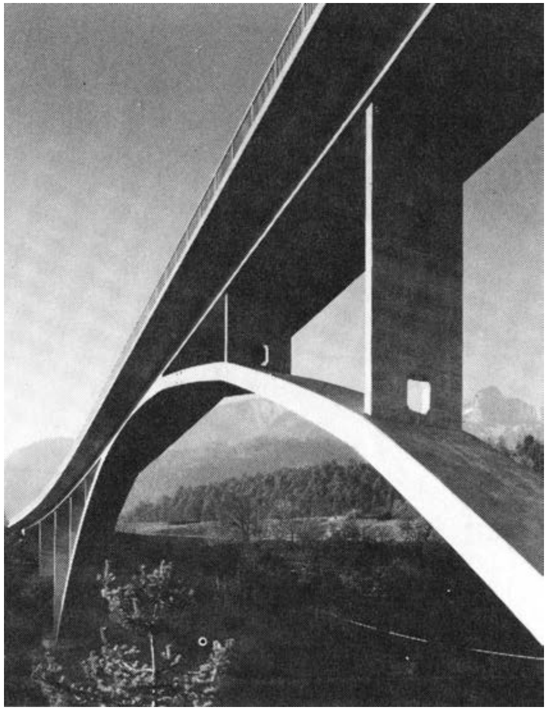
MIT Class 6.S080 (AUS)

Mechanical Invention through Computation

Mechanism Basics

Design Principles
Structure and Mechanism



Structure:

Force is resisted

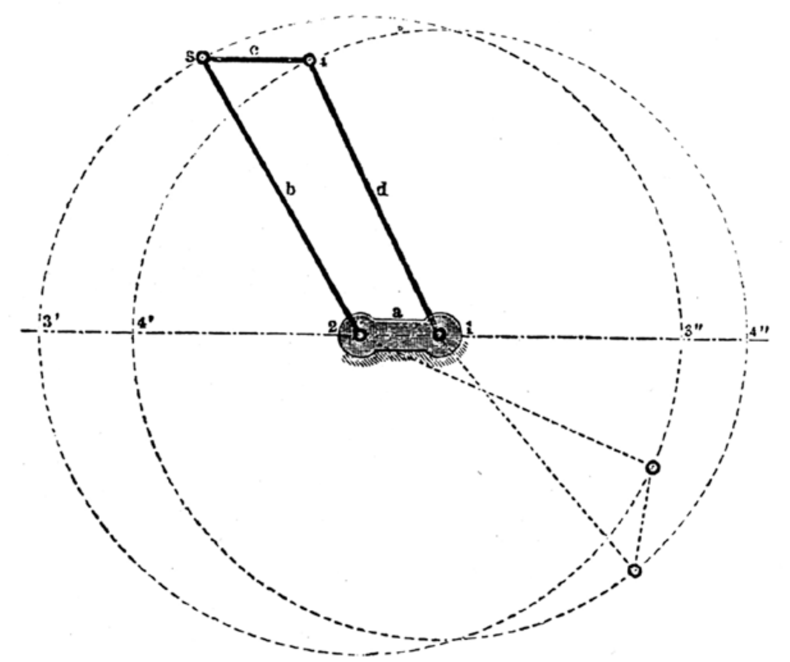


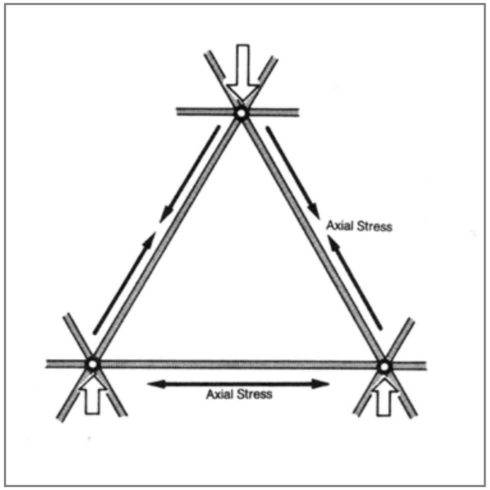
FIG. 203.

Mechanism:

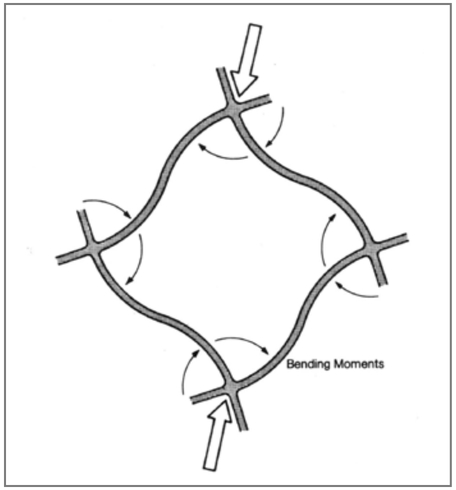
Force flows into movement

Design Principles
Structure and Mechanism

Possible Responses to Applied Force



Structural Resistance



Structural Deflection (elastic or Inelastic)

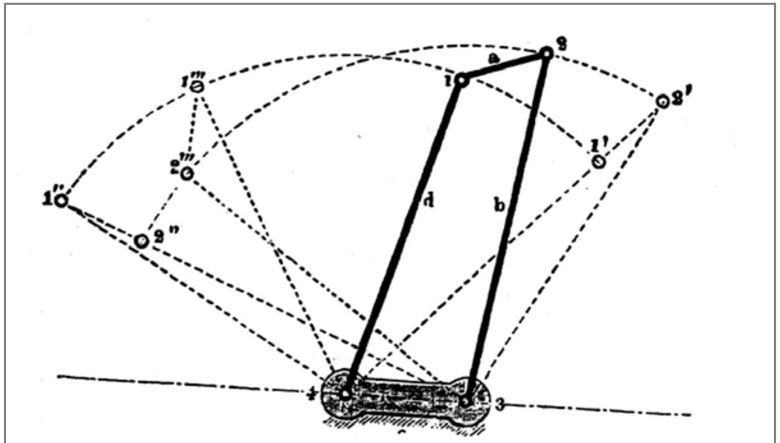
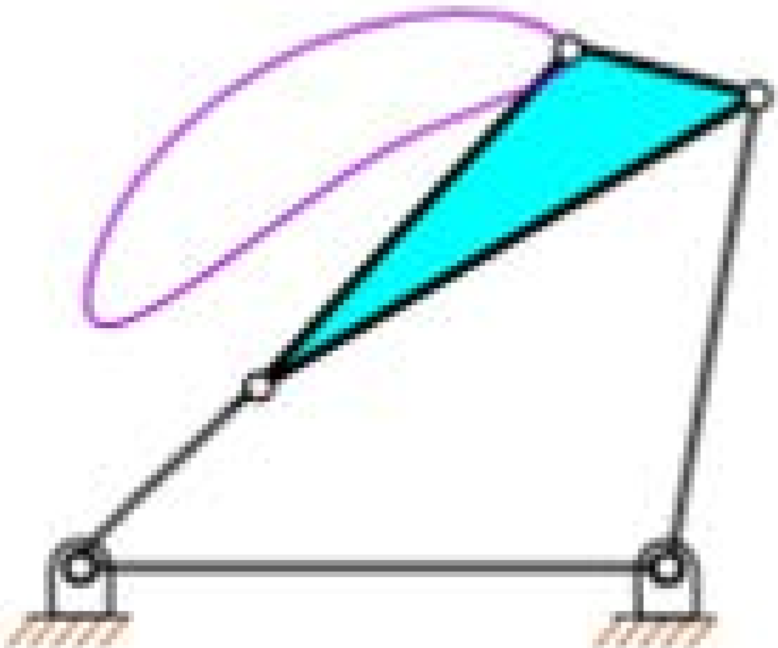


