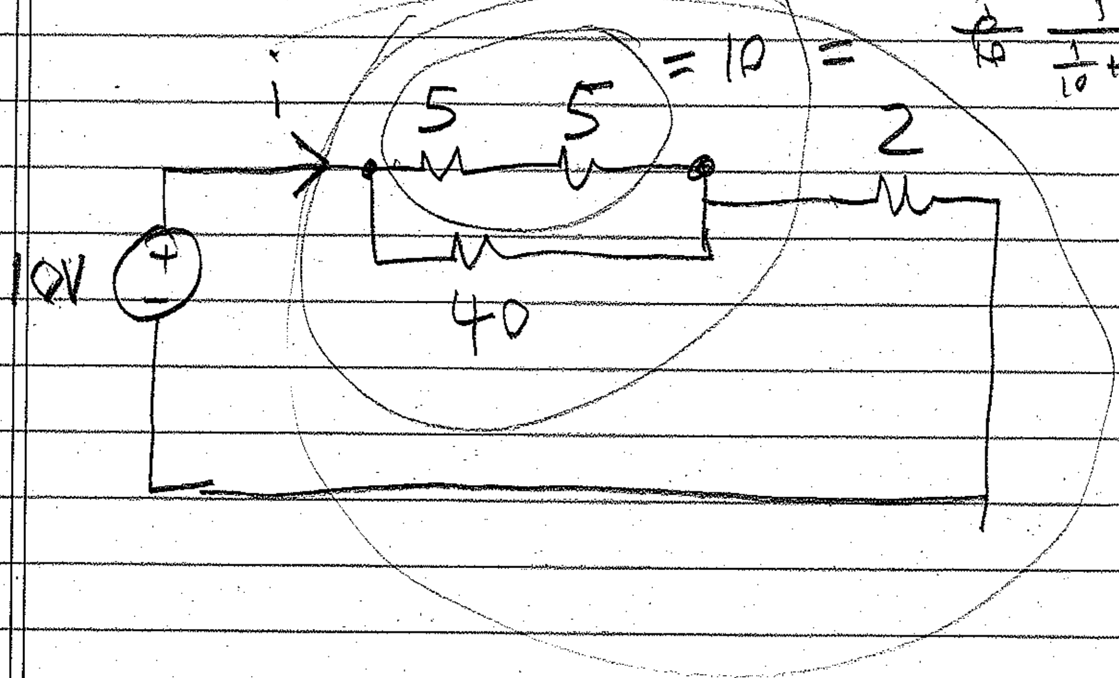
$$i_s = -i_{R_1} = -i_{R_2} = -i_{R_3} = -i_{R_4} = -i_T$$

$$\Rightarrow \quad V_{n_1} - 0 - i_T (R_1 + R_2 + R_3 + R_4) = 0$$



Resistors Add in Series

Series & Parallel Help Simplify

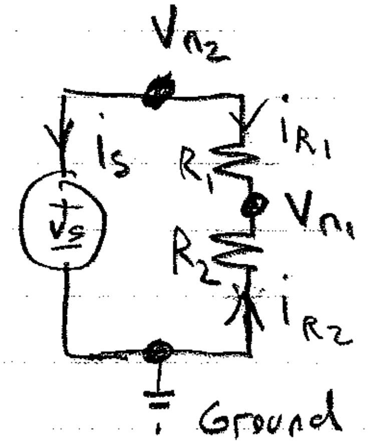


$= \frac{1}{\frac{1}{10}+\frac{1}{40}} = \frac{400}{50} = 8$

$8+2=10$

$\frac{10V}{10} = 1$ amp

Voltage Divider Example



Constit

$$(V_{n_2} - 0) - V_s = 0$$
$$(V_{n_2} - V_{n_1}) - R_1 i_{R_1} = 0$$
$$(0 - V_{n_1}) - R_2 i_{R_2} = 0$$

