Problem 1: These are the key concepts from lecture this week:

1. Counter and Stack Machines - We can simulate a Turing machine computation with two stacks, can simulate two stacks with three counters and three counters with two. Reference for this material: Hopcroft, Motwani and Ullman.


3. Rice’s Theorem

Problem 2:(Rice’s Theorem)
Classify each of the following problems as either

- (D) decidable,
- (R) recognizable but not decidable,

and indicate which undecidable examples follow from Rice’s Theorem. ¹

1. $E_{NFA}$, the Equivalence problem for NFA’s.

2. $\{ M \mid M$ is a Turing machine that accepts at least 42 different strings $\}$.

3. $\{ M \mid M$ is a Turing Machine that has at least 42 states $\}$.

4. $\{ M \mid M$ is a Turing Machine that runs for at least 42 steps when started with a blank input tape $\}$.

5. $\{ M \mid M$ accepts the string 01 in a perfect square number of steps $\}$.

6. $\{ M \mid L(M)$ is recognized by a Turing Machine that has an even number of states$\}$.

Problem 3:(Mapping Reducibility)
Answer the following True or False:

1. $E_{TM}$ is mapping reducible to $A_{TM}$.

2. $A_{TM} \leq_m 0^*1^*$.

Problem 4:(Pebble Machine)
A “pebble machine” is a TM with two tapes – an input tape and a work tape. The input tape contains the input string, and is read-only. The machine cannot write on the work-tape either!! But, it has three distinguishable pebbles which can be placed anywhere on the work tape, and moved around. The machine can determine when two pebbles are adjacent to each other. How powerful is this machine?

¹Check your answers from the back of this handout
**Problem 2 Solutions:**

1. D; recall the $E_{Q_{DFA}}$ algorithm.

2. R; Undecidable by Rice’s Theorem; Recognizable by running the TM in parallel using a dove-tailing kind of trick.

3. D; This is a simple check, given a machine’s description. Rice doesn’t apply because this is not a language property.

4. D; just simulate $M$ for up to 42 steps. Rice doesn’t apply because this is not a language property.

5. R; Undecidable, but not by Rice’s theorem. Recognizable, trivially. Just run the machine on input #/.

6. D; This is a trivial language property.

**Problem 3 Solutions:**

1. False; $A_{TM}$ is recognizable, $E_{TM}$ is not. See Corollary 5.17.

2. False; $0^{*}1^{*}$ is decidable, $A_{TM}$ is not. See Theorem 5.16.