Measuring Value at Risk(VaR) of a Portfolio of Interest Rate Derivatives with Monte-Carlo Simulation and Parallel Computing

Mid Term Progress Report

SMA5505 Applied Parallel Computing

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18th April 2002

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1 Motivation and Background

The use of formal mathematical models and optimization in finance became common practice in the 1980s and 1990s. Distributed and parallel computing is playing an increasingly important role in trading, financial engineering, and all aspects of investment and commercial banking, due to a convergence of the following factors:

- Increased volatility due to globalization of financial markets
- Global distribution of data sources
- Increase in complexity of derivatives and other risk management vehicles
- Increased volume of raw data and need to process large databases
- Increased volume on the retail side of the spectrum mainly due to on-line technologies (Internet and WWW)

For our project, we derive motivation from trends leading to pervasive deployment of parallel architectures to support market demands. In particular, we study the potential application of parallel computing concepts in determining Value at Risk (VaR). Before we delve into the details of the project, we present some background on VaR. A glossary of financial terms used in this report is attached for the reader's benefit. Value-at-Risk is an attempt to provide a single number for senior management that summarizes the total risk in a portfolio of financial assets. The VaR calculation is aimed at making a statement of the form:

We are c percent certain not to lose more than V dollars in the next m days.

The VaR is therefore an estimate, with a given degree of confidence, of how much one can lose from one's portfolio over a given time horizon The VaR is usually calculated assuming *normal* market circumstances, meaning that extreme market conditions such as market crashes are not considered. It is widely used by corporate treasurers and fund managers as well as financial institutions.

Another interesting application is in investment evaluation. Here risk is viewed in terms of the impact of the prospective change on the overall value at risk, i.e., incremental VaR, and we go ahead with the investment if the incremental VaR is low enough relative to the expected return.

2 Objective

The Objective of the project is mainly two-fold:

1. Determine the Value at Risk of Portfolios of multi-currency interest rate derivatives, namely, Interest Rate Swaps, Caps and Swaptions for US dollar (USD), Euro (EUR), Japanese Yen (JPY), Swiss Franc (CHF), Deutsche Marks(DEM) and British Pounds (GBP).

2. Use Monte-Carlo Simulation to measure the VaR of the portfolios and compare the performances of Monte-Carlo simulation, Quasi-Monte Carlo using Faure Sequences and Monte-Carlo simulation using parallel computing.

3 Research and Related Work

The approaches to VaR can be essentially divided into two groups. The first is based on local valuation of the financial instruments. The best example from this group is the **variance-covariance** method. This method relies on the assumption of normality of the market risk factors and delta-equivalent presentation of the portfolio. Due to these assumptions it is possible to show that there is an explicit formula for the quantile and, therefore, for VaR. From a computational point of view, this method consists of relatively simple computations that are based on matrix algebra. For example, under this method options are specified by their delta-equivalent representation, i.e., the change of the option is given by the product of delta of the option and the change of the underlying instrument. At-the-money options may exhibit high convexity that is not taken into account by the variance-covariance method.

The second group is based on the simulation approach that performs full valuation of the financial instruments. The best examples are **historical simulation** and **Monte Carlo simulation** methods. The historical simulation method is similar to the Monte Carlo method except that historical changes in prices are used rather than simulated changes based on a stochastic model. Although both methods solve the convexity problem of the variance-covariance approach mentioned earlier, the Monte Carlo method is more flexible and has certain advantages over the historical simulation method. Some of the criticism of the historical simulation method is due to:

- the assumption that the past represents the immediate future, i.e., the distribution is assumed to be stationary.
- the high sensitivity of the results with respect to the length of the time horizon
- the problem with collecting consistent historical data.

3.1 Monte Carlo Simulation

The approach of Monte Carlo VaR is to randomly generate scenarios based upon some assumed joint probability distribution for the risk factors. Historical market data is used to infer statistical characteristics such as volatilities and correlations for the assumed distribution.

