6.006
Rivest
L13.1
10/16/08

Admin:

Announce: Talk by me: "Security of Voting Systems", 7pm, 32-123

Reading: CLRS 22.1-22.3, B.4

Outline:
- ☐ intro - graph searching
- ☐ Breadth-first search (BFS)
- ☐ Depth-first search (DFS)

Graph searching:

Given: finite graph $(G=(V,E))$ & start vertex $s \in V$
(assume directed, although need not be)

Explore: visit every vertex reachable from s.

① s is reachable from s.
② if u is reachable from s, and $v \in Adj[u]$
then v is reachable from s
(go to u, then follow edge (u,v))
③ only vertices reachable from s are those provably so by ① & ②.

if u is reachable from s, then
there is a path

$$s \to v_1 \to v_2 \to \cdots \to v_k \to u$$

(of length k+1) leading from s to u. We'll find such paths...

Exploring a graph

Let R = set of vertices known to be reachable from start vertex s

Let Q = vertices known to be reachable from s, which we haven't yet visited (applied ②)

$Q \subseteq R$ always (Q = "frontier")


```
R = set([s])
Q = set([s])                    or  Q = queue([s])
while Q:
    u = Q.dequeue()
    for v in Adj[u]:
        if v not in R:
            R.add(v)
            Q.add(v)
```

"visit u"

now R is set of vertices reachable from s

Q could be FIFO, LIFO, other...

Running time:

- each vertex added to R and Q at most once
- each adjacency list examined at most once
- $\sum_u |Adj[u]| = |E|$ by definition
- running time is $\Theta(V+E)$ <u>linear time</u>

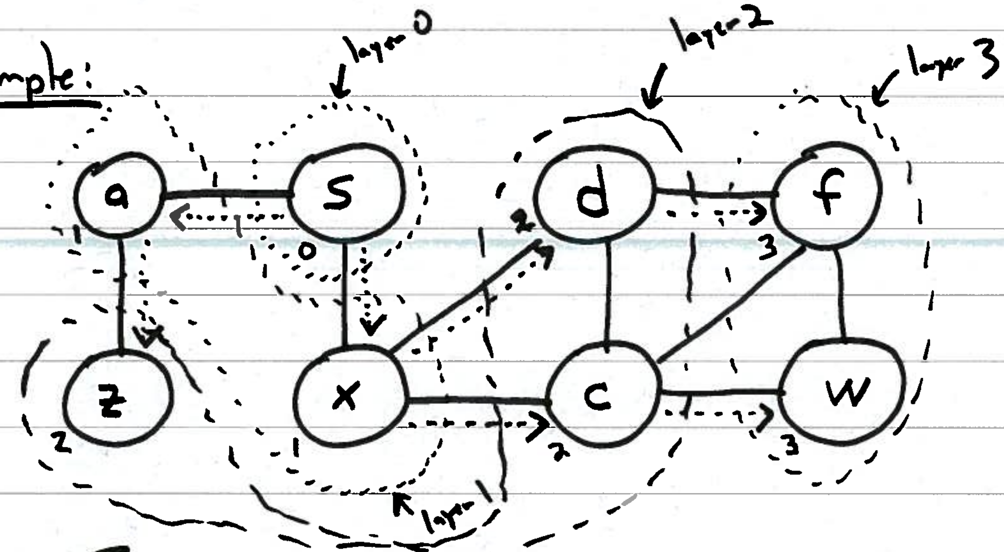
We can keep track of paths, too:

- if v is discovered from u, then call u the "parent" of v.
- Keep track of parents:

$p[s] = \text{nil}$ s has no parent

$p[v] = u$ u is parent of v $u \dashrightarrow v$

Example:



$R = \{s, a, x, z, d, c, f, w\}$

$Q = [s, a, x, z, d, c, f, w]$ (& cross out)

$\dashrightarrow$ edges form a tree, with paths leading to all reachable vertices

(Above search is actually BFS...)

BFS (Breadth-First Search)

- So far, we have been talking about generic search. It finds all reachable vertices (by induction), but we can't say much more...
- By ensuring that Q is FIFO ("first-in first-out"), we obtain BFS.
- BFS enables us to calculate shortest path distance from s to each reachable vertex.

  Let $d[v]$ be this distance.

  Initially, $d[s] = 0$

  When we add v to R, set $d[v] = d[u] + 1$
- (Show on example of page 3)
- Why does this work?
  - we visit all ~~nodes~~ vertices at distance k before any ~~nodes~~ vertices at distance k+1
  - when we visit all ~~nodes~~ vertices at distance k, we discover all vertices at distance k+1 (each vertex at distance k+1 is reachable from some vertex at distance k)
  - can think of BFS as exploring one layer after another

    layer k = all vertices at distance k from s.