FIG. 204.

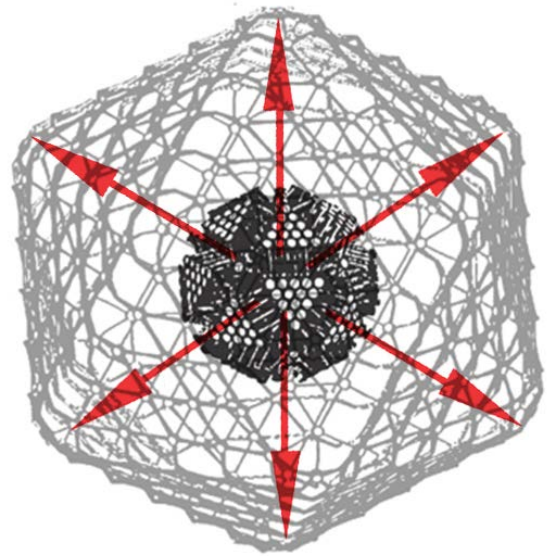
Kinematic deflection

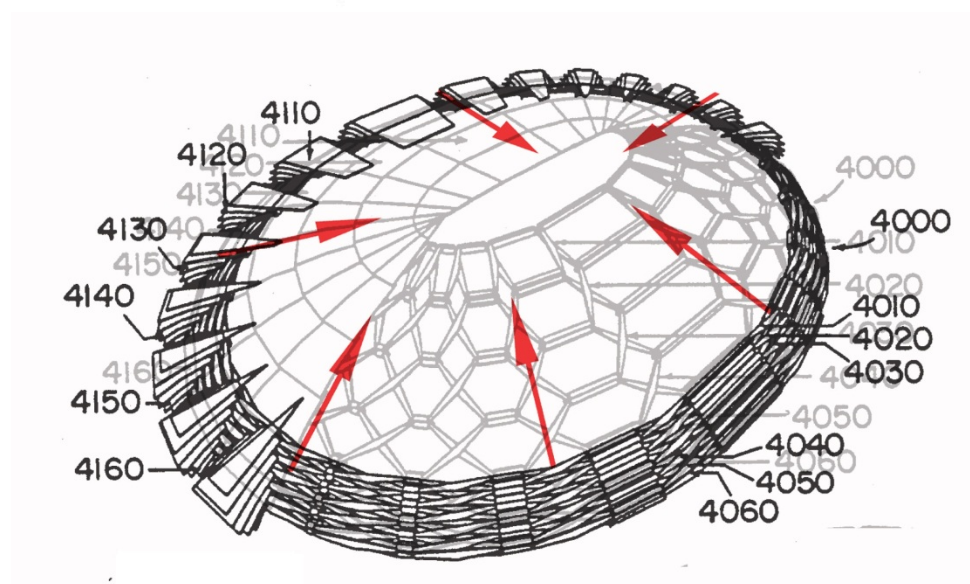
Mechanism paradigms

Synthesize a motion path



Synthesize a form change





Definitions

- Kinematics: the study of the motion of bodies without reference to mass or force
- Links: considered as rigid bodies
- Kinematic pair: a connection between two bodies that imposes constraints on their relative movement. (also referred to as a mechanical joint)
- Ground: static point of reference
- Degree of freedom (DOF): of a mechanical system is the number of independent parameters that define its configuration.

