Newton’s Method

6.006 Review Session
Problem Statement

• Given a function, $f(x)$, find its zeros, i.e. all $x$ such that:

$$f(x) = 0$$

• A close approximation should be sufficient.
Newton’s Method

• Idea: iteratively find better and better approximations.
  – Function should be “reasonably well-behaved”.

• Newton’s Method Steps:
  – Start with an approximation to the root $x_0$.
  – Iterate using:
    \[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]
Intuition

Use tangent line to get closer to the actual value of the solution.
Worked Example 1

• Approximate 1 / 2011 using Newton’s method.
  – Let $f(x) = 1/x - 2011$
  – Guess $x_0 = 0.0005$
  – Then $x_{i+1} = 2x_i - 2011x_i^2$ (after simplifying).
  – Thus:
    • $x_1 = 0.00049725$
    • $x_2 = 0.0004972650418125$
  – Actual value: 0.00049726504226752855
Worked Example 2

• Approximate 2011^{0.5}
  – Let \( f(x) = x^2 - 2011 \)
  – Guess \( x_0 = 40. \)
  – Then \( x_{i+1} = 0.5 \times x_i + 1005.5 / x_i \)
  – Thus:
    • \( x_1 = 45.1375 \)
    • \( x_2 = 44.845127734699531 \)
  – Actual value: 44.844174649557324