PROOFS

Getting started:
Pythagorean theorem

\[ a^2 + b^2 = c^2 \]

Familiar? Yes!
Obvious? No!

A Cool Proof

Rearrange into:
(i) a \( c \times c \) square, and then
(ii) an \( a \times a \) & a \( b \times b \) square
A Cool Proof

Proof by Picture

- elegant and correct
  -- in this case
- worrisome in general
  -- hidden assumptions
Bogus Proof: Getting Rich By Diagram

A False Proof: Getting Rich By Diagram

The bug:

Profit!

Getting Rich

The bug:

So the top and bottom line of the "rectangle" is not straight!
Another Bogus Proof

Theorem: Every polynomial, \( ax^2 + bx + c \)
has two roots over \( \mathbb{C} \).

Proof (by calculation). The roots are:
\[
r_1 := \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 := \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\]

Counter-examples:
The bug: divide by zero error
The fix: require \( a \neq 0 \)

1x^2 + 0x + 0 has 1 root.
The bug: \( r_1 = r_2 \)
The fix: require \( D \neq 0 \) where
\[
D := b^2 - 4ac
\]

What if \( D < 0 \)?
\[ x^2 + 1 \] has roots \( i, -i \)
--ambiguous which is \( r_1 \)
and which is \( r_2 \)?
1 = -1?

Ambiguity can cause problems:

\[1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1} \times \sqrt{-1} = (-1)^2 = -1\]

Moral:
1. Be sure rules are properly applied.
2. Thoughtless calculation no substitute for understanding.

\[\frac{1}{2} = -\frac{1}{2} \quad \text{(multiply by } \frac{1}{2}\text{)}\]
\[2 = 1 \quad \text{(add } \frac{3}{2}\text{)}\]

“Since I and the Pope are clearly 2, we conclude that I and the Pope are 1. That is, I am the Pope.”
-- Bertrand Russell

Bertrand Russell (1872 - 1970)

(Picture source: http://www.users.drew.edu/~jlenz/brs.html)