Links types

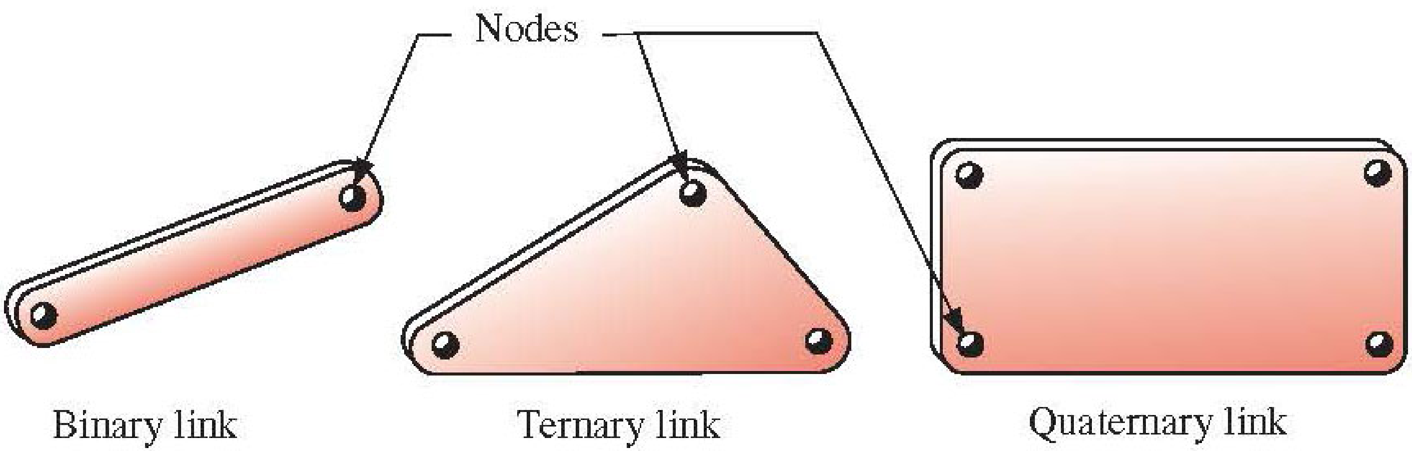
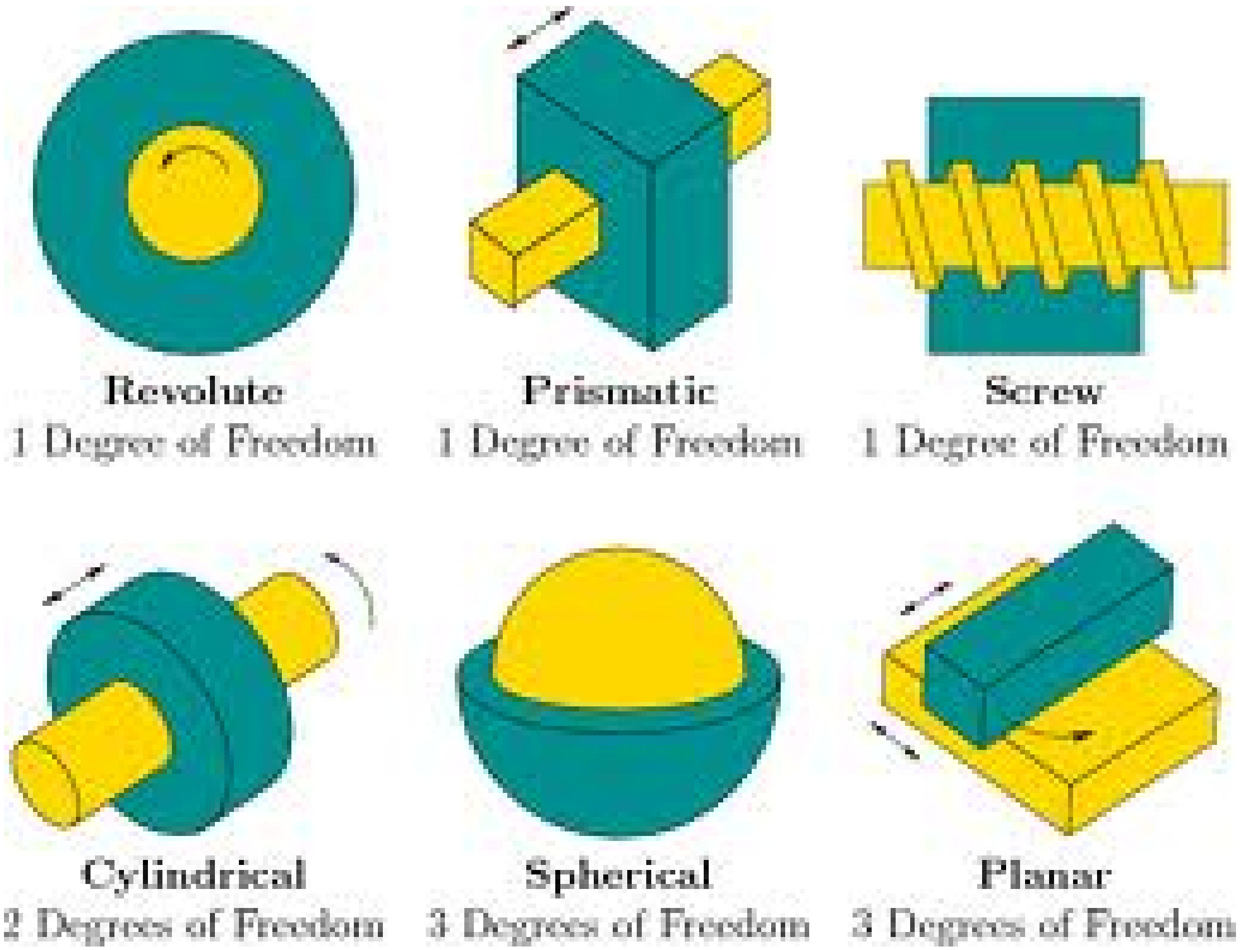


