Recitation 16

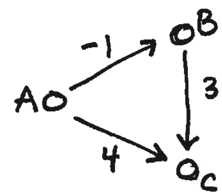
Bellman-Ford

→ idea: relax edges in rounds, each round preserves ordering.
do at most V-1 rounds.

→ runtime: [init = $\Theta(V)$] + [$\Theta(V)$ rounds of $\Theta(E)$ each] = $\Theta(VE)$

(?) can we ever terminate early?
YES! no edges relaxed in around. ($d_{i+1} = d_i$)

→ ordering: matters for speed
not for correctness.



AB, BC, AC (1rnd)
faster than
AC, BC, AB. (2rnd)

(?) what if we change the ordering between rounds?
doesn't matter.

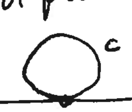
→ correctness

no-neg

- in round i (at the end) $d[v] = \min[p(s,v)]$ for all $p(s,v)$ with i or fewer nodes in it (not including s)
  - base case: round 0: $d[s] = 0$, 0 is len of a zero node path.
    $d[v] = \infty$, no path to any others of 0 nodes exist.
  - inductive case:
    at start of round $d[v] = \min[p(s,v)]$ using $i-1$ nodes or less.
    since we consider all edges - we consider all extensions of current shortest paths (which may or may not be i-1 nodes long)
    2 possibilities: min path to v had less than i-nodes - we won't change it.
    - min path has i nodes. then min to pred had to have i-1 exactly. we consider all these. OK.

[book: use path-relaxation property.

neg. cycles. - a path of len V or greater must have a cycle (pidgeon-hole princ)
- if $d[v]$ can be relaxed after $n-1$ then path of n shorter than a path of len $< n$.



$W(c) < 0$

properties of shortest paths & relaxation

- Triangle inequality

  for any $(u,v) \in E$, $\delta(s,v) \le \delta(s,u) + w(u,v)$
- Upper Bound

  $d[v] \geq \delta(s,v) \quad \forall v \in V$

  once $d[v] = \delta(s,v)$ it never changes.
- No-path

  no path from s to u then $d[u] = \delta(s,u) = \infty$
- convergence

  $s \leadsto u \to v$ is a shortest path for some $u, v \in V$
  and $d[u] = \delta(s,u)$ prior to relaxing (u,v) then
  $d[v] = \delta(s,v)$ at all times afterward.
- path relaxation

  $p = \langle v_0 \cdots v_k \rangle$ shortest and edges of p relaxed in order
  then $d[v_k] = \delta(s, v_k)$
  (holds regardless of intermixing of other relaxations)
- predecessor-subgraph.

  once $d[v] = \delta(s,v)$ $\forall v \in V$ the pred. subgraph is a
  shortest-paths tree rooted @ s.

reachable vs. non reachable.

linearity of w modifications.