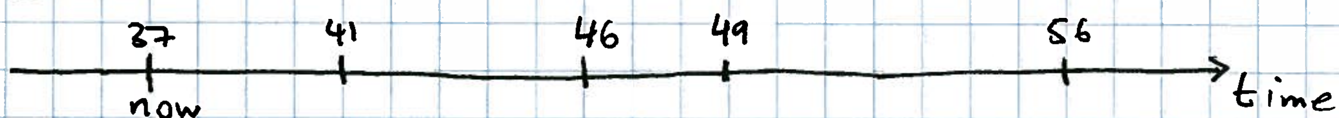
6.006 Lecture 3: Binary Search Trees

- Runway reservation problem
- Binary Search Trees (BSTs), operations
- CLRS Ch 10 + 12.1-3

Runway Reservation System

- Maintains a set R of future landing times
- When a plane lands, remove from the set
- For a request to land at time t:
    - reject request if there are reservations within <3 minutes of t,
    - otherwise add t to set

Example



$R = \{41, 46, 49, 56\}$, $n = |R| = 4$

request for time:

44 reject (too close to $46 \in R$)

53 add reservation (just right)

20 already past, not allowed.

Goal: handle requests & landings in $O(\lg n)$ time.

- Keeping R as an (arbitrarily orded) list
  - Checking a request takes O(n) time
  - Deleting upon landing takes O(n) time

- Keeping R as a sorted list

```
init : R = []
req(t) : if t < now return 'error'
         for i in range(len(R)):                      } O(n
             if abs(t-R[i]) < 3 : return 'error'      }
         R.append (t)
         R.sort()                       ← O(n lg n)
land : t = R[0]
       if (t ≠ now) return 'error'
       R = R[1:]                        ← O(n)
```

- Can we do better with a sorted list?

37	41	46	49	56			

can use binary search to determine conflicts in $O(\lg n)$ but inserting t into R and deleting R[0] still takes O(n)

⇒ better (no sorting) but not enough.

- Indicator representations
  - Represent a set R such that

    $A[t] == \text{True}$ iff $t \in R$

    - Python dictionaries
    - Python sets
    - A Python list of all possible landing times (all possible elements of R)

- Fast insertion, deletion
- Checking a request is fast if landing times are whole minutes, but expensive for hi-res
- Not a flexible data structure:

  How many planes land on or before t?

- [illegible]

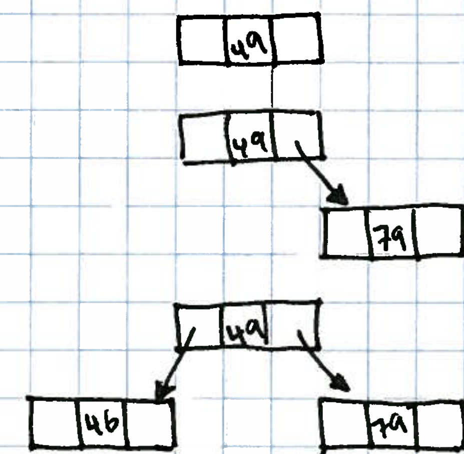
- Binary Search Trees (BSTs)

1. insert(49)

2. insert(79)

~~3. insert~~

3. insert(46)

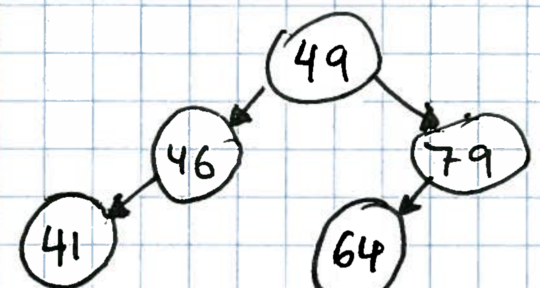


this is the root

to find elements > 4
go right

elements < 49 are
in the left subtree

A more compact graphical representation:



Same representati
in the program:
every node has
a value and
left/right pointers

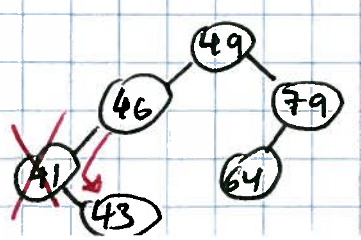
find(~~4~~64): follow left/right pointers until done

find(48): how do you tell that 48 is not in the set?

findmax(): just go right until you can't go right

findmin(): similar, go left

deletemin(): min & max nodes will have zero
or one child ⇒ find, eliminate, & patch



next-larger(t): $v = \text{find}(t)$

if right(v) $\neq$ None : return findmin(right(v))

