11/06/2007 6.01 1

Today

1) Fun With Op-amps

2) Op-amp Nonidealities

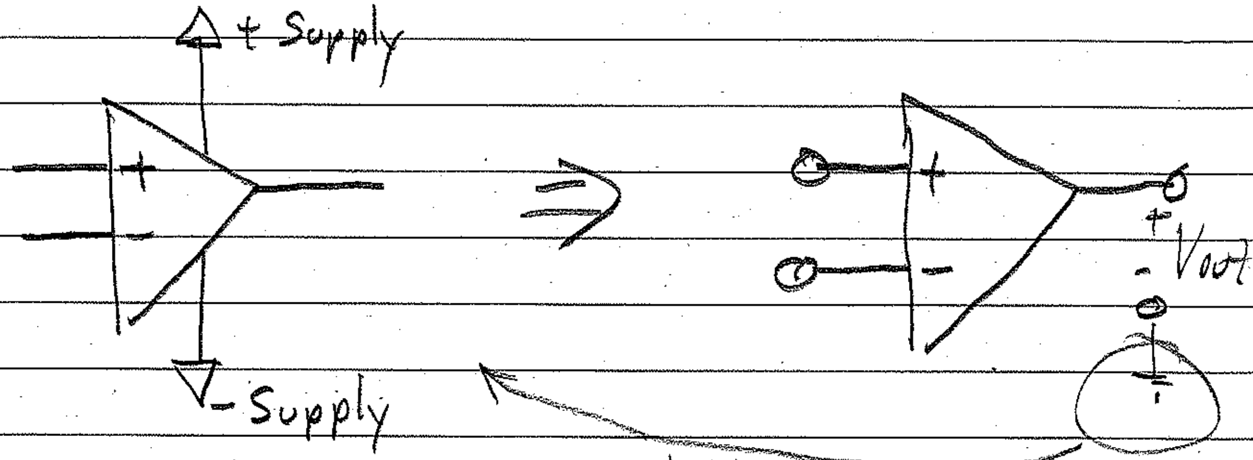
3) (If Time) Head Controller model

Announcements

1) Hand In Lab

2) Midterm Rules
   - Start on Thursday Lab today
   - Hand in Work on Thursday
   - Finish Lab on Thursday
      - Serious about Checkoffs

Op-amp

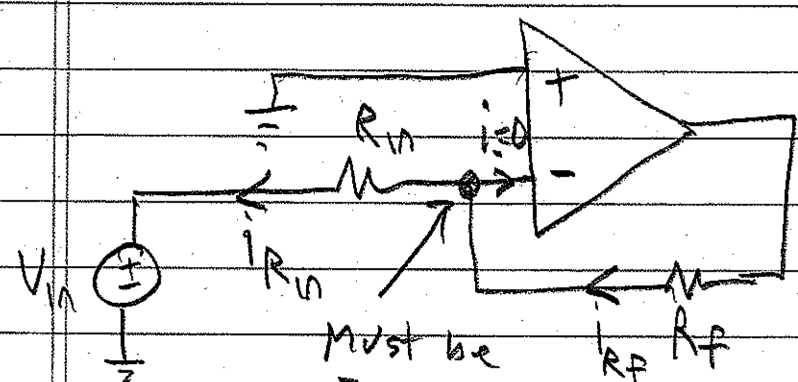


Halfway between + Supply & - Supply

Design Rule

If the Op-amp is connected "correctly": 1) Then the output will adjust so $V_+ = V_-$ 2) the current entering V_+ & V_- is zero

Inverting Amp (Output feeds back to V_-)



Must be Zero Volts

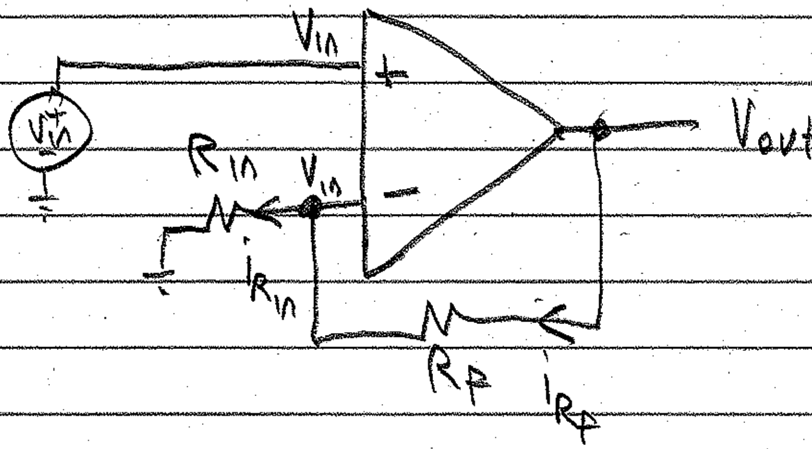
"virtual ground"

$$i_{R_{in}} = i_{R_f}$$

$$\frac{0 - V_{in}}{R_{in}} = \frac{V_{out} - 0}{R_f}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

