Manipulation: Mechanisms, Grasping and Inverse Kinematics

RSS Lectures 14 & 15
Monday / Wednesday, 31 March / 2 April 2014
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MIT HKN is hosting another study break TODAY with cupcakes from Georgetown Cupcakes! 3pm until we run out at the Chu Lounge (38-201).

Force-Direction Closure

- Under what conditions will a set of point contact forces resist arbitrary planar translation?

... What's going on?
How many contacts are needed?

• Analyze situation in c-space with DOF argument
  – First: how many c-space DOFs for object origin?

Contact A

How many contacts are needed?

• Analyze situation in c-space with DOF argument
  – What does a Cartesian point contact imply in c-space?
How many contacts are needed?

- Analyze situation in c-space with DOF argument
  - What does a Cartesian point contact imply in c-space?
**DOF Counting for Translation**

- Conclude that **3 contacts are needed in general**
  - Are there situations in which more are required?

  - Example of degeneracy / degenerate conditions

  Cartesian space

  Configuration space

**Conditions for Force-Direction Closure**

- **Force vectors must**
- **Some positive combination of forces**

  Algebraic condition?
  For force vectors $p, q, r$, there must exist $\alpha, \beta, \gamma > 0$ s.t.
Synthesizing a Force-Direction Grasp

1. Choose contact edges admitting a force center
2. Project force center onto per-edge contact points
3. Scale force magnitudes to produce zero net force

Bottom line: given a polygonal shape, compute a (pinning, frictionless) three-fingered grasp

Torque Closure

- Under what conditions will a set of point contact forces resist arbitrary planar rotations?

... What’s going on?
How many contacts to “pin” rotation?

- Use analogous DOF argument in c-space
  - First: how many c-space DOFs for object pose?

How many contacts to pin rotation?

- Introduce point contact in Cartesian space
  - Implies c-space constraint with 2D manifold boundary

Cartesian space

Configuration space
How many contacts to pin rotation?

• Introduce point contact in Cartesian space
  – Implies c-space constraint with 2D manifold boundary

• Locally, each constraint has a planar boundary
  – ... So, how many halfspaces needed to pin point?
Are There Degeneracies?

- Polygon with sides not in general position…
- Might we need more? What about circles?
- For polyhedra in 3D: need 7 contacts (6 DOF + 1)
  - Frictionless contacts cannot pin

Conditions for Torque Closure

- Each normal cone must contain the other’s apex
- Pairwise effective forces must cancel each other

Algebraic condition? For force vectors $f_1$, $f_2$, $f_3$, $f_4$, there must exist $\alpha$, $\beta$, $\gamma$, $\delta > 0$ s.t.

(Notation as in Nguyen 1986)
Synthesizing a Torque-Closure Grasp

1. Choose two edge pairs* admitting force centers
2. Choose centers inducing mutual normal cones
3. Project centers to respective edge contact points
4. Scale forces to produce alignment, cancellation

*Edge pairs need not be contiguous

Does rotation closure imply translation closure?

Kinetic and Static Friction (“Stiction”)

\[ F_f \leq \mu_s \cdot F_n \text{ (at rest): coefficient of static friction } \mu_s \]
\[ F_f \leq \mu_k \cdot F_n \text{ (moving): coefficient of kinetic friction } \mu_k \]

(Stiction makes things difficult both for humans and robots. Why?)
Point Contact with Friction

• Consider a point contact exerting force $F$ at an angle $\theta$ to the surface normal. What happens?

For contact at rest,

$$|F| < |F_t| = \mu |F_n|$$

At critical angle $\theta_{\text{crit}}$,

$$|F| =$$

Substituting gives

$$|F| \sin \theta_{\text{crit}} =$$

Which yields

$$\mu =$$

So that

$$\theta_{\text{crit}} =$$

• Produces a cone of directions, s.t. point will not slide when $F$ is applied.

Grasp Analysis With Friction

Consider forces $f_1, f_2$ at frictional contacts $p_1, p_2$

When can $f_1, f_2$ oppose one another without sliding?

Each force must

Point $p_1$ (resp. $p_2$) must
Grasp Synthesis With Friction

Choose a compatible pair of edges $e_1, e_2$

Intuition? Using what data? How to choose?

Grasp Synthesis With Friction

Choose target region for contact point $p_1$
Determine feasible target region for contact $p_2$
Orient and scale $f_1, f_2$ so as to cancel along $p_1p_2$
Forward and Inverse Kinematics

- So far, have cast computations in Cartesian space
- But manipulators controlled in configuration space:
  - Rigid links constrained by joints
  - For now, focus on joint values
- Example 3-link mechanism:
  - Joint coordinates $\theta_1, \theta_2, \theta_3$
  - Link lengths $L_1, L_2, L_3$
- End effector coordinates
  - “Reference pose” described by $x, y,$ and $\phi$ (w.r.t. vertical)
- How can we relate EE to configuration variables?

Forward Kinematics

- Given mechanism description and joint values, express end effector pose in Cartesian coordinates
  - Example: two-link arm with one sliding, one rotating joint
- Configuration variables:
  - Joint coordinates $d, \theta$
  - Link lengths (both 1)
- End effector coordinates
  - “Reference point” ($x, y$)
- Challenge: express as
  \[
  x = x(d, \theta) = \ldots
  \]
  \[
  y = y(d, \theta) = \ldots
  \]
Inverse Kinematics

• Given end effector pose in Cartesian coordinates, identify the joint values that yield the desired pose

• Challenge: solve for joint values in terms of pose

\[ \theta = \theta (x, y) \]
\[ d = d (x, y) \]

Hints:

\[ x = 1 + \cos \theta \]
\[ y = d + \sin \theta \]
\[ \cos^2 \theta = (x-1)^2 \]
\[ \sin^2 \theta = (y-d)^2 \]

1 = (x-1)^2 + (y-d)^2

Why is IK difficult?

• Nonlinear
  – Revolute joints \( \rightarrow \) inverse trigonometry

• Multi-valued
  – Often multiple solutions for a single Cartesian pose

• Discontinuities and singularities
  – Can lose one or more DOFs in some configurations

• Possibly over-constrained (no exact solution)
  – Use of approximation and iterative algorithms

• Dynamics
  – In reality, want to apply forces and torques (while respecting physical constraints), not just move arm!
Putting it All Together: Grasping

- Input workspace, obstacles, and manipuland:
  - Determine a feasible grasp (set of contact points)
  - Use IK to solve for target end-effector pose in c-space
  - Plan a collision-free reach to the computed pose
  - Control end-effector along desired trajectory

What have we swept under the rug?

- Sensing
  - Shape, pose of target object, accessibility of surfaces
  - Classification of material type from sensor data
  - Freespace through which grasping action will occur

- Material properties
  - Estimation of object’s mass, moments, friction coefficients
  - Internal, articulated, passive vs. active degrees of freedom

- Uncertainty & compliance
  - Tolerate noise inherent in sensing and actuation
  - Ensure that slight sensing, actuation errors won’t cause damage
  - Handle soft fingers making contact over a finite (not zero) area

- Dynamics
  - All of the above factors may be changing in real time