2 ways to represent variables in 3SAT:

1) Dual-rail logic:
   - variable gadget forces exclusive OR of 2 "semi-wires" (true & false)
   - semiwire connects to clauses of variable (active only when chosen)

   (e.g. Nintendo, pushing blocks, Phutball - most 3SAT reductions we've seen)

2) Binary logic: (not just Circuit SAT)
   - wire gadget has 2 (types of) solutions
   - split gadget to make copies of wire (e.g. flat-foldable crease patterns)

   - Circuit SAT also needs terminator gadget to start a variable wire

→ in both cases, may need
   - turn gadget to route (semi)wires
   - crossover gadget to cross (semi)wires
Akari/Light Up: [Nikoli 2001]
- given square grid with some obstacles
- some obstacles have a number
  → how many (0-4) edge-adjacent lights
- light illuminates like rook, up to obstacles
- goal: place lights in blanks so that
  - black space lit
  - no lights light each other
  - satisfy numbers

NP-complete by reduction from Circuit SAT: [McPhail 2005]
- wire, turn gadgets
- split/negation gadget
  → split & negation gadgets (via terminators)
- OR/XNOR gate
- crossover gadget: just XORs!
**Minesweeper:** given square grid of numbers & unknowns & possibly mines

**Consistency:** does there exist a solution?  
- e.g. see whether mine at x is consistent with (consistent) info so far: if not, play x  
  → special case of interest

*NP-complete* by reduction from Circuit SAT  
  - wire, terminator  
  - split/NOT/turn  
  - phase changer (shift by 2) via 2 NOTs  
  - AND  
  - crossover gadget: just use NANDs!

Kaye 2000

Goldschläger 1977
Winning: can I force a win? (no guessing)  
i.e. figure out all squares?  
[Hearn 2006]

Inference: can I figure out any squares?  
[Scott, Stege, van Rooij 2011]

$\in \text{CoNP}$: proof of NO = 2 differing solutions

**CoNP-complete** by reduction from  
**Circuit UNSAT**: $\exists x_1, x_2, \ldots, x_n$ s.t. $f(\bar{x})$  
$\equiv \forall x_1, x_2, \ldots, x_n$: $\neg f(\bar{x})$

- wire, turn, terminator
- NOT, OR, shifter
- split
- crossover: just use NORs!

- special care to ensure equal # mines in all cases (# mines part of puzzle) & ports aligned (middle of 3)

- unsatisfiable $\iff$ output forced to be F

**Planar Circuit SAT**: given noncrossing circuit  
- only NAND or - only NOR  
  (& splitters)  

[new?]
Candy Crush / Bejeweled (perfect information)
- given square grid of colors (among 6
- move = swap two edge-adjacent squares
- whenever 3 equal colors in a row/column:
  3 squares disappear & columns fall
  $\leftrightarrow$ “pop”

NP-complete to get $p$ points with $k$ moves by reduction from 3SAT
... in model where pops happen sequentially bottom to top

**Diagram:**
- Reward
- Clause
- Positive & negative wires
- Connectors
- Each clause duplicated $m$ times
- $\#\text{clauses}$

- Claim: worse to trigger wire (even $x$ & $\overline{x}$) directly
- only use 5 colors

[Walsh 2014]
NP-complete with simultaneous pops by reduction from 1-in-3SAT
- works for many goals:
  - $p$ points in $k$ moves
  - $p$ points
  - pop $p$ gems
  - $p$ moves
  - pop a specific gem