Non-Inverting (Output feeds back to V_-)

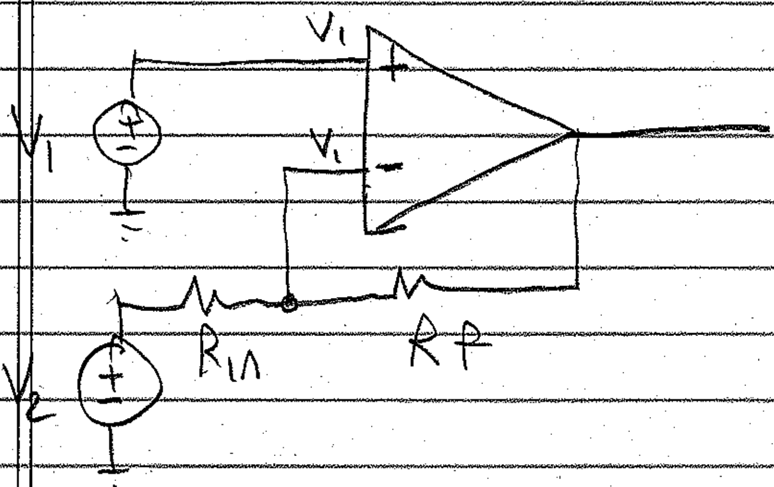


$$i_{R_f} = i_{R_{in}}$$

$$\frac{V_{out}-V_{in}}{R_f} = \frac{V_{in}-0}{R_{in}}$$

$$\frac{V_{out}}{V_{in}} = \left(1+\frac{R_f}{R_{in}}\right)$$

Superposition Works



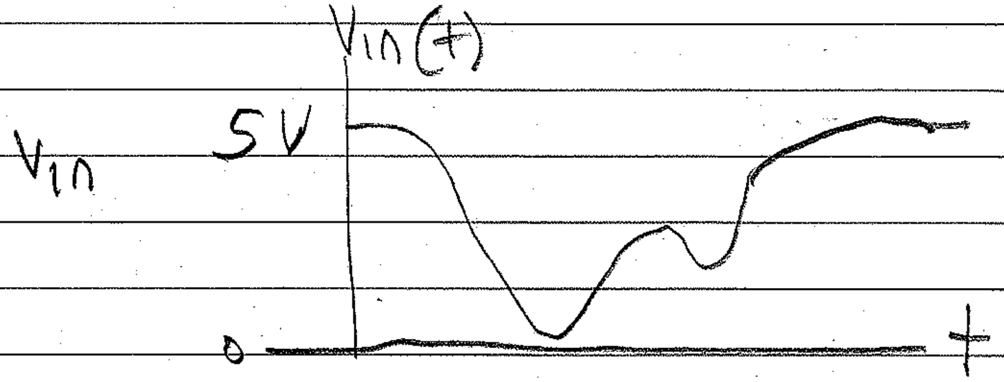
$$\frac{V_{out}-V_1}{R_f} = \frac{V_1-V_2}{R_{in}}$$

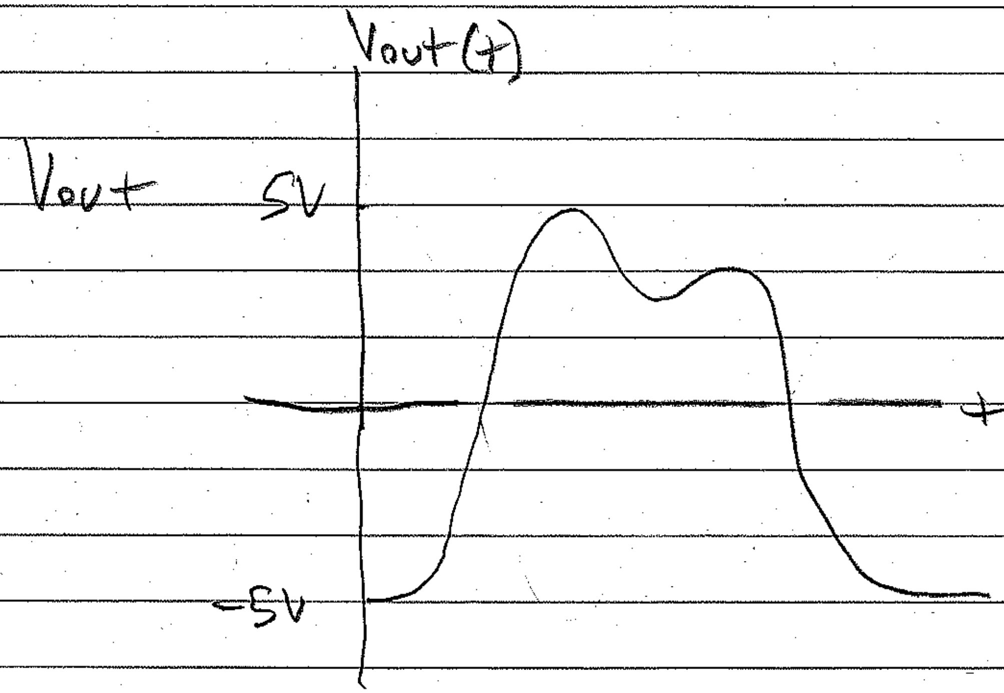
$$V_{out} = \left(1+\frac{R_f}{R_{in}}\right)V_1 - \frac{R_f}{R_{in}}V_2$$

