1. A hash table of size m is used to store n items, with $n < \frac{m}{2}$. Open addressing is used for collision resolution. Assume simple uniform hashing.

   (a) Show that for $i = 1, 2, ...n$, the probability that the ith insertion requires strictly more than k probes is at most 2^{-k}.

   (b) Show that for $i = 1, 2, ...n$, the probability that the ith insertion requires strictly more than $2lgn$ probes is at most $\frac{1}{n^2}$.

   (c) In this scenario, does looking up a key require more (or fewer) probes than inserting a key? Would this change if the table contained $n \leq m/3$ items?

2. A hash table of size m is dynamically resized as items are inserted and deleted such that $m/5 < n < 4m/5$, where n is the number of items in the table. Show that any sequence of k INSERTs and DELETEs on this table requires $\Theta(k)$ time (that is, each INSERT or DELETE operation runs in amortized constant time). Assume simple uniform hashing. Further assume that resizing (shrinking or growing) a table of n elements requires $\Theta(n)$ time. Assume that initially, $n = 2m/5$.

   If there were no initial restriction on n, does our bound of $\Theta(k)$ time for k operations still hold? (What happens if n is initially $(m + 1)/5$ and we perform only $k = 1$ deletion?) How many operations do we have to perform in order to guarantee that our $\Theta(k)$ bound is valid?