Planar 3SAT:
- NP-hard special case of 3SAT
- variable-clause bipartite graph is planar
  - edge \((v_i, c_j)\) whenever \(v_i\) or \(\overline{v}_i\) is in \(c_j\)
- remains planar after connecting variables
  in a cycle: \(v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_n \rightarrow v_1\)
  - OR after connecting variables & clauses
    in a cycle \([\text{Lichtenstein-SICOMP 1982}]\)
- remains planar if we require \(v_i\)'s positive
  connections separated from negative connections
  i.e. split \(v_i\) into \(v_i\) \(\overline{v}_i\)
  positive connections negative connections
  \([\text{Dyer & Frieze 1986}]\)
- remains planar if we require all positive
  connections on one side of cycle & negative
  connections on other side \(\Rightarrow\) monotone 3SAT
  \([\text{De Berg & Khosravi - CoCoon 2010}]\)
- reductions from 3SAT
Planar rectilinear 3SAT: (essentially Lichtenstein 1982)
- variable = horizontal segment on x axis
- clause = horizontal segment (off x axis) + 3 vertical connections to variables
- no crossings/overlap (other than connections)

Planar monotone rectilinear 3SAT: as above
+ monotone 3SAT: each clause all positive or all negative
+ positive clauses above x axis
+ negative clauses below x axis
[de Berg & Khosravi - COCOON 2010]
- reduction from planar rectilinear 3SAT

Careful:
- if all clauses on one side of variable cycle (above x axis in planar rectilinear 3SAT)
then EP via tree dynamic program
⇒ if clauses also connected in a path
then EP (would force clauses on same side)
(wanted this e.g. for Push-1/Nintendo)
Planar 1-in-3SAT: [Dyer & Frieze 1986]
- NP-hard special case of 1-in-3SAT
- variable-clause bipartite graph is planar
+ remains planar after connecting variables in a cycle: $v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_n \rightarrow v_1$
- OR after connecting variables & clauses in a cycle

Reduction from Planar 3SAT:
- clause gadget

Planar positive 1-in-3SAT: no negations [Mulzer & Rote-J.ACM 2008]
+ remains planar after connecting variables in a cycle: $v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_n \rightarrow v_1$

Rectilinear ...
- variable = horizontal segment on x axis
- clause = horizontal segment (off x axis)
  + 3 vertical connections to variables

Reduction from Planar 3SAT:
- equal & not-equal gadgets
- remove negations
- expand clauses (2 cases: $u=0$ or $1$)
Careful: Planar NAE 3SAT is polynomial!  
[Moret - SIGACT News 1988]

Reduction to Planar Max Cut: 2-color vertices of planar graph to maximize red-blue edges
\[ s \in P \]  
[Orlova & Dorfman 1972]  
(in dual, red-blue edges are non-doubled edges in Chinese Postman problem)
- variable gadget / wire
- NAE clause

Planar X3C:  
[Dyer & Frieze 1986]
- bipartite graph of elements vs. 3-sets is planar
- reduction from planar 1-in-3SAT

Planar 3DM:  
[Dyer & Frieze 1986]
- special case where elements are 3-colored & each 3-set is trichromatic
- remains planar if elements connected in cycle
- reduction from planar 1-in-3SAT
Planar vertex cover:
- given a planar graph
- choose \( k \) vertices to hit all edges
- reduction from planar 3SAT
  - variable gadget: even cycle
  - clause gadget: triangle
- maximum degree 3

Planar (directed) Hamiltonian cycle:
- reduction from planar 3SAT
  - visit cycle through variables
  - variable gadget = ladder
  - clause gadget
  - can't jump var. \( \rightarrow \) clause \( \rightarrow \) other var.
- same reduction claimed for undirected

Shakashaka
- reduction from Planar 3SAT

Flattening fixed-angle chains:
- reduction from Partition
  [Soss & Toussaint 2000]
- reduction from planar monotone rectilinear 3SAT
  [Demaine & Eisenstat 2011]