Result assuming $V_2=0$

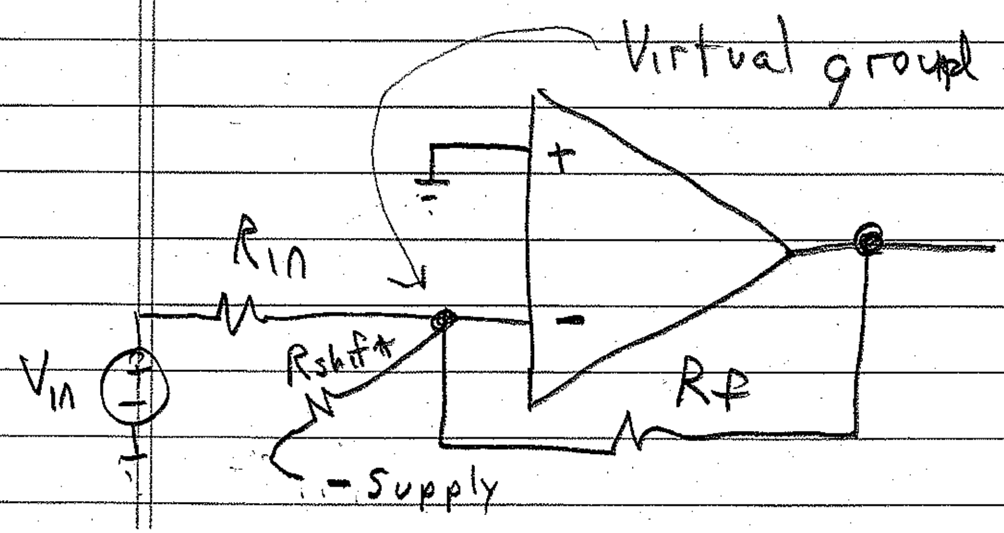
result assuming $V_1=0$

Level Shifting





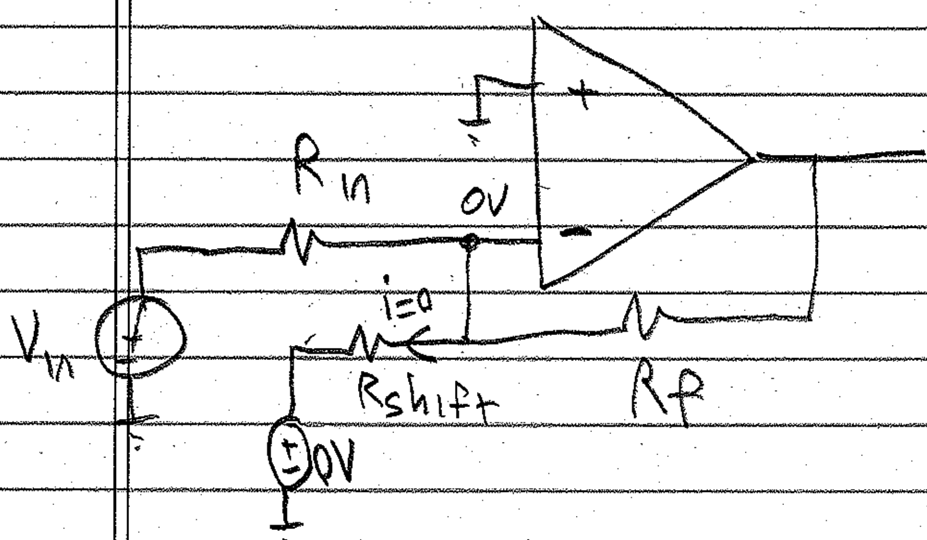
$V_{out} = -2 * V_{in} + 5$ ⟸ Level Shift



