Icosian Game  [Hamilton 1857]
Planar (Directed) Hamiltonian Cycle

[ Lichtenstein 1982 ]

\[ a \lor b \lor c \]
Planar Directed Max-Degree-3

[Planar Directed Max-Degree-3]

clause gadget

XOR gadget

$$(x_1 \lor \bar{x}_2) \land (x_1 \lor x_2 \lor x_3) \land (x_2 \lor \bar{x}_3)$$
Planar Directed Max-Degree-3

[Plesník 1979]
Planar Bipartite Max-Degree-3

[Itai, Papadimitriou, Szwarcfiter 1982]
Grid Graphs

[Itai, Papadimitriou, Szwarcfiter 1982]

figure by Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009
Planar Bipartite Graph Drawing
[Itai, Papadimitriou, Szwarcfiter 1982]
Hamiltonicity in Grid Graphs
[Itai, Papadimitriou, Szwarcfiter 1982]

edge gadget
Hamiltonicity in Grid Graphs
[Itai, Papadimitriou, Szwarcfiter 1982]

vertex gadget

vertex-edge connections
Hamiltonicity in Grid Graphs
[Itai, Papadimitriou, Szwarcfiter 1982]
Hamiltonicity in Grid Graphs

[Itai, Papadimitriou, Szwarcfiter 1982]
Max-Degree-3 Grid Graphs

[Papadimitriou & Vazirani 1984]
Max-Degree-3 Grid Graphs

[Papadimitriou & Vazirani 1984]

vertex gadget

vertex-edge connections
Max-Degree-3 Grid Graphs

[Panadimitriou & Vazirani 1984]

forced-edge vertex-edge connections
solid  Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009  polygonal
Hamiltonicity in Triangular Grid Graphs

Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009
Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009

vertex gadget

edge gadget

vertex-edge connections
turn gadget

vertex-edge connections
Hamiltonicity in Hexagonal Grid Graphs

Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009
Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009
vertex gadgets

Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009
Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009
The SETTLERS of CATAN

> 15 million sold

[Klaus Teuber 1995]
Settlers of Catan Mate-in-1 is NP-Complete [Demaine, van Eyckke, McKay 2011]

- Reduction from Hamiltonicity in hex grids
  [Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2008]

× ∞ (buys roads)

2 VP to win

opponent has road of n – 1

opponents form obstacles
Settlers of Catan Mate-in-0 is NP-Complete [Demaine, van Eyck, McKay 2011]

- Reduction from Hamiltonicity in hex grids [Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009]

In fact, deciding whether you’ve already won is NP-complete!

2 VP to win

opponent has road of $n - 1$
Slitherlink  [Nikoli 1989]
Slitherlink is NP-complete

optional vertex

required vertex

[Yato 2000]
Slitherlink is NP-complete

[Yato 2000]
Hashiwokakero is NP-Complete

[Andersson 2009]
Lawn Mowing
Laser Cutting

George Hart at MIT, 2003
3D Printing
Milling & Lawn Mowing
[Arkin, Fekete, Mitchell 2000]
Hamiltonicity in Unit Orthogonal Segment Intersection Graphs

[Arkin, Bender, Demaine, Fekete, Mitchell, Sethia 2005]
Minimum-Turn Milling

[Arkin, Bender, Demaine, Fekete, Mitchell, Sethia 2005]