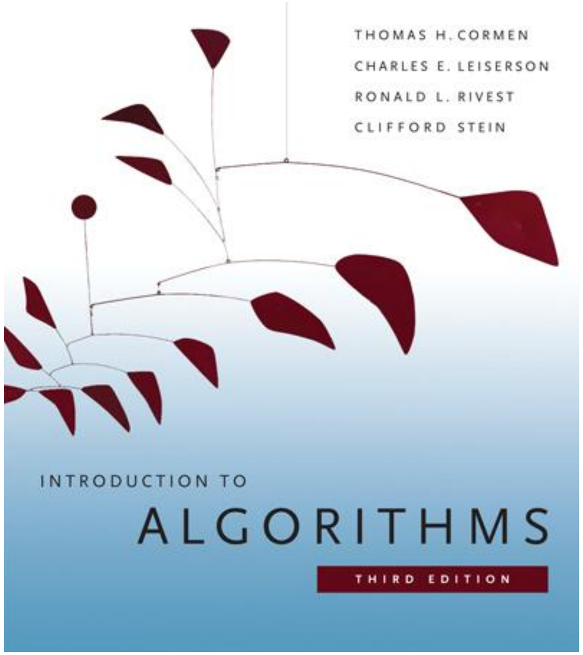
# 6.006- *Introduction to Algorithms*




## *Lecture 3*

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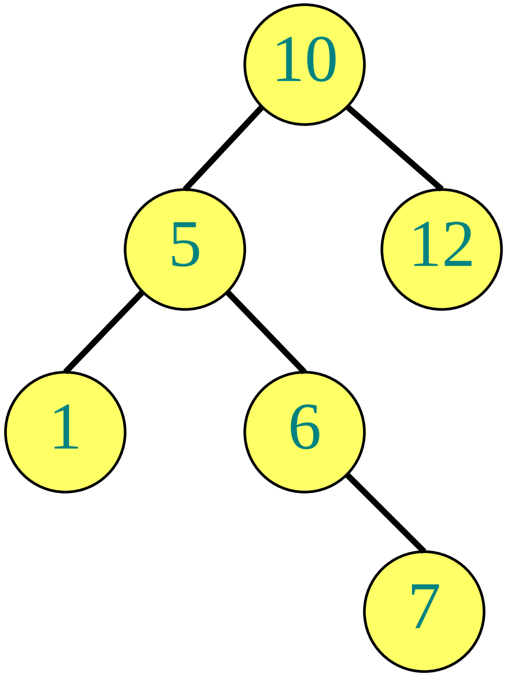
# Overview

- Runway reservation system:
  - Definition
  - How to solve with lists

- Binary Search Trees
  - Operations

**Readings: CLRS 10, 12.1-3**

http://izismile.com/tags/Gibraltar/

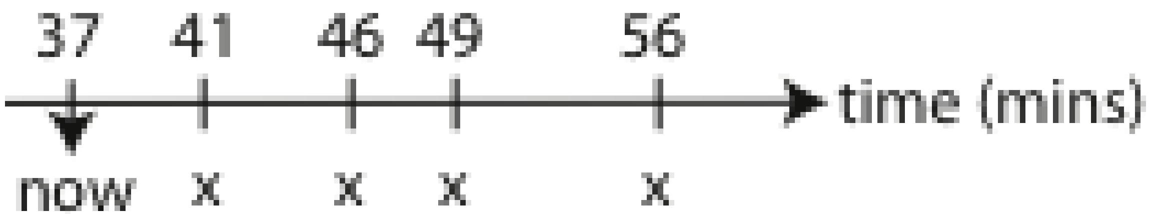


# Runway reservation system

- Problem definition:
  - Single (busy) runway
  - Reservations for landings
    - maintain a set of future landing times
    - a new request to land at time $t$
    - add $t$ to the set if no other landings are scheduled within $< 3$ minutes from $t$
    - when a plane lands, removed from the set

# Runway reservation system

- Example




  - R = (41, 46, 49, 56)
  - requests for time:
    - 44 => reject (46 in R)
    - 53 => ok
    - 20 => not allowed (already past)
- Ideas for efficient implementation ?