KCL

$$-i_{R_1} - i_{R_2} = 0$$
$$i_{R_1} + i_s = 0$$

Solving

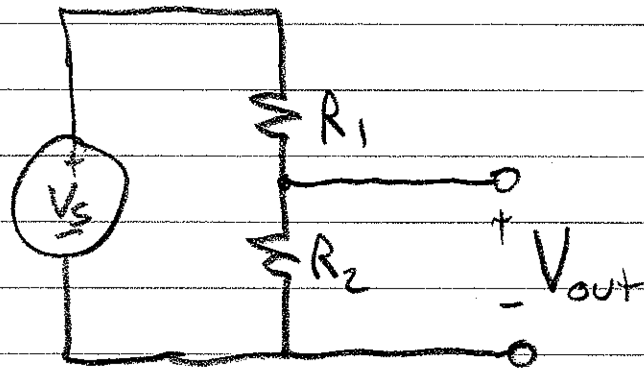
$+i_{R_2}$

$$\frac{V_s - V_{n_1}}{R_1} - i_{R_1} = 0 \Rightarrow \frac{V_s - V_{n_1}}{R_1} - \frac{V_{n_1}}{R_2} = 0$$

$$(R_1 R_2)\left(+\frac{1}{R_1} + \frac{1}{R_2}\right)V_{n_1} = \left(\frac{V_s}{R_1}\right)R_1 R_2$$

$$\underbrace{V_{n_1} = V_S \frac{R_2}{R_1 + R_2}}_{\text{Voltage Divider Formula}}$$

Reasoning About Dividers



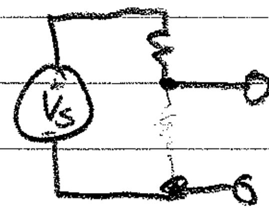
$$V_{out} = \frac{R_2}{R_2 + R_1} V_s$$

$R_2 = 10{,}000\,\Omega = 10K$

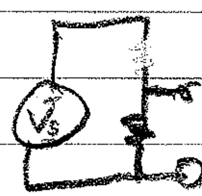
$\Rightarrow V_{out} \simeq V_s$

$R_1 = 100\,\Omega$

$\frac{10k}{10.1k} V_s$



$R_2 = 100\,\Omega$



$V_{out} \simeq 0$

$R_1 = 10K\,\Omega$

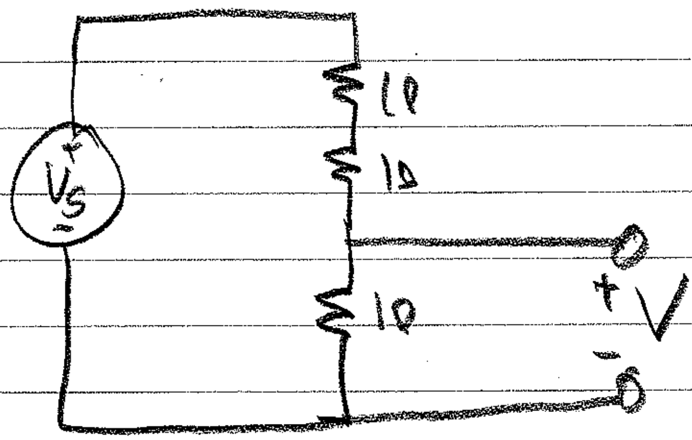
$$V_{out} = \frac{100}{10.1k} \simeq \frac{R_1}{R_2} V_s = .01\, V_s$$