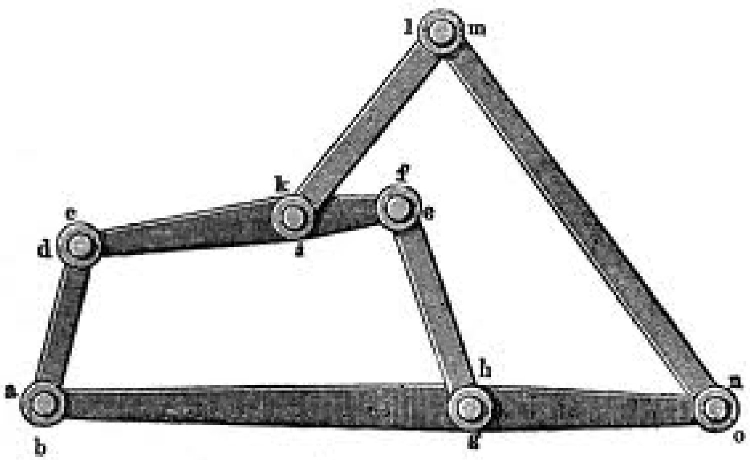
FIGURE 2-2

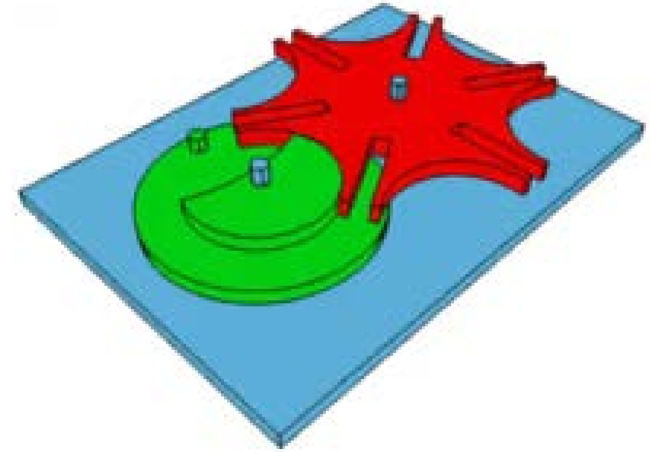
Links of different order

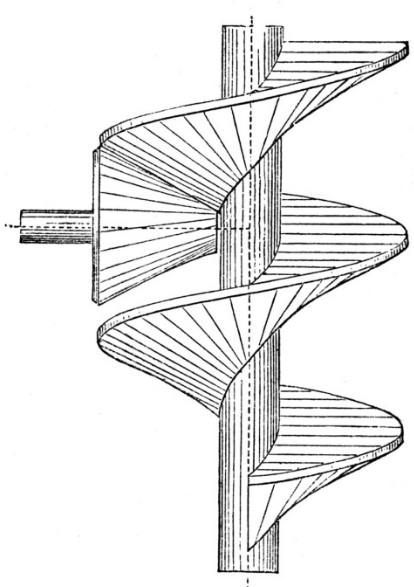
Kinematic pairs



Historic Mechanisms







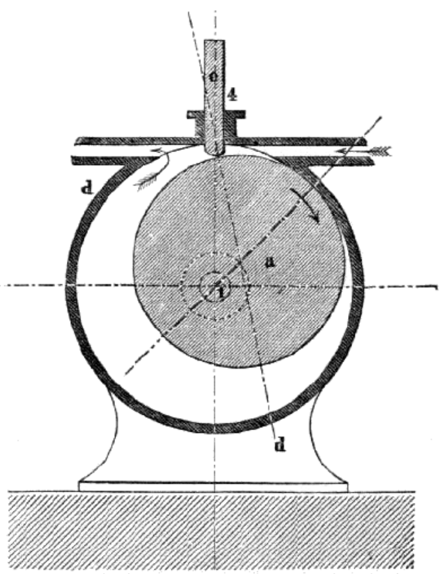


FIG. 4.—Yule, Hall. (Eng.)

$(C_3'' P^{\perp})^{\frac{d}{a}} - b - \frac{c}{2}$; $(V^{\pm}) = a, d.$

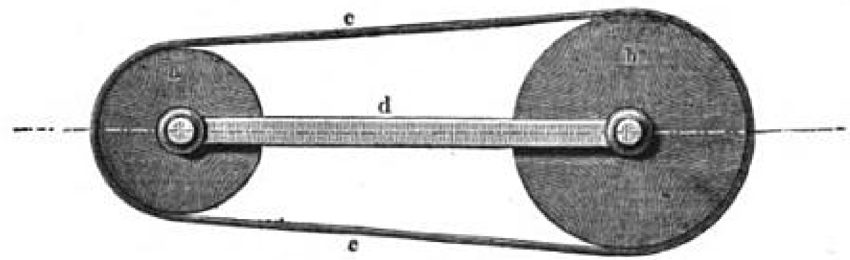


FIG. 182.

the same as that running off the other. The band for the ulley a is identical—coincident—with that for b, the corre-