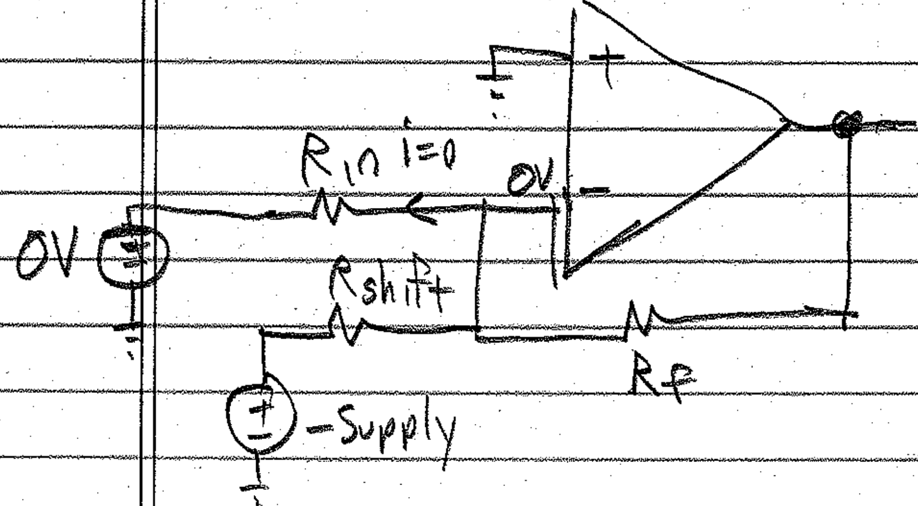
Superposition to the Rescue

$$V_{out} = -2 * V_{in}\Big|_{\substack{\text{assuming} \\ -\text{Supply} \\ \text{to } R_{shift} \\ = 0}} + 5\Big|_{\substack{\text{assuming} \\ V_{in} = 0}}$$

$$\Rightarrow \frac{R_f}{R_{in}} = 2 \quad \text{e.g.} \quad \frac{R_f = 10k}{R_{in} = 5k}$$



$$V_{out}\Big|_{s=0} = -2 V_{in} \Rightarrow \frac{R_f}{R_{in}} = 2 = \frac{10k}{5k}$$