$R_1 = 100\,\Omega$

$R_2 = 100\,\Omega$

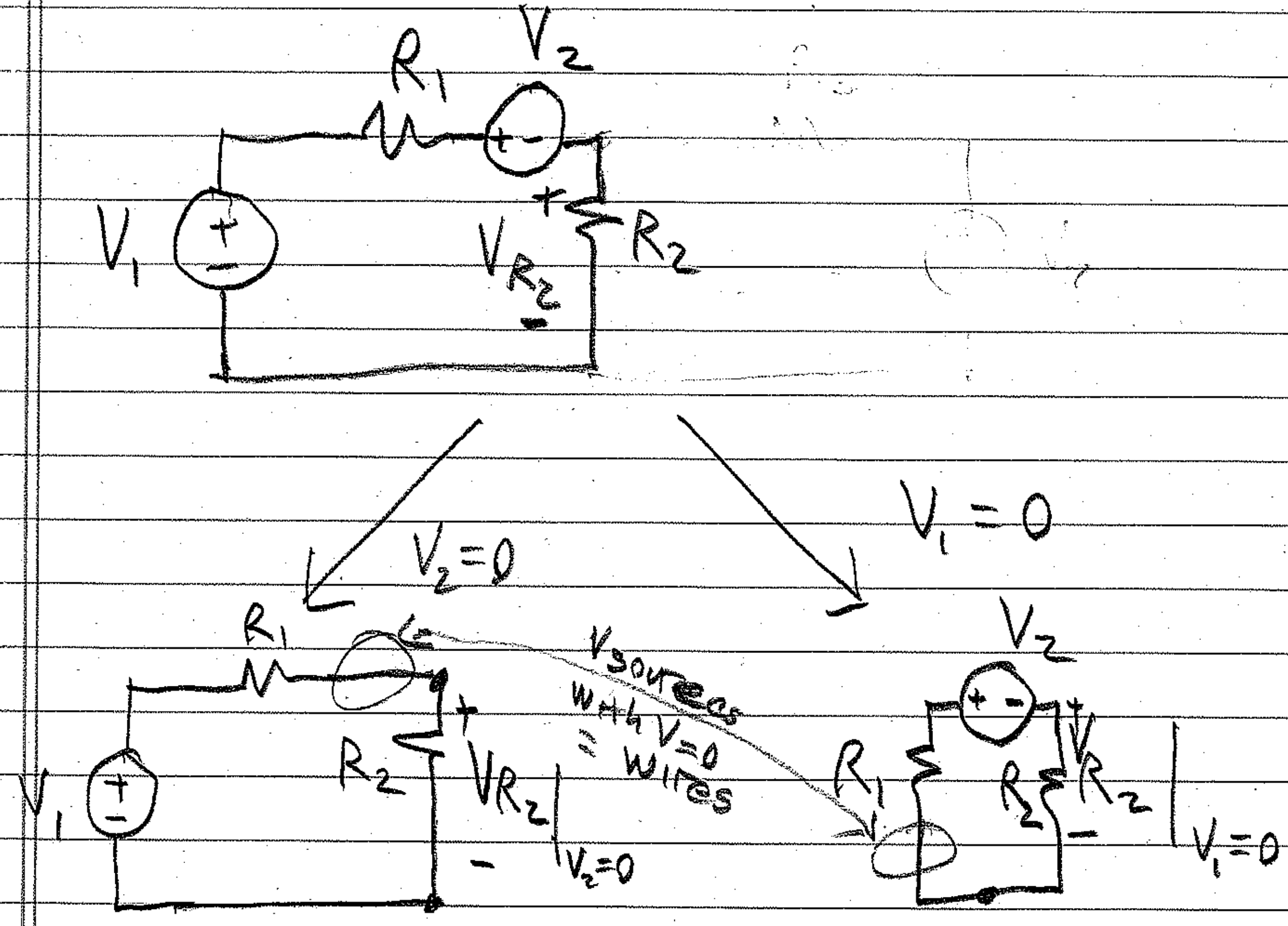
$V_{out} = \frac{1}{2} V_s$

Circuit Example



$V = ?$

Linearity and Superposition



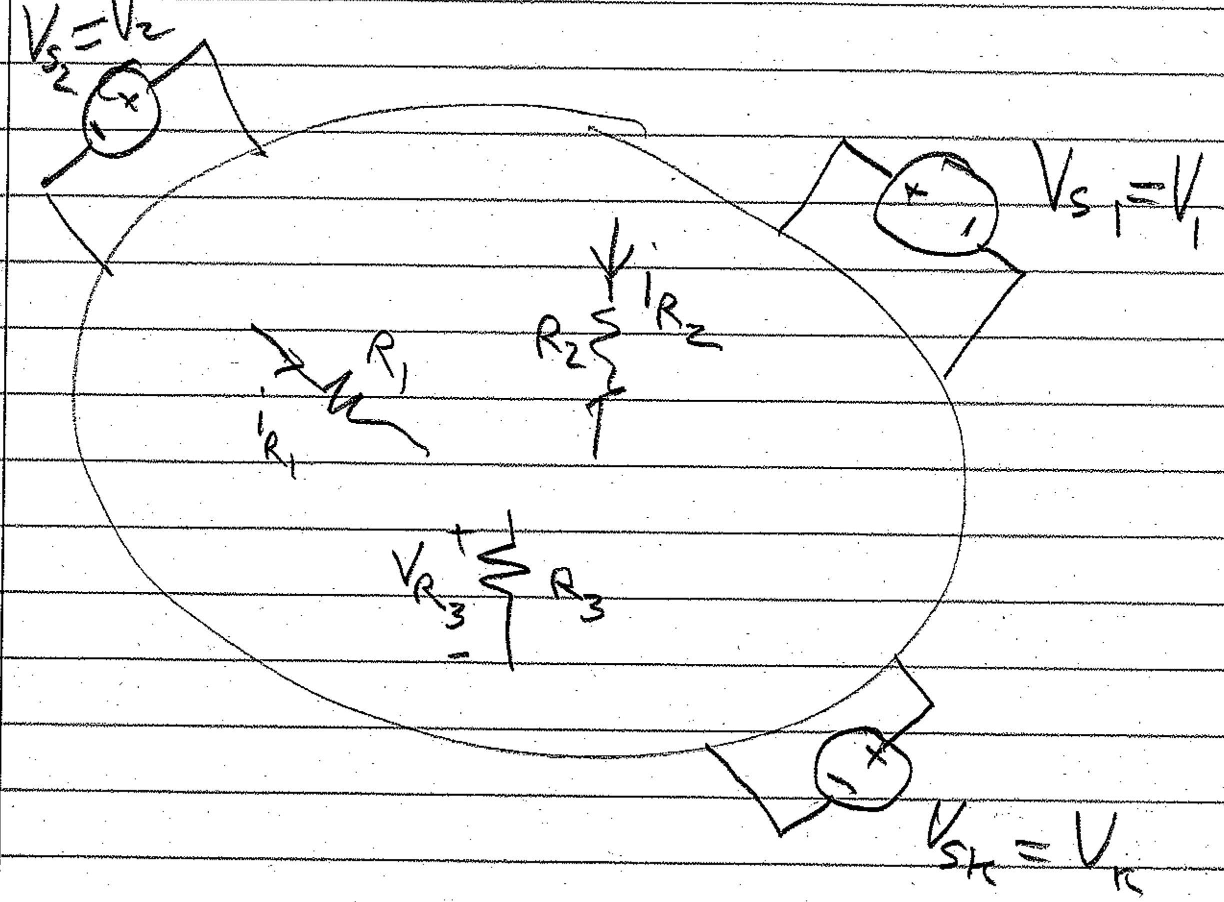
$$= \frac{R_2}{R_1+R_2}V_1 \qquad\qquad = -\frac{R_2}{R_1+R_2}V_2$$

$$V_{R_2} = \frac{R_2}{R_1+R_2}V_1 + \left(-\frac{R_2}{R_1+R_2}\right)V_2$$

In General

1) Analyze circuit k times with $V_{s_j}=0 \quad k \neq j$
2) Sum the results

Linearity & Super Position



All voltages and currents in the network can be determined by turning on one voltage source at a time