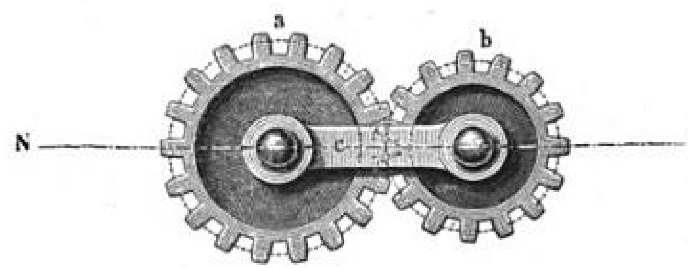
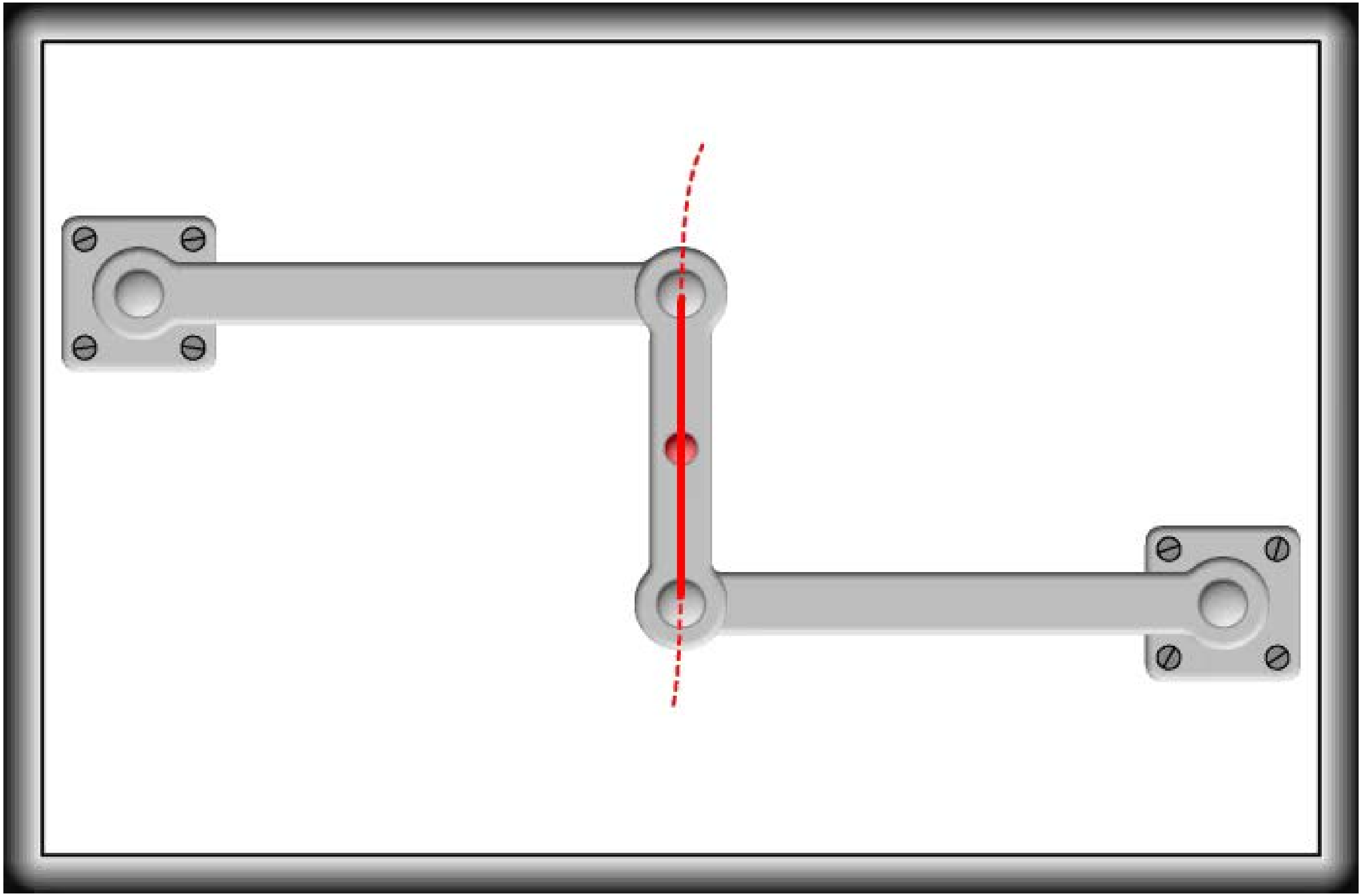
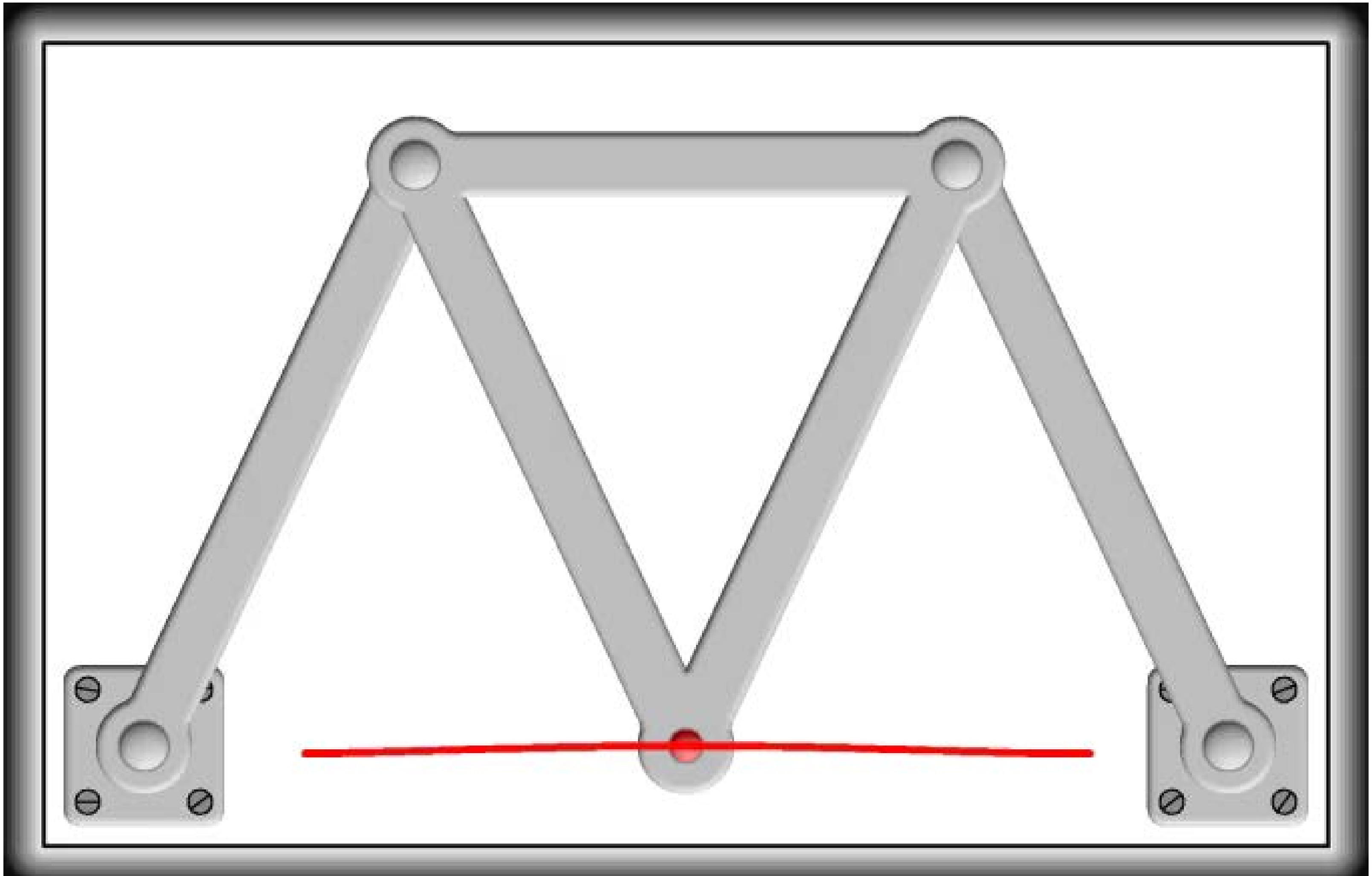


FIG. 183.

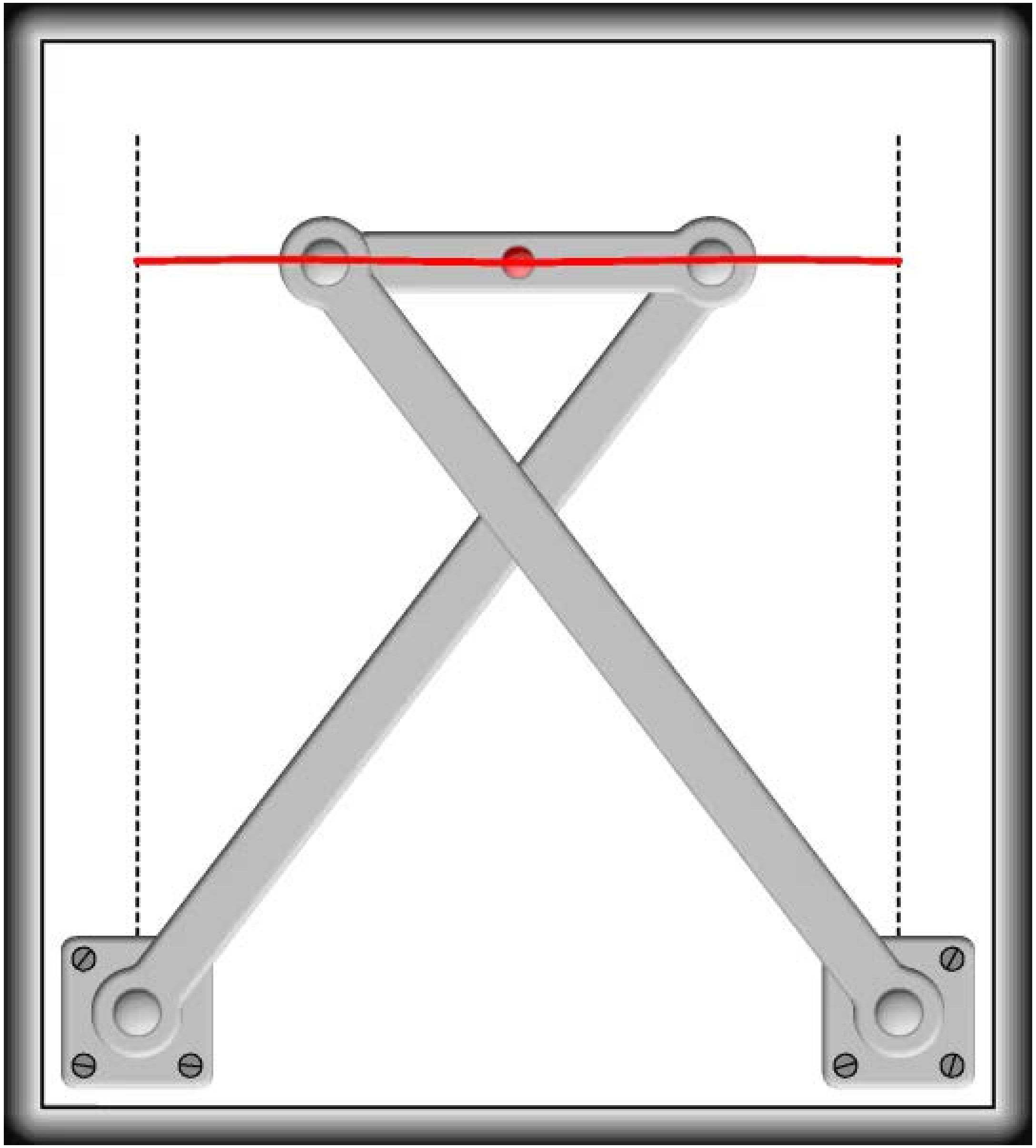
Straight- line linkages (James Watt)