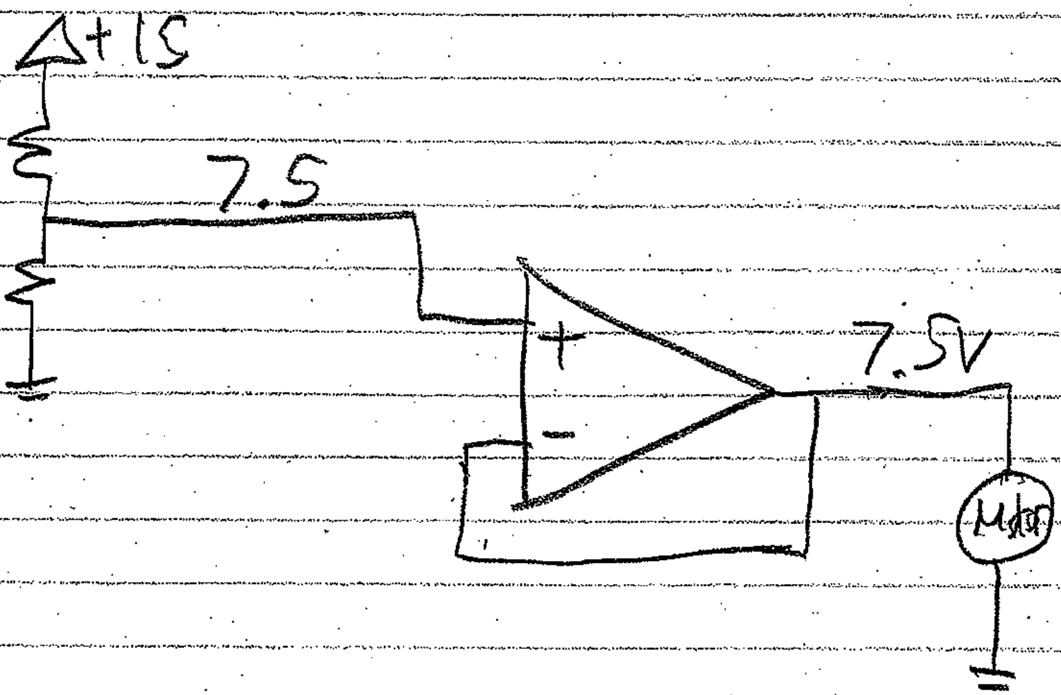


$$V_{out}\Big|_{V_{in}=0} = 5$$

$$\Rightarrow -\frac{R_f}{R_{shift}}(-\text{Supply}) = 5$$

$$R_{shift} = \frac{-R_f(-\text{Supply})}{5}$$

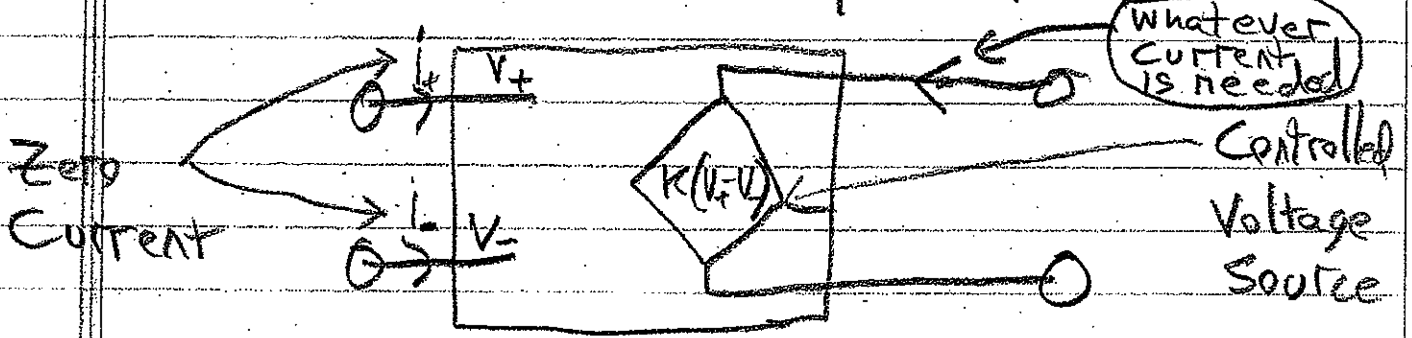
Use an op-amp Follower



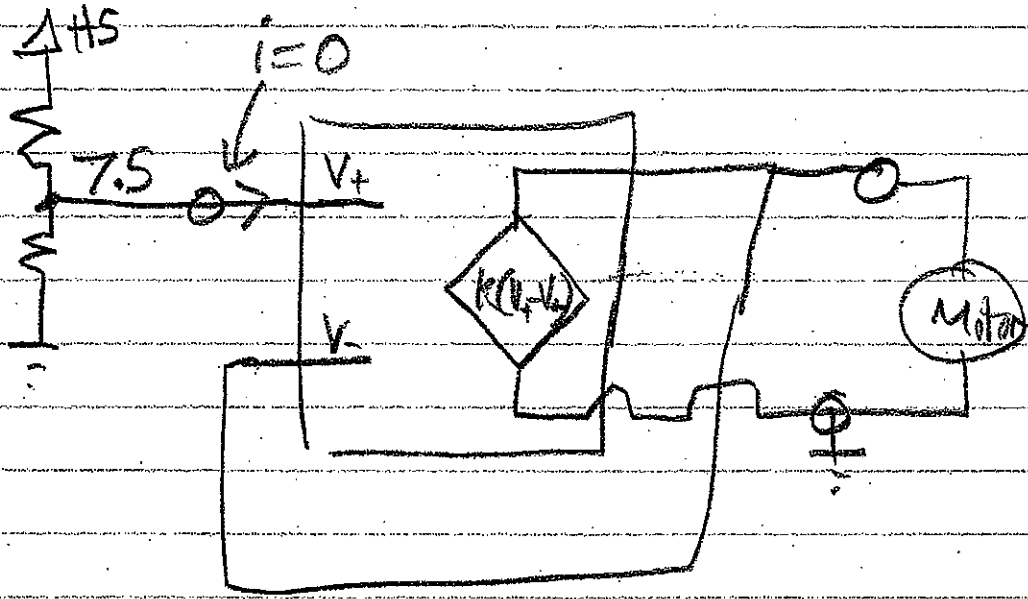
Why does follower help?

Need a better Model of op-amp!