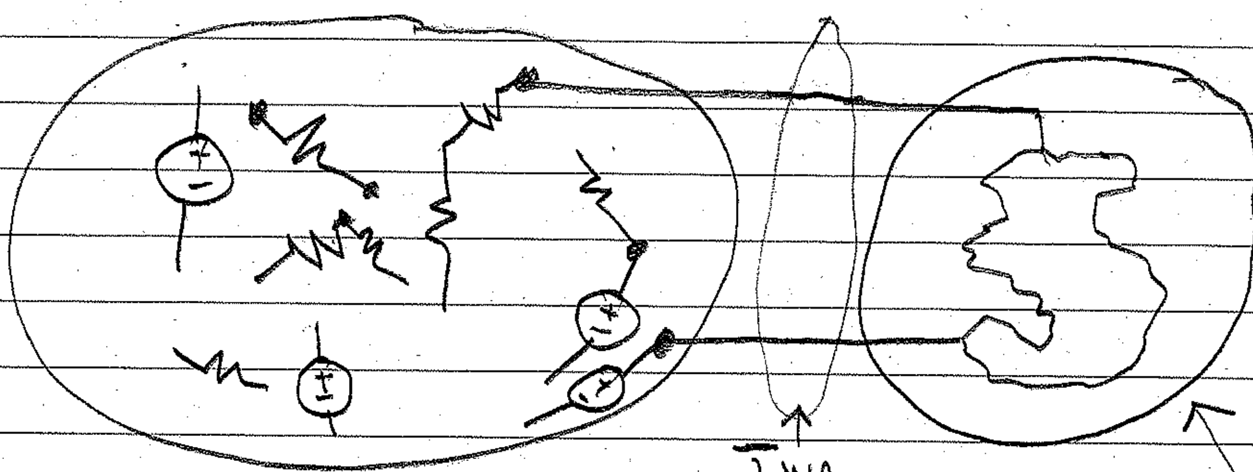
i.e.

$$\overset{V_{R_3}}{i_{R_2}} = \overset{V_{R_3}}{i_{R_2}}\Bigg|_{\substack{V_{S_1}=V_1 \\ V_{S_2}=0 \\ \vdots \\ V_{SK}=0}} + \overset{V_{R_3}}{i_{R_2}}\Bigg|_{\substack{V_{S_1}=0 \\ V_{S_2}=V_2 \\ V_{S_3}=0 \\ V_{SK}=0}} + \ldots \overset{V_{R_3}}{i_{R_2}}\Bigg|_{\substack{V_{S_1}=0 \\ V_{S_2}=0 \\ V_{SK-1}=0 \\ V_{SK}=V_K}}$$

Another Aspect of Linearity

Thevenin Equivalents

Given



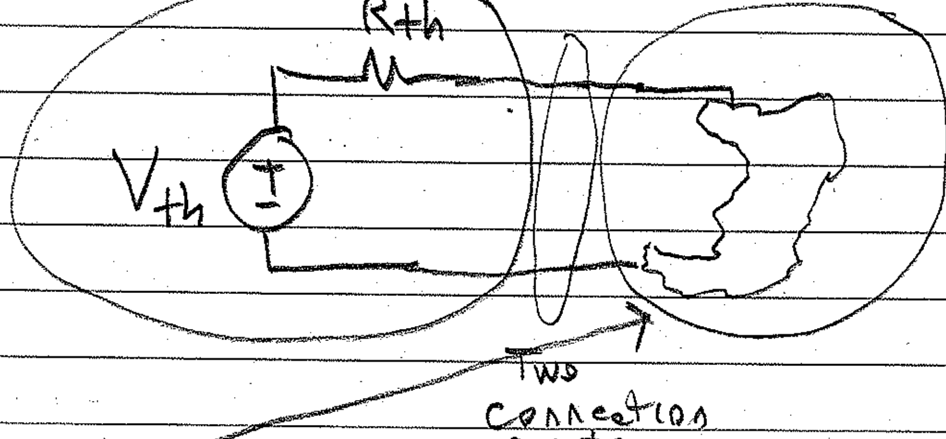
Two connection points

Circuit A (Arbitrary Collection of resistors and voltage sources)