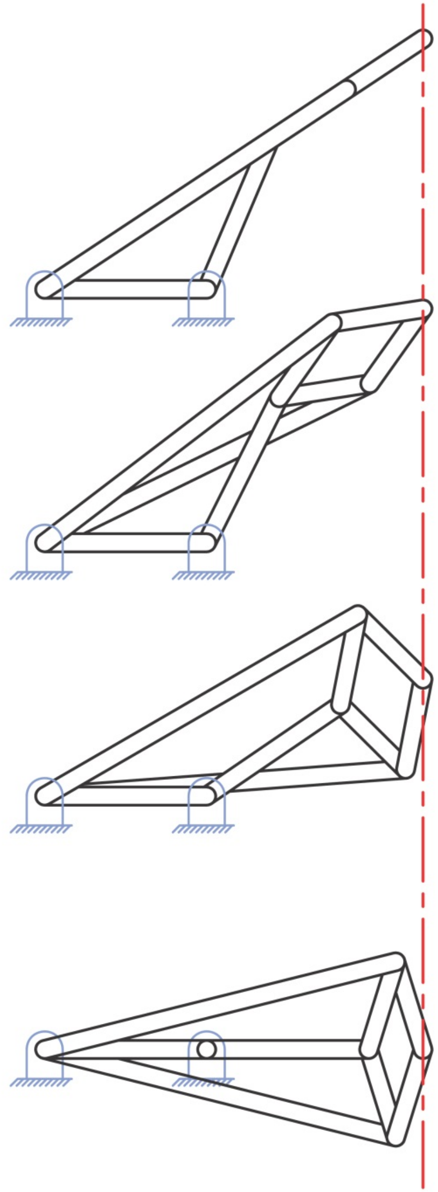
Straight- line linkages (Richard Roberts)

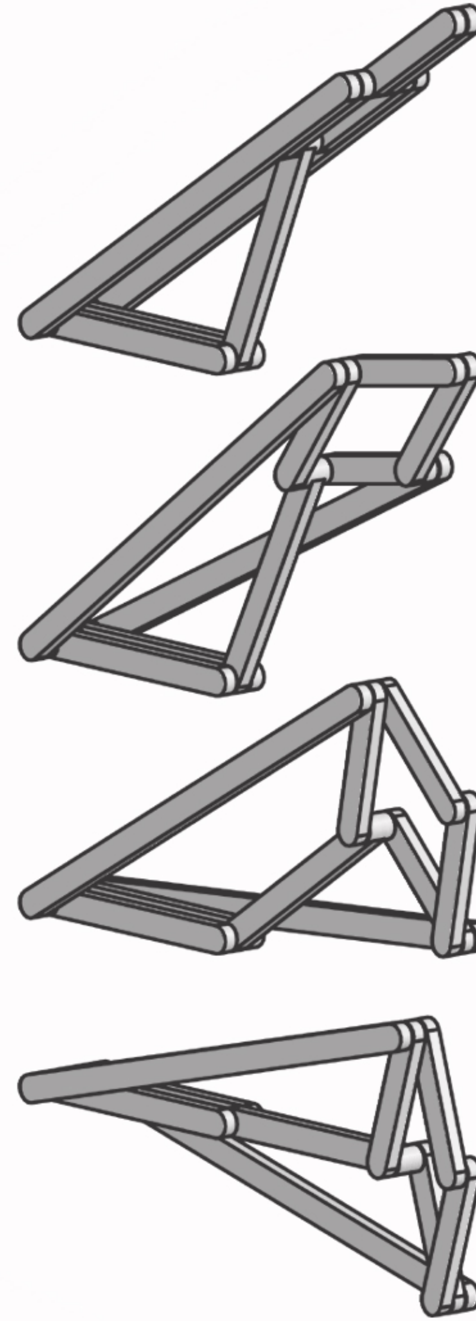


Straight- line linkages (Tchebicheff)