# Proposed algorithm

- (keep R as a sorted list)

```
init: R = [ ]
req(t): if t < now: return "error"
for i in range (len(R)):
if abs(t-R[i]) < 3: return "error"
R.append(t)
R = sorted(R)
land: t = R[0]
if (t != now) return error
R = R[1: ] (drop R[0] from R)
```

- Complexity?
- Can we do better?

# Some options:

- Keep R as a sorted list:
  - takes linear time to insert element in proper place
  - a 3 minute check can then be done in O(1)

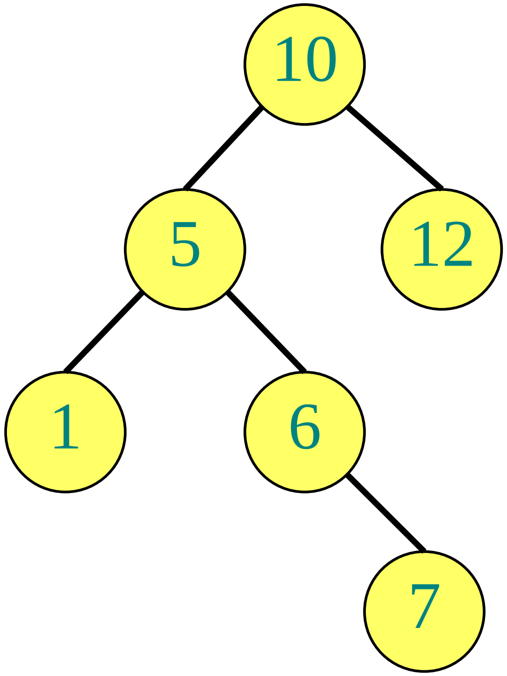
- Keep R as a sorted array:
  - takes O(logn) to find a place to insert new time ...
  - but still requires linear time to actually insert (requires shifting of elements)

- Keep R in unsorted order
  - takes linear time to search for collisions

**Need: *fast* insertion into *sorted* list**

# Binary Search Trees (BSTs)

- Each node x has:
  - key[x]
  - Pointers:
    - left[x]
    - right[x]
    - p[x]



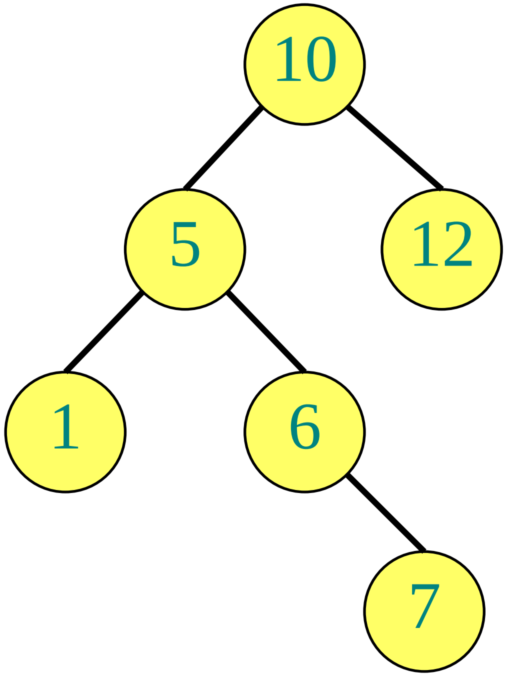
# Binary Search Trees (BSTs)

- Property: for any node x:
  - For all nodes y in the left subtree of x:

    $$key[y] \leq key[x]$$

  - For all nodes y in the right subtree of x:

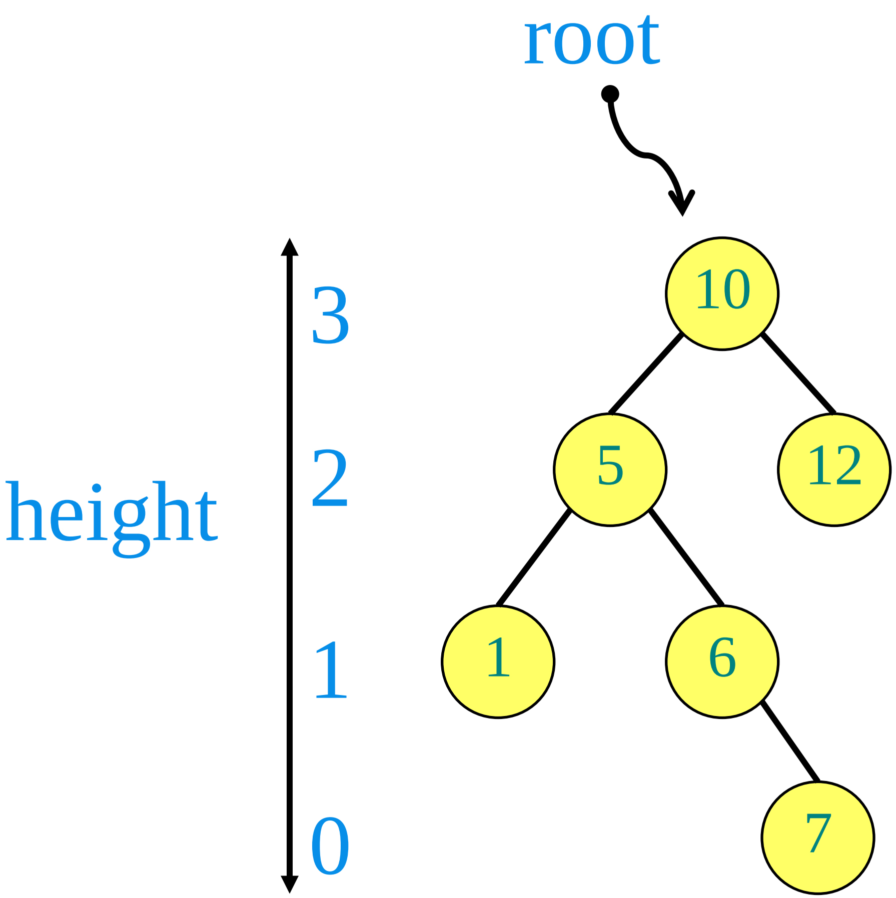
    $$key[y] \geq key[x]$$



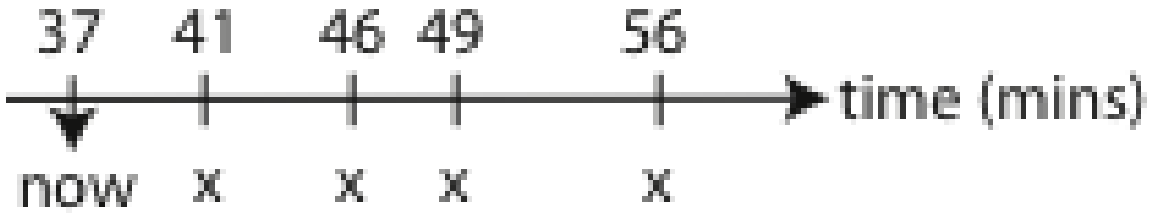

- How are BSTs made ?

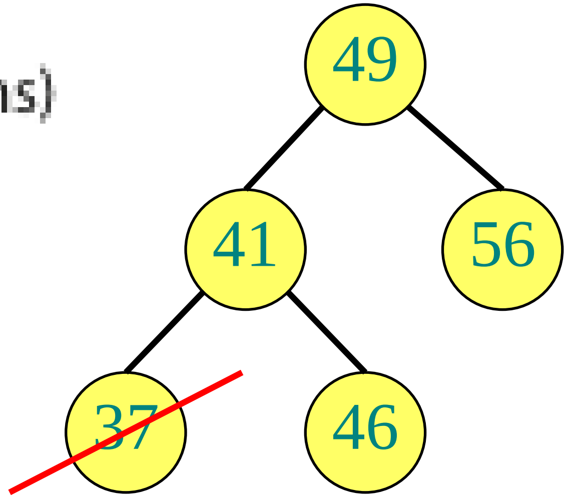
# Growing BSTs

- Insert 10
- Insert 12
- Insert 5
- Insert 1
- Insert 6
- Insert 7

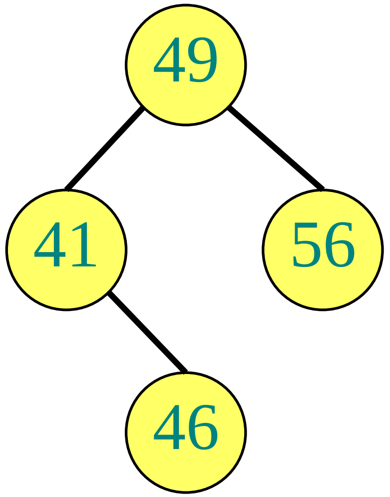


# BST as a data structure




- Operations:
  - insert(k) (note: can do the "within 3" check for reservation during insertion)
  - find(k): finds the node containing key k (if it exists)
  - findmin(x): finds the minimum of the tree rooted at x
  - deletemin(): finds the minimum of the tree and delete it
  - next-larger(x): finds the next element after element x