More Detailed Op-amp Model



Follower

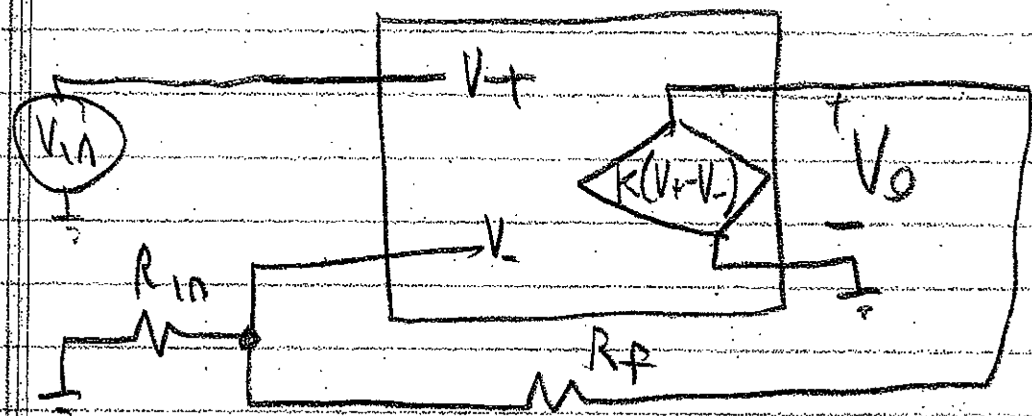


$$V_0 = K(\overset{=7.5}{V_+} - \underbrace{V_-}_{=V_0})$$

$$V_0 = K(7.5 - V_0) \Rightarrow V_0 = \frac{K}{K+1} 7.5$$

$\frac{K}{K+1} \simeq 1$ for large K

Non-inverting Amp Using Detailed Model



$$V_o = K\,(\overset{V_{in}}{V_+} - V_-)$$

$$V_- = V_o \frac{R_{in}}{R_{in}+R_f}$$

$$V_o = K\left(V_{in} - \left(\frac{R_{in}}{R_{in}+R_f}\right) V_o\right)$$

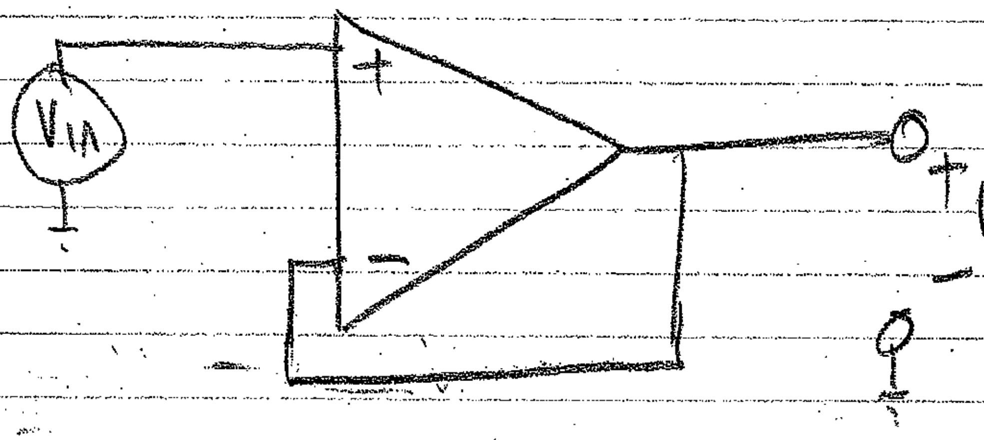
$$\left(1 + K\frac{R_{in}}{R_{in}+R_f}\right) V_o = K\,V_{in}$$

$$V_o = \frac{K}{1 + K\frac{R_{in}}{R_{in}+R_f}} V_{in}$$

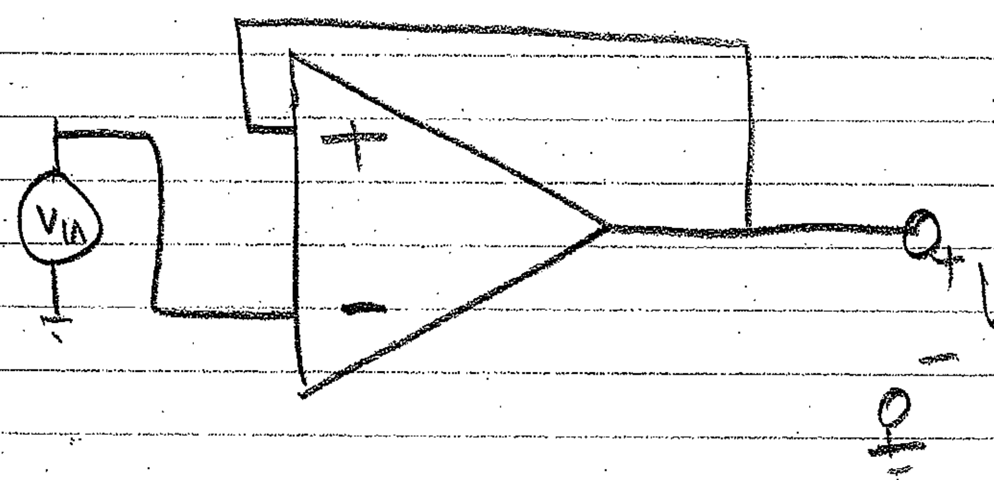
$$= 1 + \frac{R_f}{R_{in}} \quad \underline{\text{if } K \text{ large}}$$

