

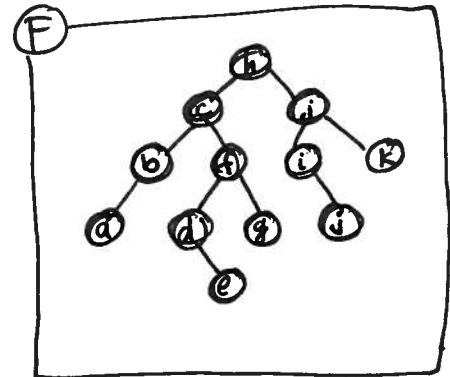
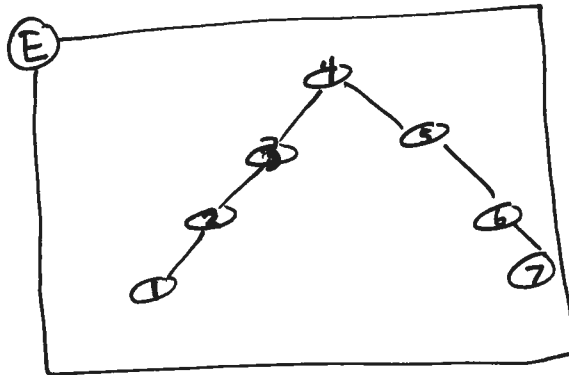
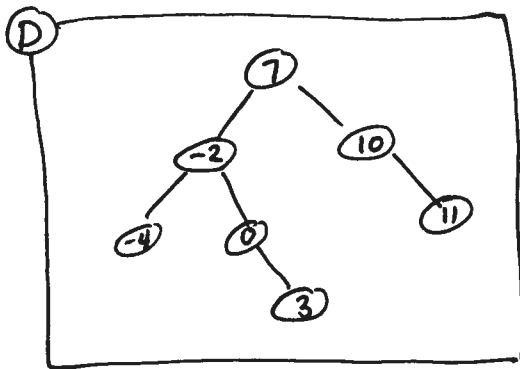
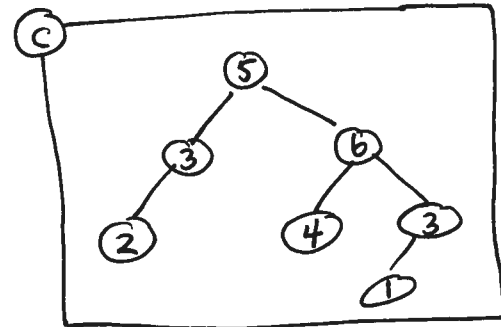
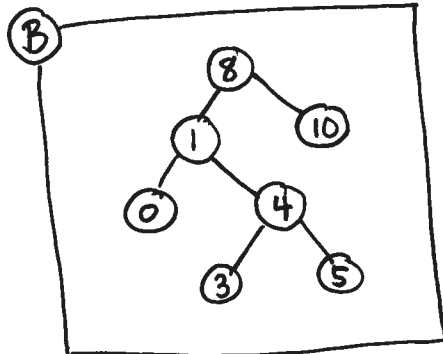
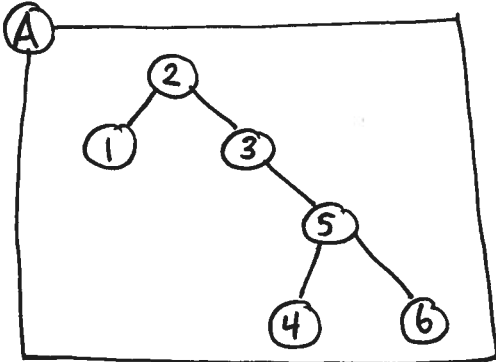
WARMUP

R04

Playing with AVL trees: <http://www.site.uottawa.ca/~stan/esc2514/applets/avl.../BT.h>

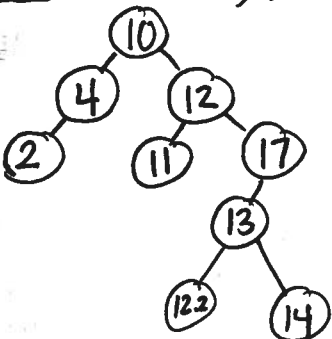
Rotations: [CLRS p 277-279] for psuedocode.