For example, for a given portfolio we might assume that each risk factor is log normally distributed with a mean equal to today's value for that factor.

Recent market data would then be analyzed to infer volatility for each risk factor as well as a correlation for each pair of risk factors.

Once an assumed joint distribution is specified, standard techniques for generating correlated random numbers are used to select a set of scenarios. In this way, the selected scenarios are literally drawn from the assumed distribution. They reflect the statistical characteristics-volatilities and correlations-inferred from the historical data.
This approach has four sources of specification error:

1. The standard distribution we assume for risk factors may imperfectly reflect the "true" distribution q.

2. Because we estimate volatilities and correlations from a limited set of historical data, those estimates entail a sampling error

3. Because market conditions are non-stationary (vary over time), the historical data upon which we base volatility and correlation estimates may imperfectly reflect today's market conditions.

4. Imperfections in the random number generator we use for selecting scenarios may introduce a bias.

We can reduce first source of error by basing volatility and correlation estimates on a lot of historical data. We can reduce the second by basing those estimates on only the most recent data. We cannot reduce both the errors.

In addition to specification error, Monte Carlo VaR also entails convergence error. There is, however, no limit to the number of scenarios that can be used with Monte Carlo VAR. Accordingly, this error can be made as small as desired.

3.2 Quasi Monte Carlo Method

The most serious disadvantage of Monte Carlo is its high computational cost. Quasi Monte carlo method is similar to Monte Carlo except that the samples are taken at well-devised deterministic points rather than at random points. The deterministic points are uniformly spread because they belong to low discrepancy sequence. Hence, they help us obtain faster convergence with known error bounds. The use of Quasi-Monte Carlo methods leads to more efficient numerical procedures with the added benefit of deterministic error bounds.

4 System Design

As mentioned in the Objective, the scope of the project covers three different interest rate derivatives, namely: Interest Rate Swaps, Swaptions, and Caps. Each derivative has different base product and risk factors.

Our approach at this stage is to focus on one derivative first, i.e. Interest Rate Swaps. This is because it is simpler and easier to implement as compared to the others. In addition, the design and code for Interest Rate Swaps can be extended easily for other two interest rate derivatives.

4.1 Main Algorithm

We use Monte Carlo simulation out of the three approaches mentioned in the previous section as it is most widely used in empirical finance to estimate the VaR, particularly for derivatives.

4.2 Design Overview

The overview of the design is explained as follows:
The input of the program is:

- A list of historical data of interest rate swaps (e.g. daily interest swaps price starting from year 1990 until 2000).
- Historical information of the currency rate against US dollars in the same period is also needed.
- Time period for Value at Risk measurement (e.g. one week).
- Level of confidence for Value at Risk measurement.

The output of the program is a single number that describes the value at risk of a certain amount of money invested in the portfolio over a certain period of time with a particular level of confidence. The brute-force Monte-Carlo version is presented in figure 1. This algorithm will be parallelized in the remaining duration of the project.

5 Conclusion and Future Work

There were essentially four phases identified in the project proposal for the successful completion of the project. The first phase entailed an understanding of risk management from a computation perspective. In this phase, we scaled the learning curve in finance and familiarized ourselves with various financial concepts that would be used in the project. The second phase that we had identified involved research on seminal work in the realm of risk management from the computational perspective.

In section 2 of this report, we have described briefly the three most popular approaches used to determine VaR. In the third phase we identified our approach to the problem and presented the algorithm that we intend to parallelize. The final phase involves implementing a small-scale prototype parallel solution using C and MPI, to quantify risk management.