Question About Followers

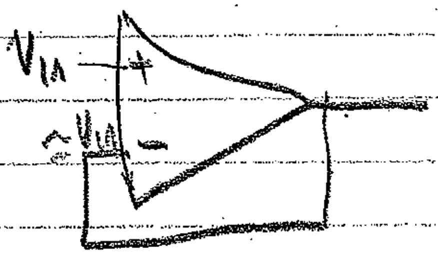
"Good" Follower



"Bad" Follower

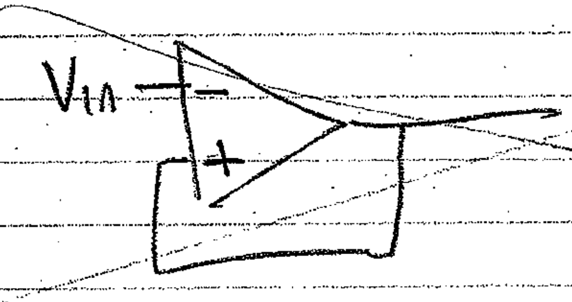


Simple Model



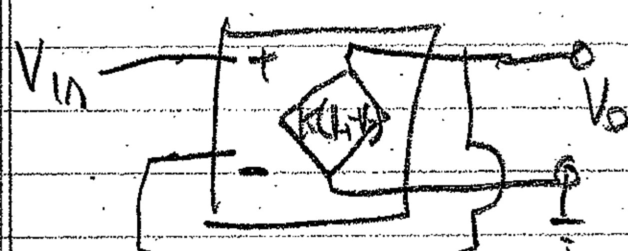
$$V_o \cong V_{in}$$

Wrong



$$V_o \cong V_{in}$$

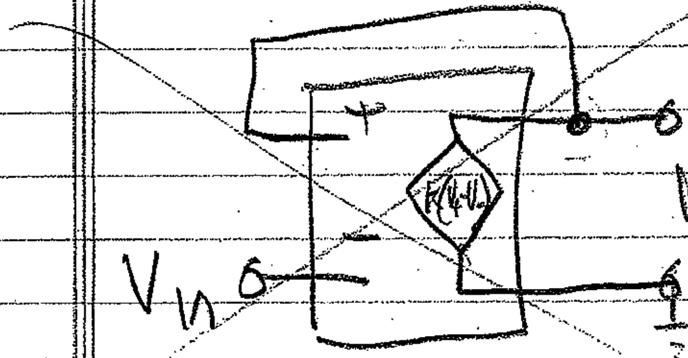
More Detailed Model Does not help



$$V_o = K(\underbrace{V_+}_{=V_{in}} - \underbrace{V_-}_{=V_o})$$

Wrong

$$V_o = \frac{K}{K+1} V_{in} \underset{K\ \text{large}}{\cong} V_{in}$$



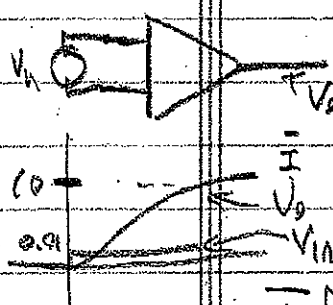
$$V_o = K(\underbrace{V_+}_{=V_o} - \underbrace{V_-}_{=V_{in}})$$

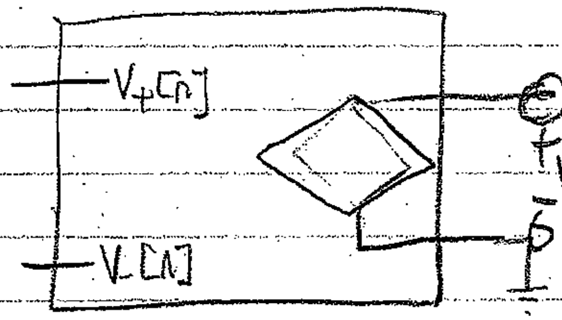
$$(1-K)V_o = -K V_{in}$$

$$V_o = \frac{K}{K-1} V_{in} \cong V_{in}$$

More Detailed Op amp model

What to Model





$$V_0[n+1] = \frac{K-1}{K} V_0[n] + \left(V_+[n] - V_-[n]\right)$$

If $K = 1000$

$$V_0[n] = \frac{999}{1000} V_0[n-1] + \left(V_+[n-1] - V_-[n-1]\right)$$

What if $V_+[n] - V_-[n] = 0.1 \ \forall n$

If $V_0[0] = 0$

$$V_0[n] = \sum_{k=0}^{n-1} \left(\frac{999}{1000}\right)^{((n-1)-k)} \left(V_+[k] - V_-[k]\right)$$

$$V[1] = \left(V_+[0] - V_-[0]\right)$$

$$V[2] = \frac{999}{1000}\left(V_+[0] - V_-[0]\right) + \left(V_+[1] - V_-[1]\right)$$

