

The following are a set of reduction problems in approximate order of difficulty.

1. Consider the two problems:

- FACTOR: Given a number  $n = pq$ , the product of two distinct primes  $p$  and  $q$ , find  $p$  and  $q$ .
- EULERTOTIENT: Given a number  $n = pq$ , where  $p$  and  $q$  are unknown primes, find  $\phi(n) = (p-1)(q-1)$ , the number of numbers less than  $n$  relatively prime to  $n$ .

Reduce FACTOR to EULERTOTIENT.

2. Consider the following two problems:

- FEEDBACKNODESET: Given a directed graph  $G = (V, E)$ , find a set of vertices  $U \subseteq V$  of minimal size such that if  $U$  is removed from  $V$ ,  $G$  has no cycles.
- VERTEXCOVER: Given an undirected graph  $G = (V, E)$ , find a minimum set of vertices  $U \subseteq V$  such that for all  $(u, v) \in E$ , either  $u \in U$  or  $v \in U$  or both.

Reduce VERTEXCOVER to FEEDBACKNODESET.

3. Consider the following problem:

- SETCOVER: Given a set of sets  $S = \{\{a_{11}, a_{12}, \dots, a_{1n_1}\}, \{a_{21}, \dots, a_{2n_2}\}, \dots, \{a_{m1}, \dots, a_{mn_m}\}\}$  where the  $a_{ij}$ 's may not necessarily be distinct, find the minimum number of sets,  $A_1, \dots, A_k$ , necessary such that every element in any set in  $S$  is in at least one of the  $A_i$ .

Reduce VERTEXCOVER to SETCOVER.

4. Consider the following two problems:

- PARTITION: Given a set  $n$  of non-negative integers  $\{a_1, \dots, a_n\}$ , decide if there is there a subset  $P \subseteq [1, n]$  such that  $\sum_{i \in P} a_i = \sum_{i \notin P} a_i$ .
- KNAPSACK: Given a set  $S = \{a_1, \dots, a_n\}$  of non-negative integers, and an integer  $K$ , decide if there is there a subset  $P \subseteq S$  such that  $\sum_{a_i \in P} a_i = K$ .

Reduce KNAPSACK to PARTITION.

5. Consider the two problems:

- CNFSATISFIABILITY: Given a set of boolean variables (i.e. variables that can only be TRUE or FALSE),  $\{v_1, \dots, v_n\}$  and the boolean operators  $\wedge$  (AND),  $\vee$  (OR),  $\neg$  (NOT), a boolean expression written in conjunctive normal form (CNF) has the form  $(x_{11} \vee x_{12} \vee \dots x_{1m_1}) \wedge (x_{21} \vee \dots \vee x_{2m_2}) \wedge \dots \wedge (x_{k1} \vee \dots \vee x_{km_k})$  where each  $x_{ij}$  is either a boolean variable or the negation of a boolean variable. Given a boolean expression written in CNF, determine if there is some assignment of TRUE/FALSE to each of the boolean variables  $v_1, \dots, v_n$  such that the expression evaluates to TRUE. Assume that  $m_i \geq 3$  for all  $i$ .
- 3-SAT: A boolean expression written in 3-CNF form has the form  $(x_{11} \vee x_{12} \vee x_{13}) \wedge \dots \wedge (x_{k1} \vee x_{k2} \vee x_{k3})$  where each clause has exactly 3 literals. Given a boolean expression written in 3-CNF form, determine if there is some assignment to each of the boolean variables such that the expression evaluates to TRUE.

Reduce CNFSATISFIABILITY to 3-SAT.