Proof by **Cases:**

**Friends & Strangers**

Six people. Every two are either friends or strangers.

**Claim:** there is a set of
3 mutual friends or
3 mutual strangers

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Friends & Strangers

People are circles
3 mutual strangers
3 mutual friends

**red line shows friends**
**blue line shows strangers**

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Friends & Strangers

Take 3 minutes to find a counter-example
--or convince yourself there isn’t any counterexample, that is, the **Claim** is true.
A Proof of the Claim

• Person has a line to each of the 5 other people.
• Lines are red or blue, so at least 3 must be the same color.

Case 1: some pair of these friends are friends of each other, then we have 3 mutual friends:

Case 2: no pair of these friends are friends of each other, so we have 3 mutual strangers:

Since the Claim is true in either case, and one of these cases always holds, the Claim is always true.

QED
Ramsey’s Theorem

For any \( k \), every large enough group of people will include either \( k \) mutual friends, or \( k \) mutual strangers.

Let \( R(k) \) be the large enough size. So we’ve proved that \( R(3) = 6 \).

Ramsey’s Numbers

Turns out that \( R(4) = 18 \) (not easy!)
\( R(5) \) is unknown!
Paul Erdös considered finding \( R(6) \) a hopeless challenge!
So in our second class, we have reached a research frontier!