Determine if each of the following is ① a valid BST ② maintains the AVL ~~invar~~ invariant and label the nodes w/ their height, indicate where AVL violation is

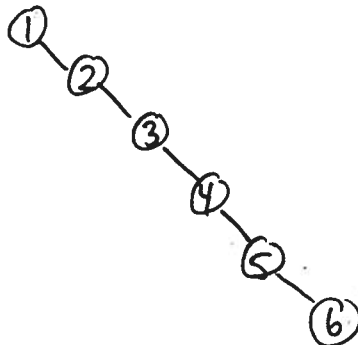


Node deletion in a BST - no need to balance!

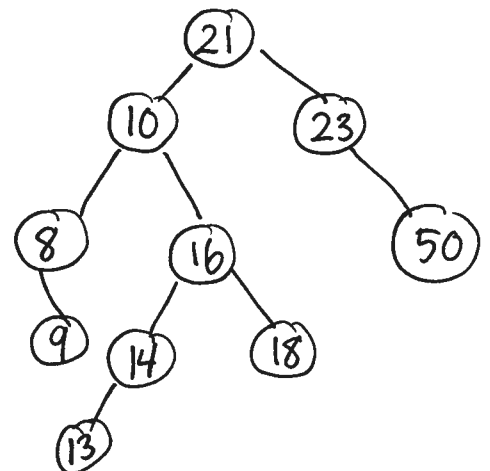
2 **A** delete 11, del 17



B delete 3



C delete 10



Recitation 4

Agenda

Reminders

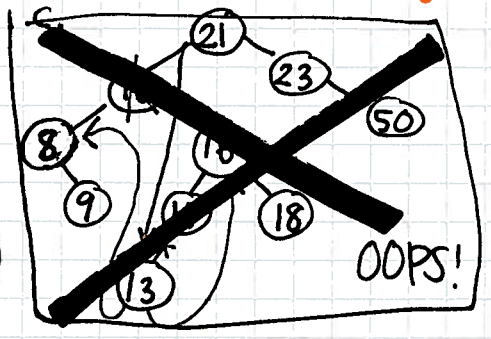
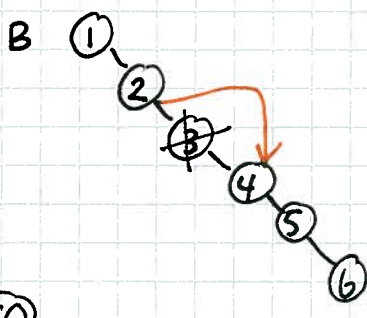
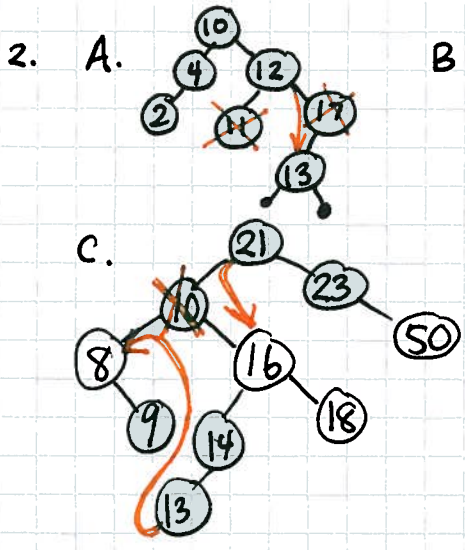
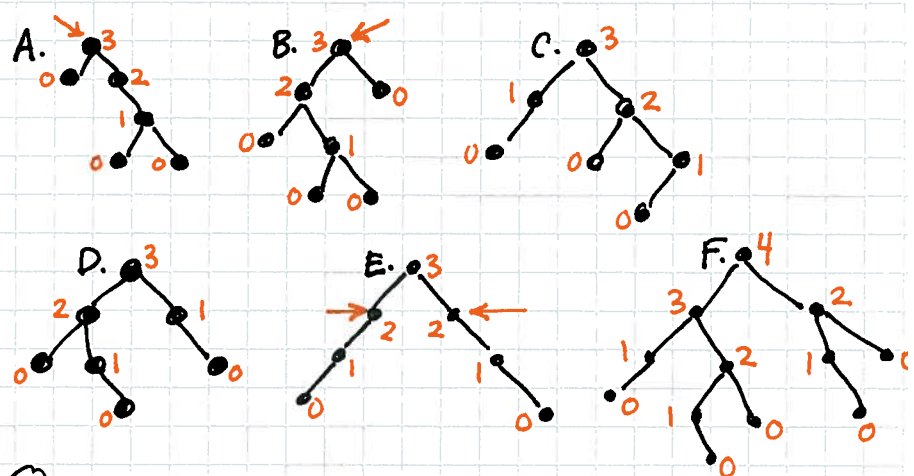
- notes + code posted & will be in future
- pset 1: idea for timing, timeit
- run test code!