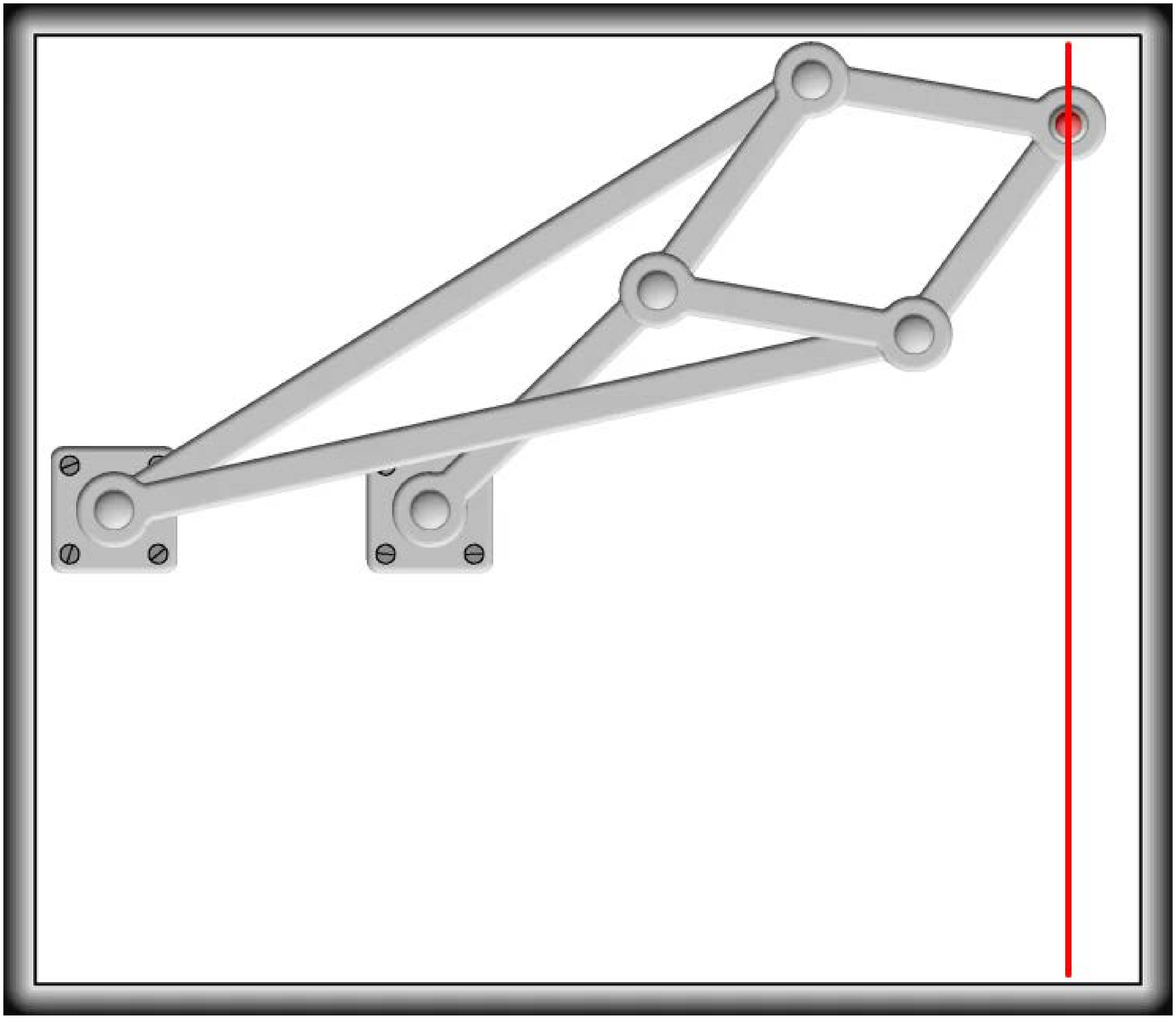


Peaucellier Linkage

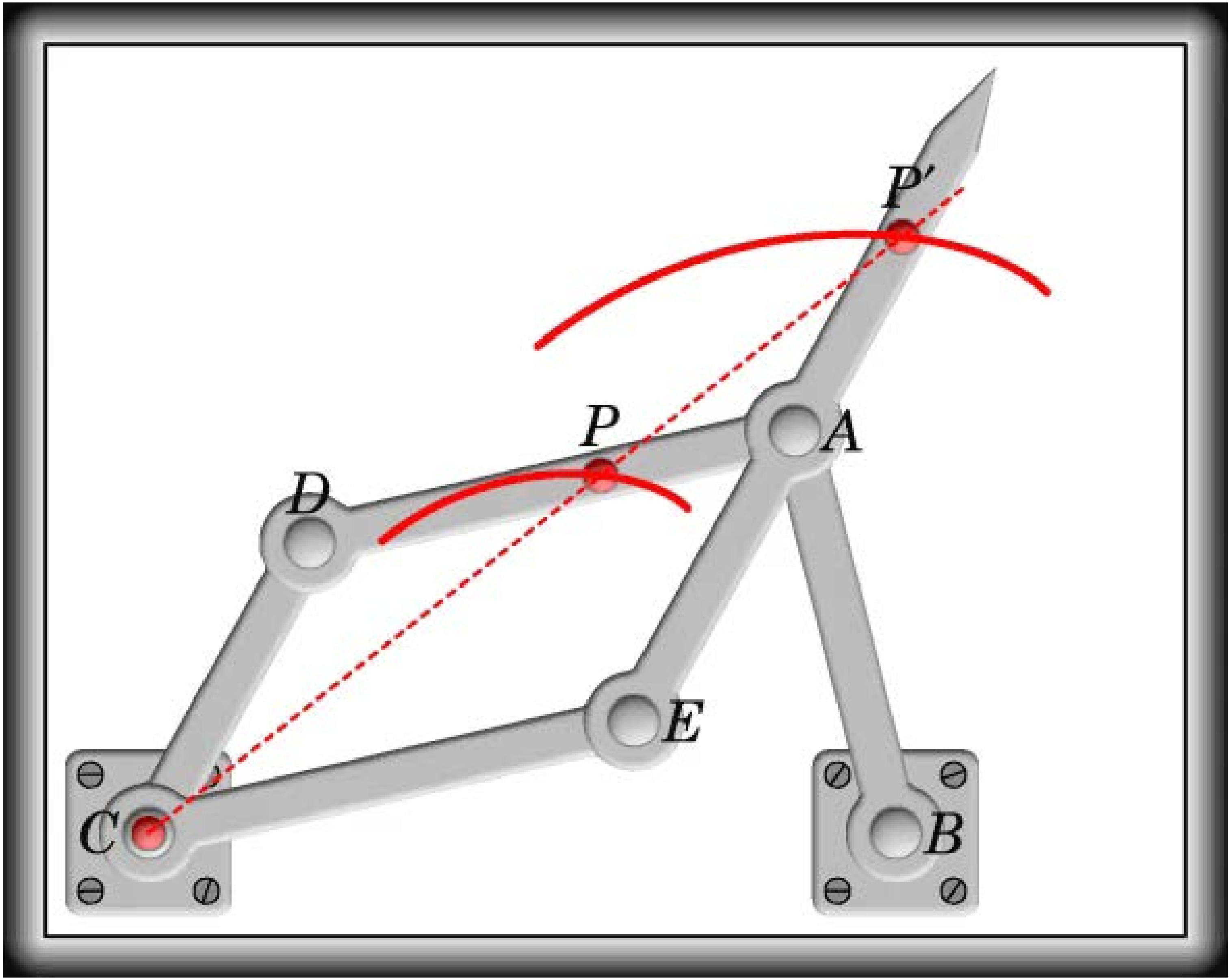




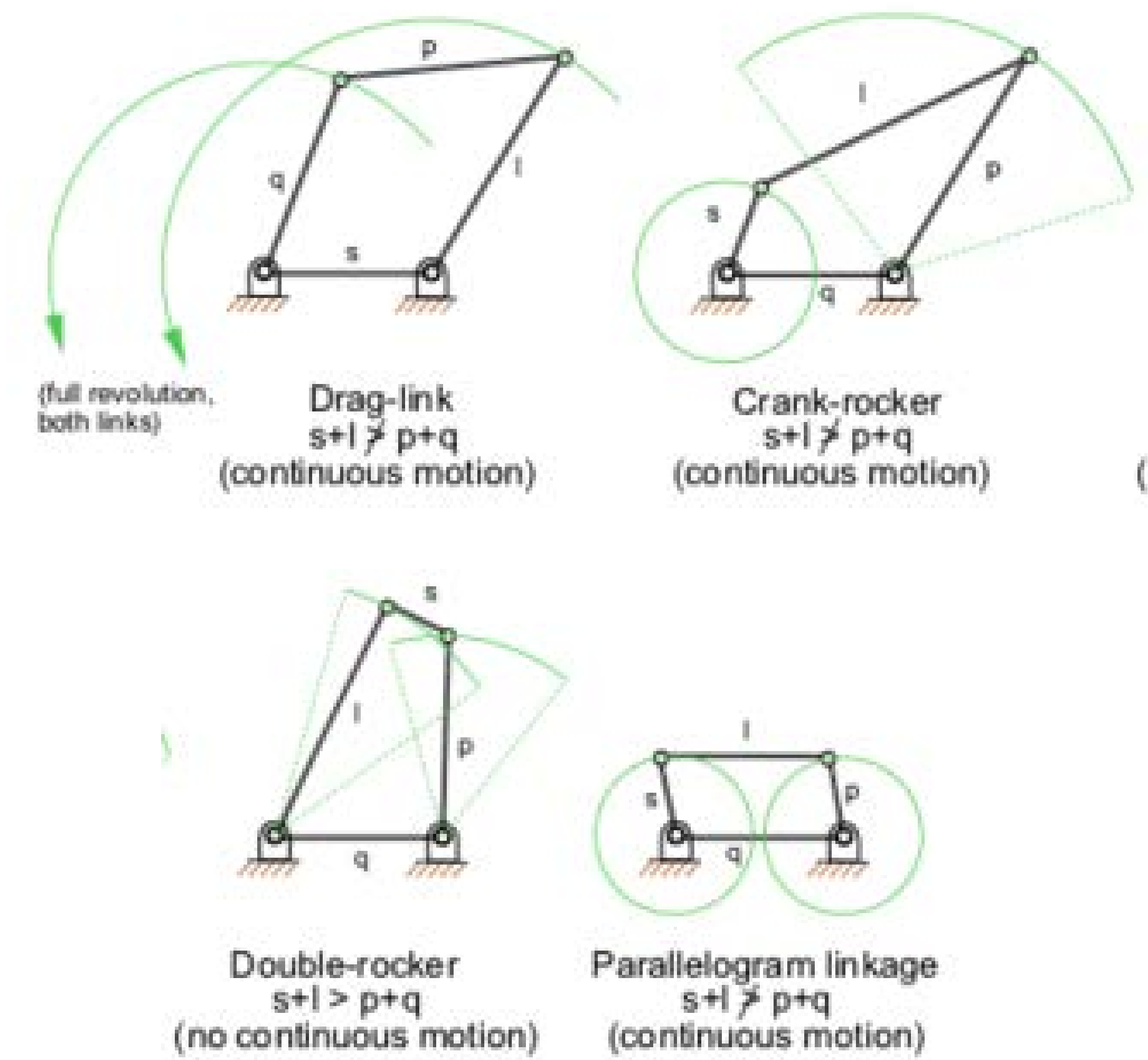