Circuit B (Circuit under Analysis)

Then

There exists V_{th} and R_{th} so that

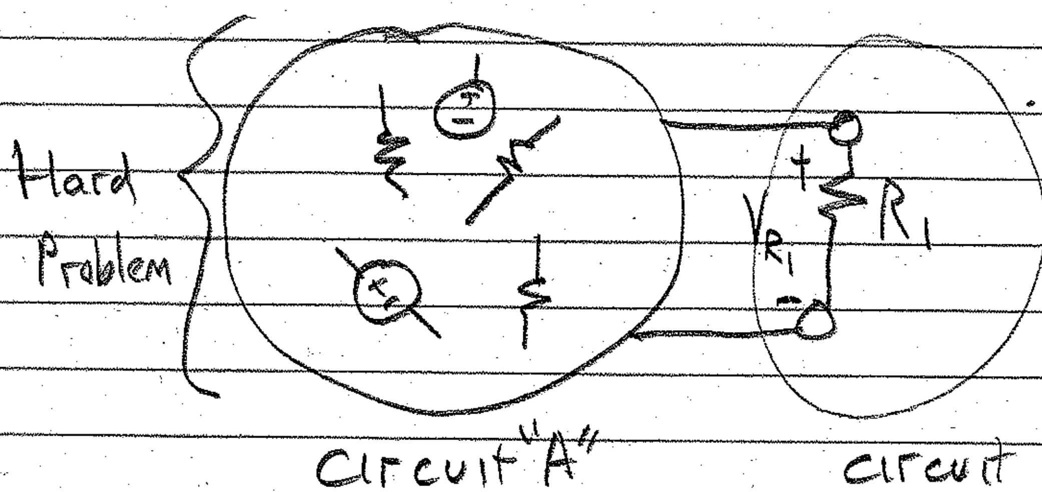


Two connection points

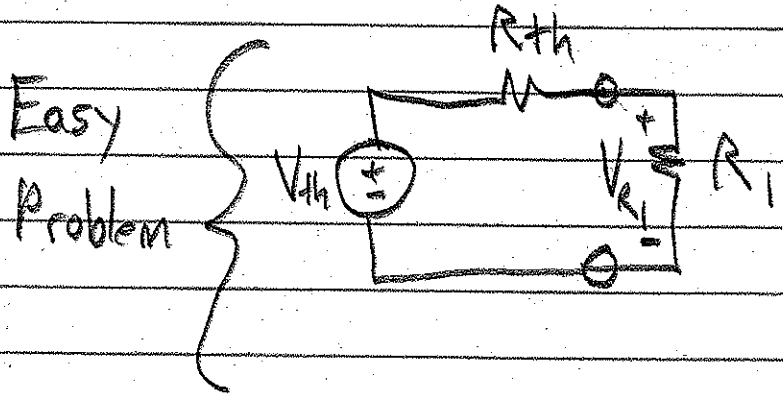
Circuit B will "behave" just like original

Why use Thevenin Equivalents

Question: For what value of R_1 will $V_{R_1} = 1V$?



$$\Downarrow$$



$$V_{R_1} = \frac{R_1}{R_{th}+R_1} V_{th}$$

$$= 1V$$

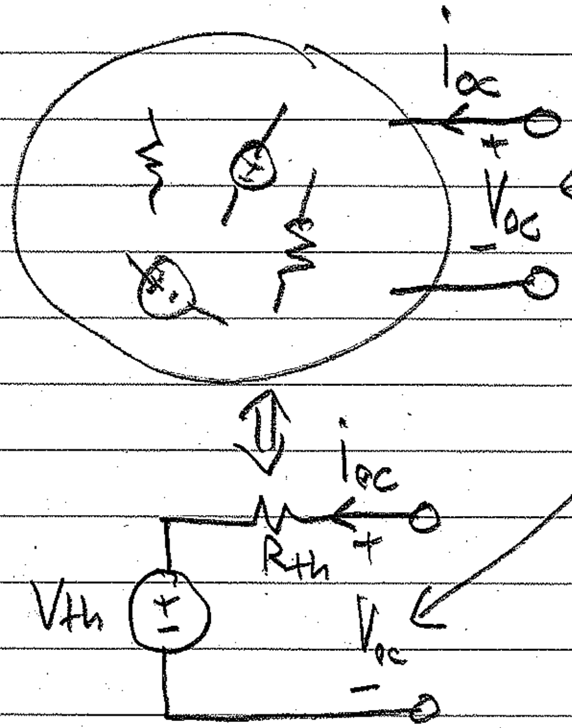
$$\Rightarrow R_{th} + R_1 = R_1 V_{th}$$