  "breadth first" ≈ all of layer k, before any of layer k+1

- BFS finds shortest path from s to each reachable vertex. (Good for Pocket Cube!)
- (We'll generalize BFS later in course, when edges have weights (i.e. lengths).)

- Note: book uses color at each vertex:

  white : unseen

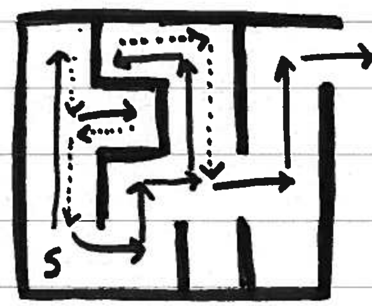
  gray : in Q

  black : R−Q

DFS (Depth-First Search)

- Searches deeper before returning to explore earlier alternatives.
- For undirected graph, corresponds to feasible "physical search" (exploring a maze). With BFS, you need to "teleport" to get to each vertex removed from Q. With DFS, you just retrace your steps (backtrack).
- Can implement DFS by changing Q from FIFO to LIFO, although we won't explore that insight here...

- DFS example:
  - each square a vertex
  - ↑ explore ↓ backtrack



  - follow path until you get stuck
  - backtrack until you reach an unexplored edge; explore it (recursively)
  - careful not to repeat a vertex

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- Code for DFS is very simple

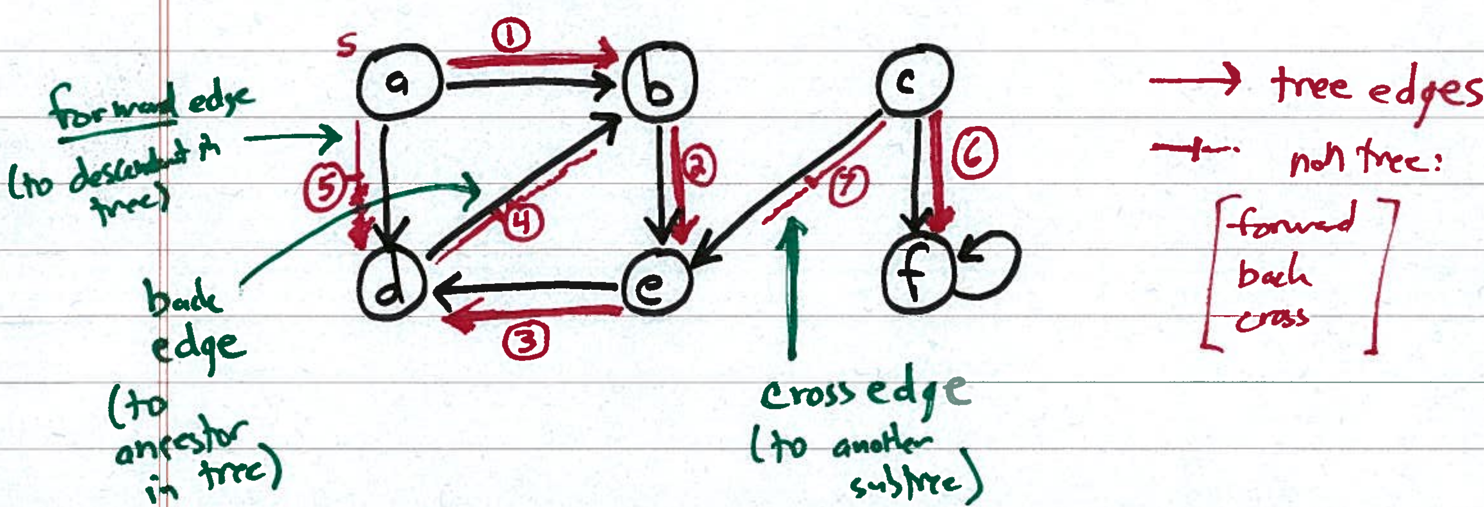
```
def visit(u):
    for v in Adj[u]:
        if v not in R:
            R.add(v)
            visit(v)

R = set([s])
visit(s)
```

or

```
R = set()
for u in V:
    if u not in R:
        R.add(u)
        visit(u)
```

- 2^{nd} version guarantees exploring entire graph...
- Can keep track of each node's parent, as we did for BFS. Get tree of edges leading from s (root) to all vertices reachable from s. Or, a set of trees containing all vertices
- Running time is $O(V+E)$ [each vertex visited at most once]
- Example on directed graph,



- 2 trees generated: one rooted at a, one at c

- Next time: applications of DFS