- overhead
- Warmup
- AVL
 - recap lecture
 - rotation + rebalance
 - excersizes

there were bugs in code - fixed.

Warmup Answers

	BST	AVL
1. A.	✓	X
B.	✓	X
C.	X	✓
D.	✓	✓
E.	✓	X
F.	✓	✓

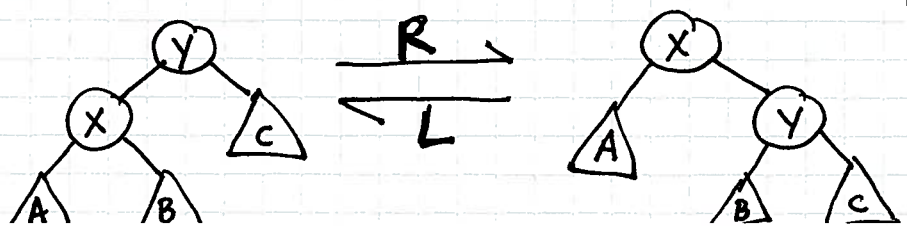


AVL trees

idea: balance trees are $\ddot{\smile}$. ^{constant} pay extra to keep balanced to maintain good order of growth running time for operations.

force: height(left child) and height(right child) to differ by at most 1

how? perform BST operations then rebalance trees through rotations.



only 2 rotations needed to rebalance tree!

Exercises

R04

- 1] write a method to perform left (or right) rotation on a node x .
- you may assume x is a python object w/ fields $x.left$ and $x.right$
 - return the new root to the subtree previously rooted at x .

(2) write a recursive function to determine the height (as defined in class) of a given node.

(3) if I replaced the subtree rooted at x with another arbitrary AVL subtree, which nodes would potentially break the AVL invariant?

(4) when the AVL invariant is broken, why do we correct it starting at the deepest node that violates the invariant rather than working from the root down?

Exercise 1:



```
def right_rot(x)
  y = x.left
  A = y.left
  B = y.right
  C = x.left
  y.right = x
  x.left = B
```

OR

```
y = x.left
x.left = y.right
y.right = x
```

return y

- error checking omitted !!
- similar for left.

Exercise 2:

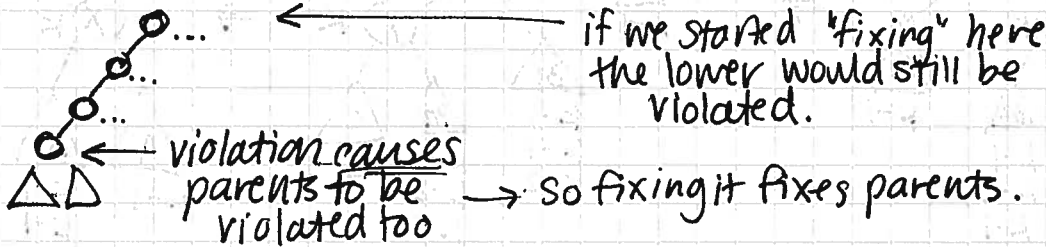
```
def height(x):
```

```
  if leaf(x): return 0
  return max(height(x.right), height(x.left)) + 1
```

ex3:

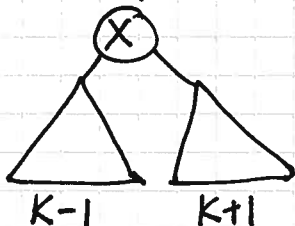
all parents of x may break the invariant

ex4: it takes fewer rotations to fix the tree from the bottom up. We will need to do at most 2, because once the location of the change is made AVL compliant, its parents will be too.



review from lecture: violation fixing.

x violates AVL invariant then

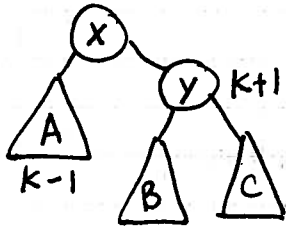


- why only a difference of 2? not 3 or 4?

because we assume the violation occurred b/c of another operation like insert or delete.

uhoh!
use mutable nodes.

so then we break down the tree more:

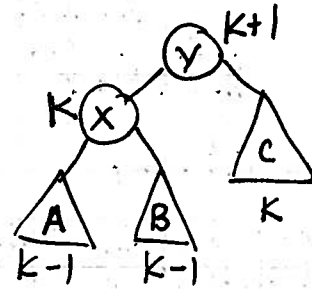


← one must be k to make (Y) have height $k+1$

case 1:

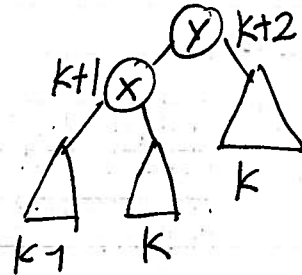
$k-1$	k
k	k
k	$k-1$

⇒ left rotate



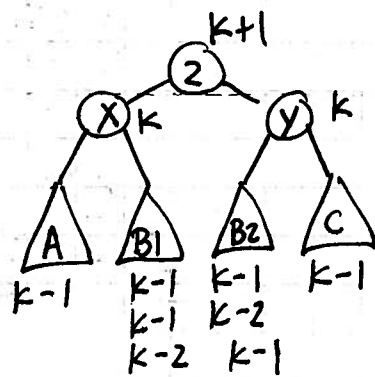
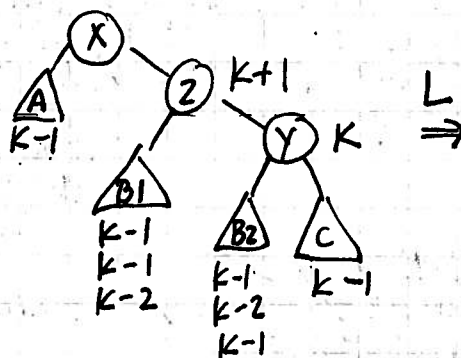
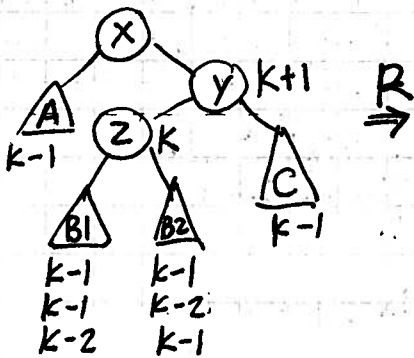
case 2:

⇒ left rotate

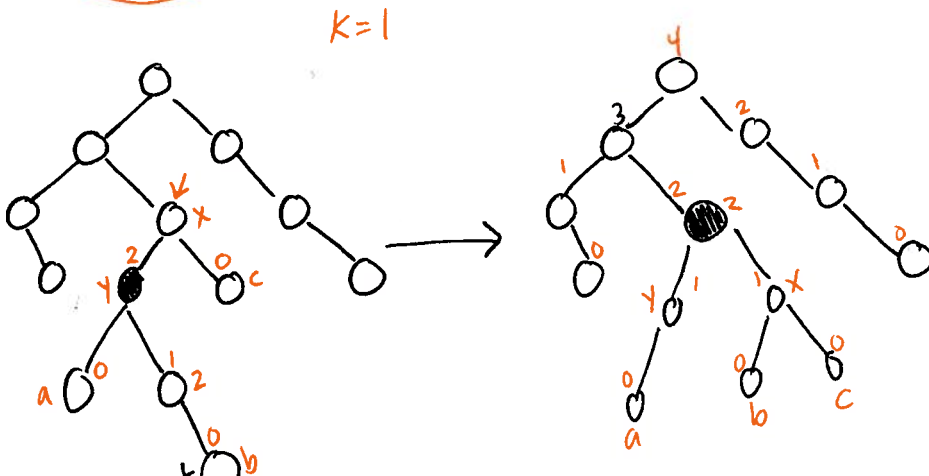
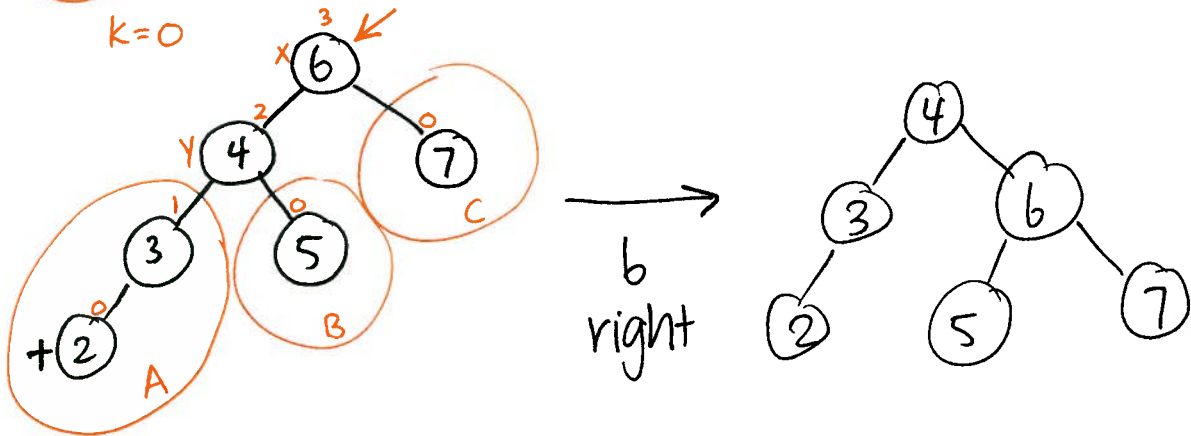
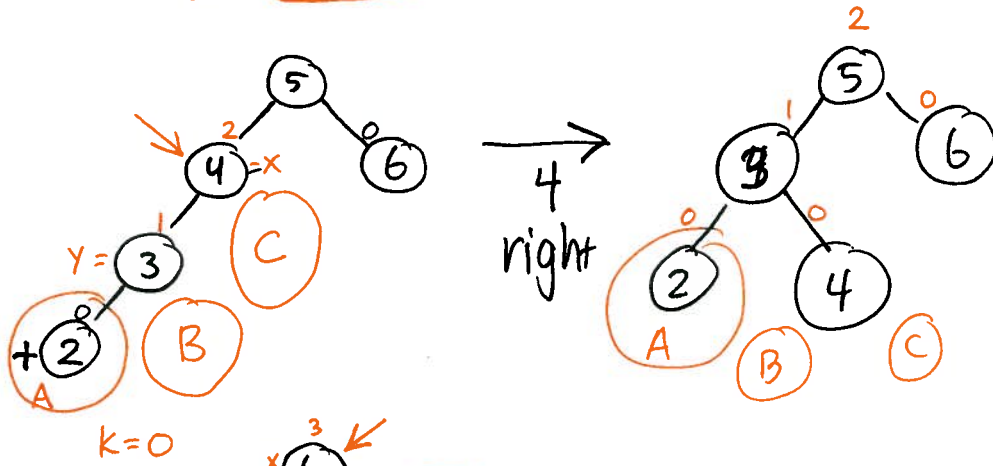
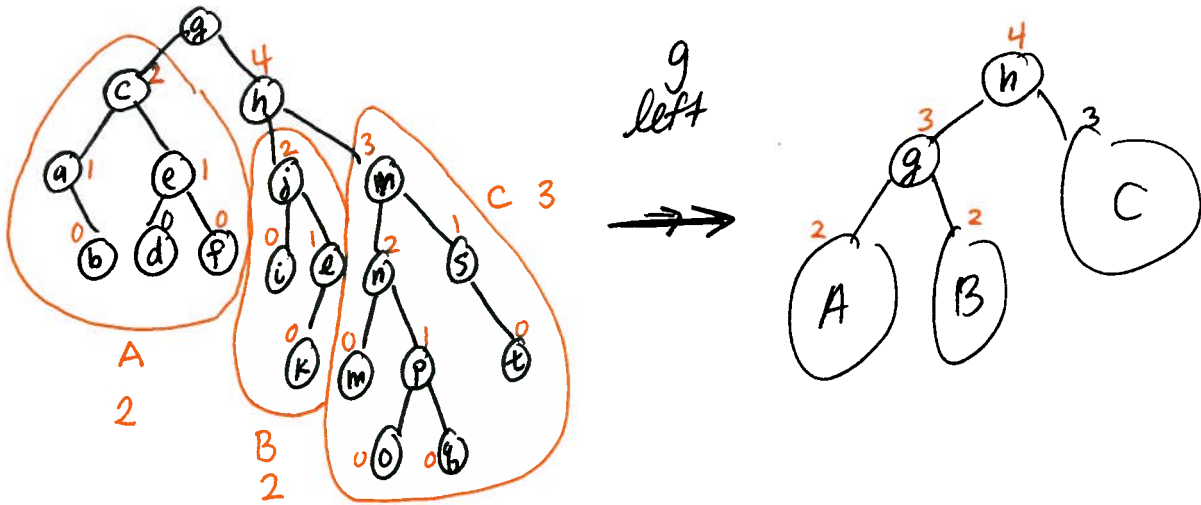


case 3:

⇒ break down further



Answers



oops. 2 steps.
write out each.