Newton’s Method

6.006 Review Session

Problem Statement

- Given a function, f(x), find its zeros, i.e. all x such that:

$$f(x) = 0$$

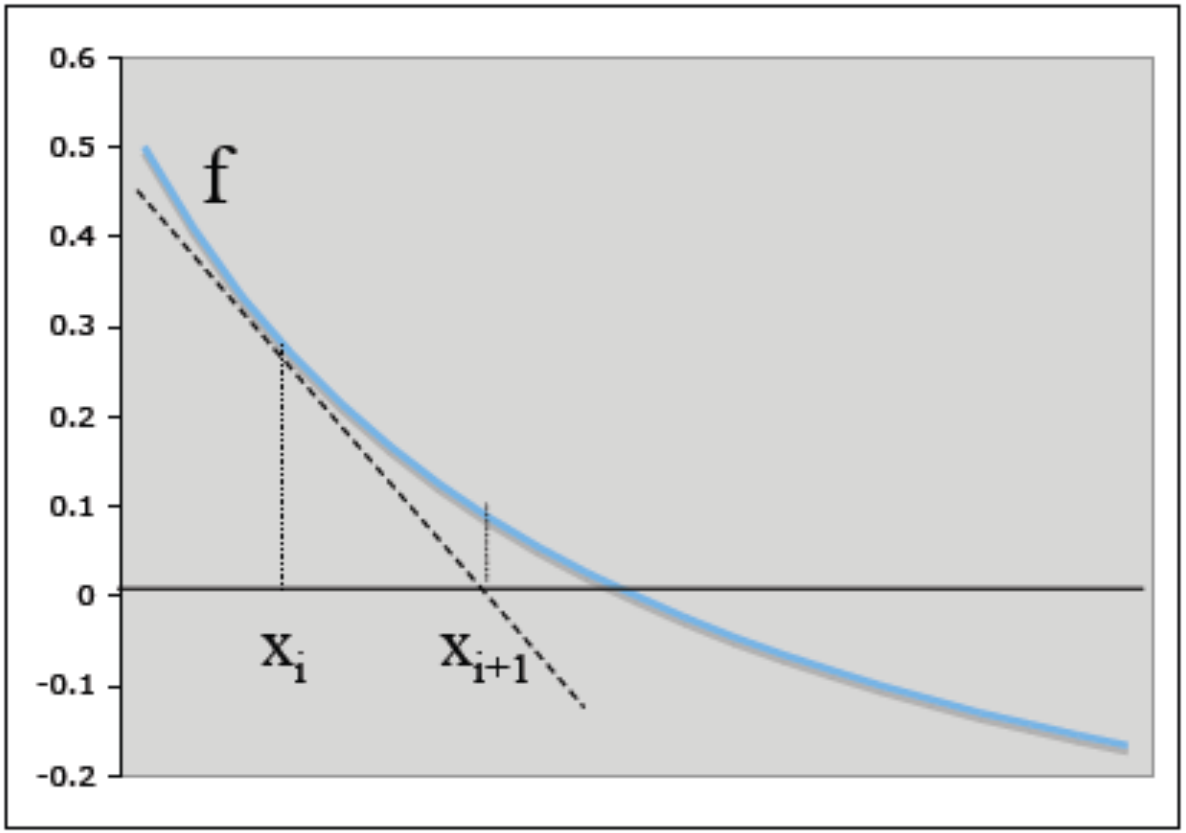
- A close approximation should be sufficient.

Newton's Method

- Idea: iteratively find better and better approximations.
  - Function should be "reasonably well-behaved".
- Newton's Method Steps:
  - Start with an approximation to the root x_0.
  - Iterate using:

$$x_{n+1} = x_n - f(x_n) / f'(x_n)$$

Intuition



Use tangent line to get closer to the actual value of the solution.

Worked Example 1

- Approximate 1 / 2011 using Newton's method.
  - Let $f(x) = 1/x - 2011$
  - Guess $x_0 = 0.0005$
  - Then $x_{i+1} = 2 x_i - 2011 x_i^2$ (after simplifying).
  - Thus:
    - $x_1 = 0.00049725$
    - $x_2 = 0.0004972650418125$
  - Actual value: 0.00049726504226752855

Worked Example 2

- Approximate $2011^{0.5}$
  - Let $f(x) = x^2 - 2011$
  - Guess $x_0 = 40$.
  - Then $x_{i+1} = 0.5\ x_i + 1005.5 / x_i$
  - Thus:
    - $x_1 = 45.1375$
    - $x_2 = 44.845127734699531$
  - Actual value: 44.844174649557324