# Next-larger

next-larger(x):

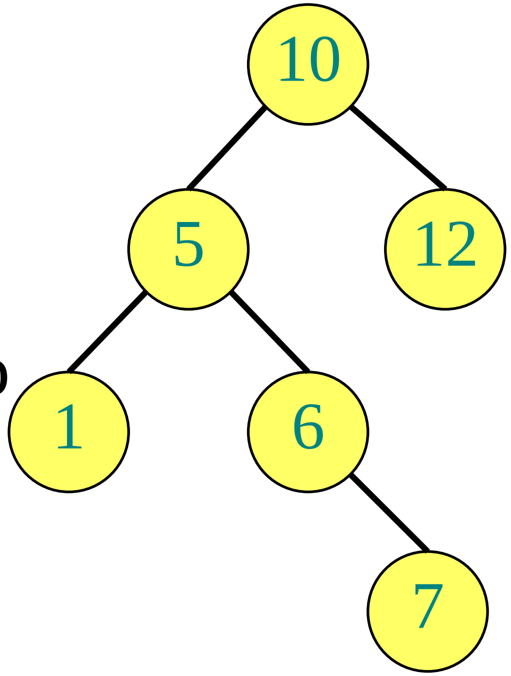
- If right[x] ≠ NIL then return findmin(right[x])
- Otherwise

  y ← p[x]

  While y≠NIL and x=right[y] do

  - x ← y
  - y ← p[y]

  Return y



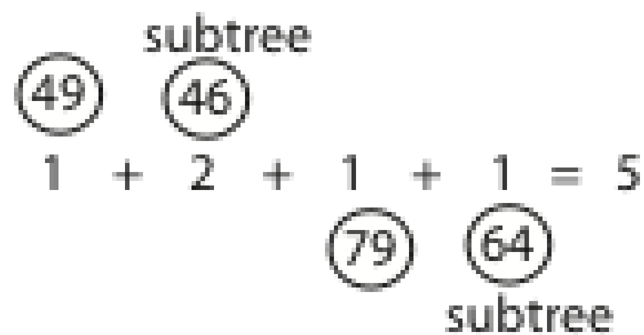

next-larger(5)

next-larger(7)

# Back to runway reservation system

- New requirement: How many planes are scheduled to land at times ≤ t ?

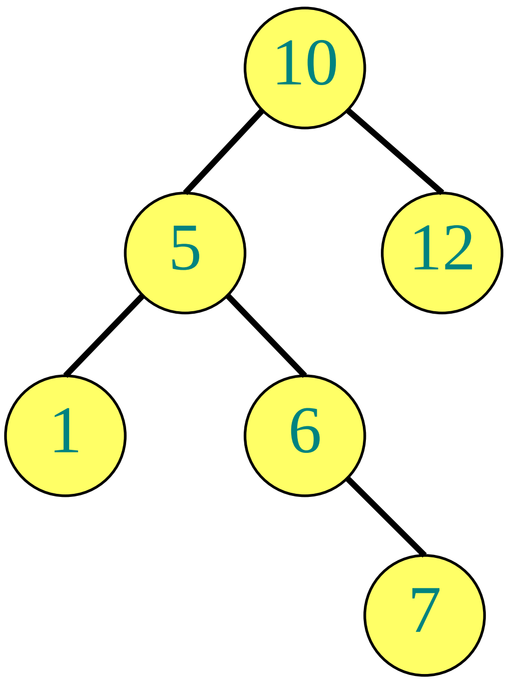

- Augment the BST structure by keeping track of size of subtrees:
- Walk down tree to find desired time
  - Add in nodes that are smaller
  - Add in subtree sizes to the left



# Analysis

- We have seen insertion, deletion, search, findmin, etc.
- How much time does any of this take ?
- Worst case: O(height)

  => height really important
- After we insert n elements, what is the worst possible BST height ?



# Analysis

- n-1
- so, still O(n) for the runway reservation system operations
- Next lecture: balanced BSTs
- **Readings: CLRS 13.1-2**