$$R_1 = \frac{R_{th}}{(V_{th} - 1)}$$

<u>Note</u>: if $V_{th} < 1$, no positive resistor exists!

Computing Thevenin Equivalents

1) Use to Compute V_{th}

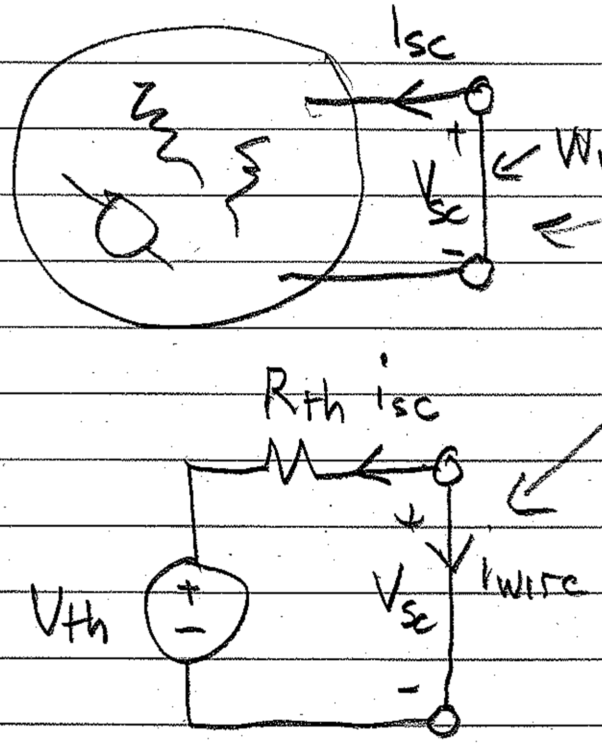


Nothing Connected

$V_{oc} = V_{open\ circuit} = V_{th}$

$(i_{oc} = 0)$

2) Use to Compute R_{th} given V_{th}



Wire Connected

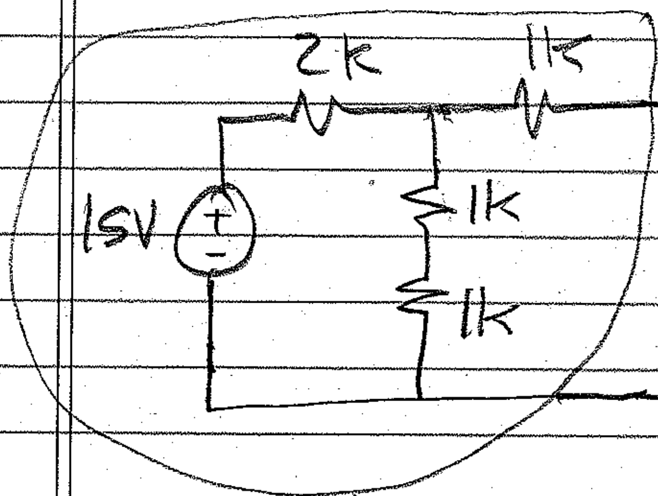
$V_{sc} = V_{short\ circuit} = 0$

$i_{sc} = -\frac{V_{th}}{R_{th}}$

$i_{wire} = -i_{sc}$

$i_{wire} = \frac{V_{th}}{R_{th}}$

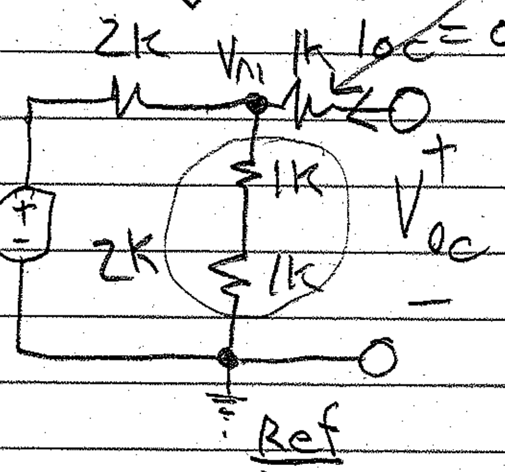
Example



For what value of R_1 is $V_{R_1} = 1V$

Thevenin

Compute V_{th}



$V_{n_1} - 0 = V_{oc}$

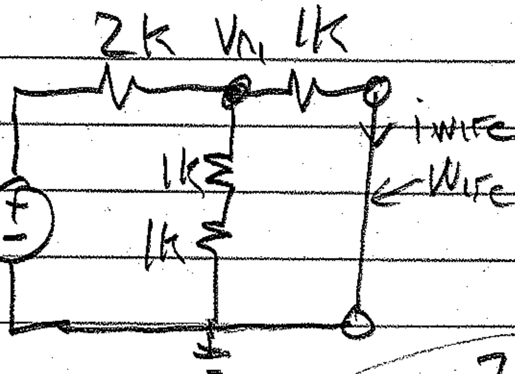
why?