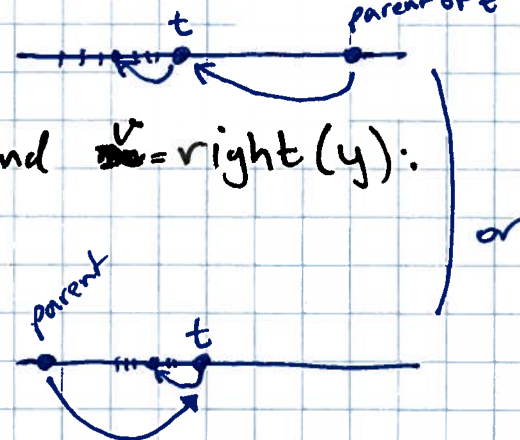
else: $y = \text{parent}(v)$

while $y \neq$ None and $v = \text{right}(y)$:

$v = y$

$y = \text{parent}(v)$

return y

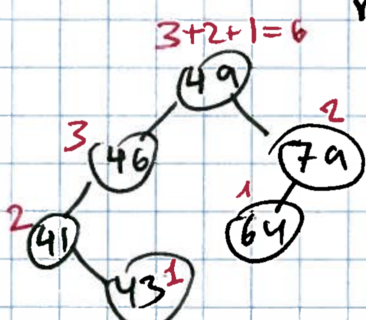


- All of these operations on the BST take $O(h)$ time, where h is the height of the tree

- Other operations that we can do in $O(h)$ time:

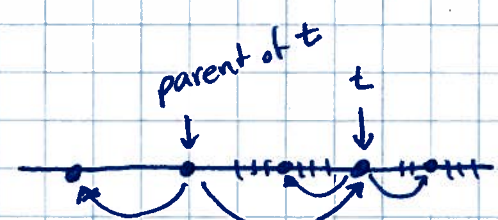
rank(t) how many planes land $\leq t$?

must augment each node with size of subtree, update during insertions & deletions.



delet(t) more tricky if t has 2 children, but still $O(h)$.

(delete next-larger(t) than put it in the node that stored t; see textbook).



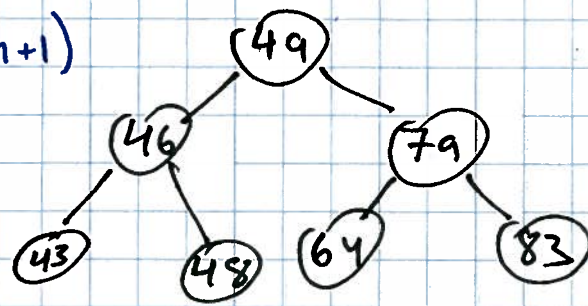
- So how high is a binary tree with n vertices?

- If we're lucky, $h = O(\lg n)$

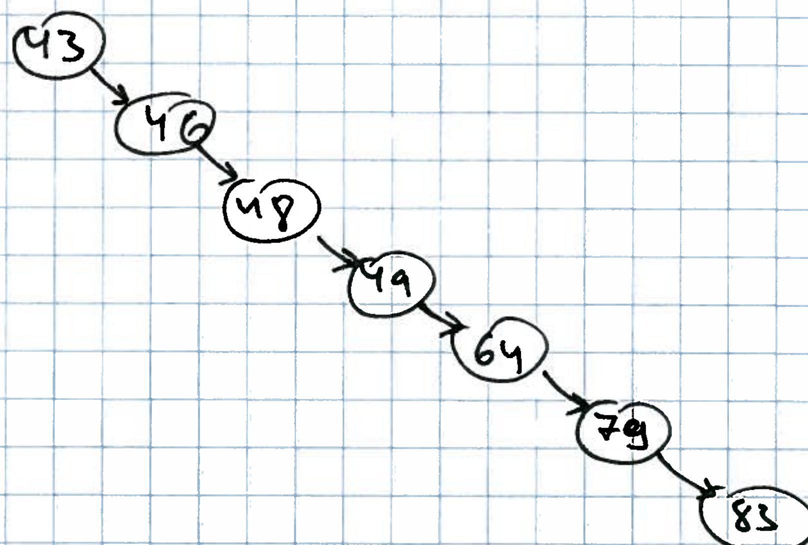
$h = 3 = \lg(n+1)$

or

$n = 2^h - 1$



- But h can be large, $h = n$



```python
# Sivan Toledo, 6.006

#--- Sorted Lists ---

def sortedlist_init():
    return []

def sortedlist_print(R):
    print R

def sortedlist_insert(R,t):
    R.append(t)
    R.sort()
    return R

def sortedlist_deletemin(R):
    return R[1:]

def sortedlist_runway_insert(R,t):
    for i in range(len(R)):
        if abs(t-R[i]) < 3:
            print 'error: ' + str(t) + ' is too close to ' + str(R[i])
            return R
    R.append(t)
    R.sort()
    return R

#--- Binary Search Trees ---

def bst_init():
    return None

def bst_insert(T,t):
    if T == None:
        return [None, t, None]
    if t <= T[1]:
        T[0] = bst_insert(T[0],t)
        return T
    else:
        T[2] = bst_insert(T[2],t)
        return T

def bst_str(T):
    if T == None:
        return ''
    return bst_str(T[0]) + ' ' + str(T[1]) + ' ' + bst_str(T[2])

def bst_print(T):
    print '[ ' + bst_str(T) + ' ]'
```