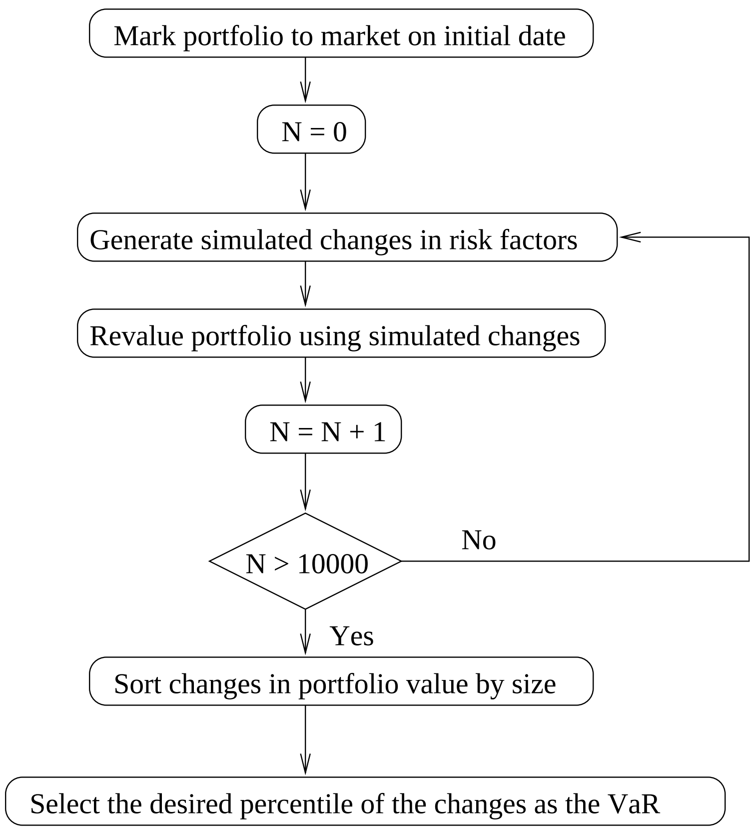

Figure 1: Flowchart to Illustrate Monte Carlo Simulation to Calculate VaR

Acknowledgement

We are very fortunate in having the support and guidance of Associate Professor Ng Kah Hwa, Centre for Financial Engineering Director, and Master of Financial Engineering. Program Director.

Glossary

- **Cap:**
  An upper limit on the interest rate on a floating-rate note (FRN) or an adjustable-rate mortgage (ARM).

- **Derivative instrument:**
  A type of financial instrument that derives its value from the value of other financial instruments.

- **Floor:**
  The price at which a stop order is activated (when the price drops low enough to activate such an order).

- **Interest Rate:**
  The monthly effective interest rate.

- **Interest Rate Swap:**
  A binding agreement between counterparties to exchange periodic interest payments on some predetermined dollar principal, which is called the notional principal amount. For example, one party will pay fixed and receive variable.

- **LIBOR:**
  London Interbank Offered Rate.

- **Portfolio:**
  A collection of investments, real and/or financial.

- **Return:**
  The change in the value of a portfolio over an evaluation period, including any distributions made from the portfolio during that period.

- **Risk:**
  Often defined as the standard deviation of the return on total investment. Degree of uncertainty of return on an asset.

- **Risk Management:**
  The process of identifying and evaluating risks and selecting and managing techniques to adapt to risk exposures.

- **Swap:**
  An arrangement in which two entities lend to each other on different terms, e.g., in different currencies, and/or at different interest rates, fixed or floating.

- **Swaption:**
  Options on interest rate swaps. The buyer of a swaption has the right to enter into an interest rate swap agreement by some specified date in the future. The swaption agreement will specify whether the buyer of the swaption will be a fixed-rate receiver or a fixed-rate payer. The writer of the swaption becomes the counterparty to the swap if the buyer exercises.

- **Value-at-Risk:**
  The probability of portfolio losses exceeding some specified proportion based on a statistical analysis of historical market price trends, correlations, and volatilities.

- **Volatility:**
  A measure of risk based on the standard deviation of the asset return. There are volatility indexes. Such as a scale of 1-9; a higher rating means higher risk.

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