Eventually

$$V_o[n] = V_o[n-1] = V[*] \text{ (for large } n)$$

$$V[*] - \frac{999}{1000} V[*] = 0.01$$

$$= \frac{1}{1000} V[*] = 0.01$$

$$V[*] = 1000$$

Now Back to the question

$$V[n] = \frac{999}{1000} V[n-1] + \left(V_+[n-1] - V[n-1]\right)$$

$$V[n] = \underbrace{-\frac{1}{1000}}_{\text{Nat freq } < 1} V[n-1] + V_+[n-1]$$

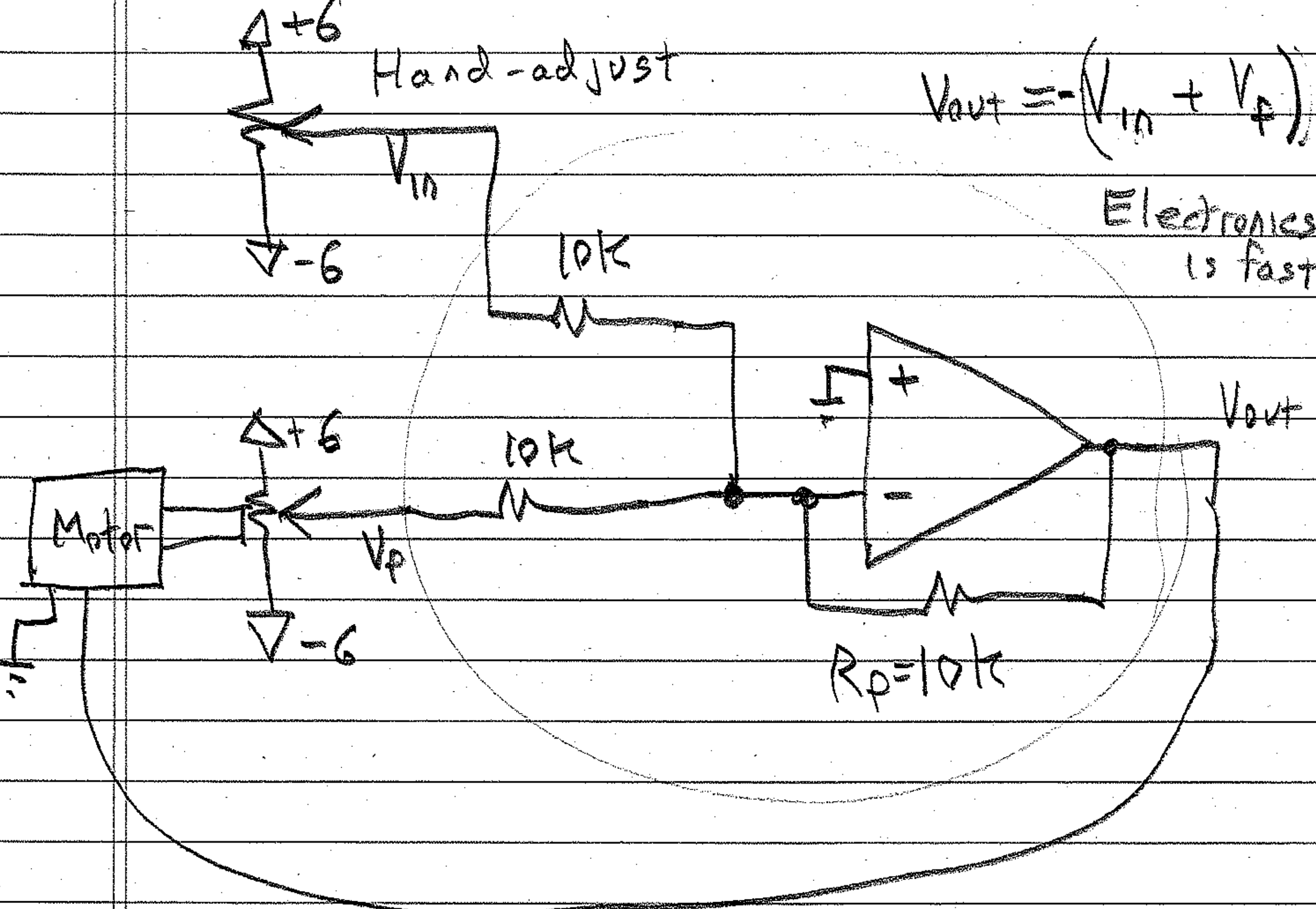
$$V[n] = \frac{999}{1000} V[n-1] + \left(V[n-1] - V_+[n-1]\right)$$

$$V[n] = \frac{1.999}{1000} V[n-1] - V_+[n-1]$$

$\Leftarrow$ Nat freq > 1

Robot Head Controller

$$V_{out} = -(V_{in} + V_f)$$



Motor Model

$$V_f(n) = f\left(V_{out}(n),\ V_{out}(n-1),\ V_{out}(n-2)\right)?$$

<u>What we know</u> Voltage across Motor → Speed

Very Simple

$$\underbrace{\frac{V_f(n) - V_f(n-1)}{\Delta t}}_{\text{Motor Speed}} = \underbrace{K_1}_{?} V_{out}(n-1)$$

$$V_f(n) = V_f(n-1) + \Delta t K_1 V_{out}(n-1)$$

$$V_f(n) = V_f(n-1) + (K_1 \Delta t)(-V_{in}(n-1) + V_f(n-1))$$

$$V_f(n) = \underbrace{(1 - K_1 \Delta t)}_{\substack{\text{Natural} \\ \text{frequency}}} V_f(n-1) - K_1 \Delta t V_{in}(n-1)$$

What happens if we swap leads?