Planar Vertex Cover

[Lichtenstein 1982]

Example: $B = (a + \overline{b} + c)(b + b + \overline{d})$
Planar Connected Vertex Cover

[Garey & Johnson 1977]
Rectilinear Steiner Tree

[Garey & Johnson 1977]
Vertex 3-Coloring
[Garey, Johnson, Stockmeyer 1976]
Vertex 3-Coloring

[Garey, Johnson, Stockmeyer 1976]

clause gadget

variable gadget

colors gadget

\( x_i \)

\( x_j \)

\( x_k \)
Vertex 3-Coloring
[Garey, Johnson, Stockmeyer 1976]
Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]

crossover gadget

[Michael Paterson]
Planar 3-Coloring
[Garey, Johnson, Stockmeyer 1976]

crossover gadget
[Michael Paterson]
Planar 3-Coloring
[Garey, Johnson, Stockmeyer 1976]

crossover gadget
[Michael Paterson]
Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]

crossover gadget

[Michael Paterson]
Planar 3-Coloring, Max Degree 4

[Garey, Johnson, Stockmeyer 1976]

high-degree gadget
Planar 3-Coloring, Max Degree 4

[Garey, Johnson, Stockmeyer 1976]

high-degree gadget
## Pushing 1 × 1 Blocks Complexity

<table>
<thead>
<tr>
<th>Name</th>
<th>Push</th>
<th>Fixed</th>
<th>Slide</th>
<th>Goal</th>
<th>Complexity</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Push-(k)</td>
<td>(k \geq 1)</td>
<td>no</td>
<td>min</td>
<td>path</td>
<td>NP-hard</td>
<td>D, D, O’Rourke 2000</td>
</tr>
<tr>
<td>Push-(*)</td>
<td>(\infty)</td>
<td>no</td>
<td>min</td>
<td>path</td>
<td>NP-hard</td>
<td>Hoffmann 2000</td>
</tr>
<tr>
<td>Push-Push-(k)</td>
<td>(k \geq 1)</td>
<td>no</td>
<td>max</td>
<td>path</td>
<td>PSPACE-complete</td>
<td>D, Hoffmann, Holzer 2004</td>
</tr>
<tr>
<td>Push-(*)F</td>
<td>(\infty)</td>
<td>yes</td>
<td>min</td>
<td>path</td>
<td>NP-hard</td>
<td>Hoffmann 2000</td>
</tr>
<tr>
<td>Push-1(F)</td>
<td>1</td>
<td>yes</td>
<td>min</td>
<td>path</td>
<td>NP-hard</td>
<td>DDO 2000</td>
</tr>
<tr>
<td>Push-(k)F</td>
<td>(k \geq 2)</td>
<td>yes</td>
<td>min</td>
<td>path</td>
<td>PSPACE-complete</td>
<td>D, Hearn, Hoffmann 2002</td>
</tr>
<tr>
<td>Push-(*)F</td>
<td>(\infty)</td>
<td>yes</td>
<td>min</td>
<td>path</td>
<td>PSPACE-complete</td>
<td>Bremner, O’Rourke, Shermer 1994</td>
</tr>
<tr>
<td>Push-(k)X</td>
<td>(k \geq 1)</td>
<td>no</td>
<td>min</td>
<td>simple path</td>
<td>NP-complete</td>
<td>D, Hoffmann 2001</td>
</tr>
<tr>
<td>Push-(*)X</td>
<td>(\infty)</td>
<td>no</td>
<td>min</td>
<td>simple path</td>
<td>NP-complete</td>
<td>Hoffmann 2000</td>
</tr>
<tr>
<td>Sokoban</td>
<td>1</td>
<td>yes</td>
<td>min</td>
<td>storage</td>
<td>PSPACE-complete</td>
<td>Culberson 1998</td>
</tr>
</tbody>
</table>
Planar Euler Tours

[Demaine, Demaine, Hoffmann, O’Rourke 2003]
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O’Rourke 2003]
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O’Rourke 2003]
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O’Rourke 2003]
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O’Rourke 2003]
Push-1G is NP-complete
[Friedman 2002]

fork

one way

XOR crossover

NAND
Graph Orientation
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]
Packing L Trominoes into Polygon
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

edge gadget
Packing L Trominoes into Polygon

[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

crossover
Packing L Trominoes into Polygon

[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]
Packing I Trominoes into Polygon

[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]

crossover

dgadget
Packing I Trominoes into Polygon
[Horiyama, Ito, Nakatsuka, Suzuki, Uehara 2012]
<table>
<thead>
<tr>
<th>Problem</th>
<th>NP-complete</th>
<th>Reference</th>
</tr>
</thead>
</table>
| **Bandwidth** | in general  
for trees with maximum degree 3  
for caterpillars with hair-length $\leq 3$  
for caterpillars with $\leq 1$ hair per backbone vertex  
for cyclic caterpillars with hair-length 1  
for grid graphs and unit disk graphs | [Papadimitriou 1976]  
[Garey et al. 1978]  
[Monien 1986]  
[Monien 1986]  
[Muradyn 1999]  
[Díaz et al. 2001a] |
| **MinLA**  | in general  
for bipartite graphs | [Garey et al. 1976]  
[Even and Shiloach 1975] |
| **Cutwidth** | in general  
for graphs with maximum degree 3  
for planar graphs with maximum degree 3  
for grid graphs and unit disk graphs | [Gavril 1977]  
[Makedon et al. 1985]  
[Monien and Sudborough 1988]  
[Díaz et al. 2001a] |
| **ModCut** | for planar graphs with maximum degree 3 | [Monien and Sudborough 1988] |
| **VertSep** | in general  
for planar graphs with maximum degree 3  
for chordal graphs  
for bipartite graphs  
for grid graphs and unit disk graphs | [Lengauer 1981]  
[Monien and Sudborough 1988]  
[Gustedt 1993]  
[Goldberg et al. 1995]  
[Díaz et al. 2001a] |
| **SumCut** | in general  
for cobipartite graphs | [Díaz et al. 1991]  
[Lin and Yuan 1994b]  
[Golovach 1997]  
[Yuan et al. 1998] |
| **EdgeBis** | in general  
for graphs with maximum degree 3  
for graphs with maximum degree bounded  
for $d$-regular graphs | [Garey et al. 1976]  
[MacGregor 1978]  
[MacGregor 1978]  
[Bui et al. 1987] |
Bipartite Crossing Number

[Garey & Johnson 1983]

\(|E|^2\)
Crossing Number is NP-Complete

[Garey & Johnson 1983]

$3k + 1$
How To Solve Rubik's Cube Faster
[Demaine, Demaine, Eisenstat, Lubiw, Winslow 2011]

• Kill $\Theta(\log n)$ birds with $\Theta(1)$ stones
• Look for cubies arranged in a grid that have the same solution sequence
  ▪ $X \times Y$ grid can be solved in $\Theta(X + Y)$ moves instead of the usual $\Theta(X \cdot Y)$ moves
  ▪ Can always find $\Theta(\log n)$-factor savings like this
Optimal Rubik’s Cube Solutions

[Demaine, Demaine, Eisenstat, Lubiw, Winslow 2011]

- NP-hard to solve a specified subset of $n \times n \times 1$ “Rubik’s Square” using fewest possible moves

- **Open**: NP-hard if all cubies are important?

[Erickson 2010]