$\left\{ \begin{array}{l} i_{oc} = 0 \Rightarrow = 0 \\ V_{n_1} - V_{oc} - \cancel{i_{oc}} 1k = 0 \end{array} \right\}$

$$V_{n_1} = \underbrace{\frac{2k}{2k+2k} 15}_{\text{Divider}}$$

$V_{n_1} = V_{oc} = V_{th} = 7.5 \longrightarrow$

Compute R_{th}

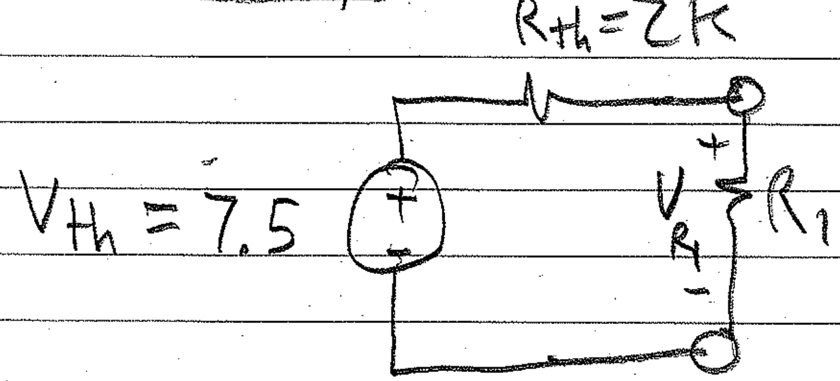


$$i_{wire} = -i_{sc} = \frac{V_{n_1}}{1k}$$

$$V_{n_1} = 15 \frac{1k \,||\, (1k+1k)}{2k + (1k \,||\, (1k+1k))}$$

$$R_{th} = \frac{7.5}{-15/4/1k} = 2k \qquad V_{n_1} = \frac{15}{4} \Rightarrow i_{sc} = -\frac{15/4}{1k}$$

Finally



For what value of R_1 is $V_{R_1} = 1V$?

$$V_{R_1} = \frac{R_1}{R_1 + R_{th}} V_{th}$$

$$1 = \frac{R_1}{2K + R_1} 7.5$$

$$R_1 = \frac{2K}{7.5 - 1} = \frac{2K}{6.5}$$