Straight- line linkages (Peaucellier)



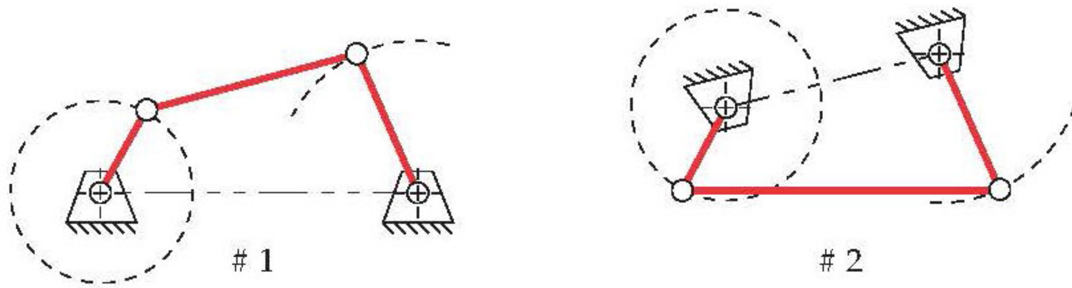
Straight- line linkages (Kempe)



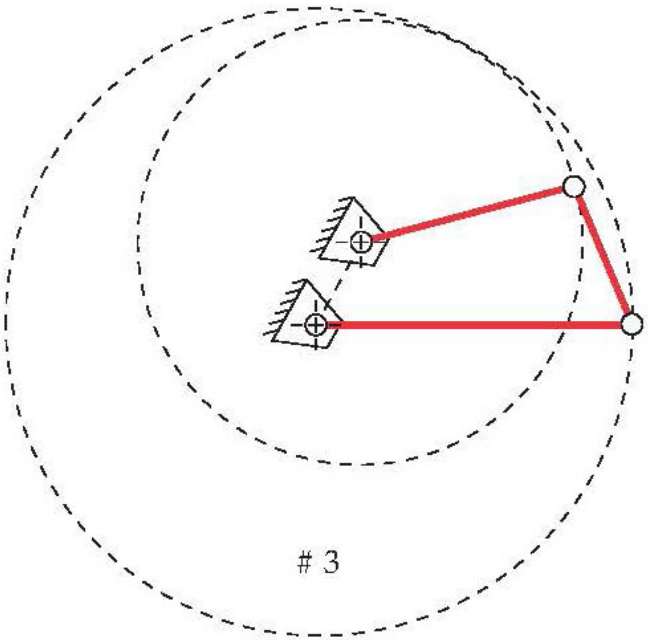
4-bar linkage types



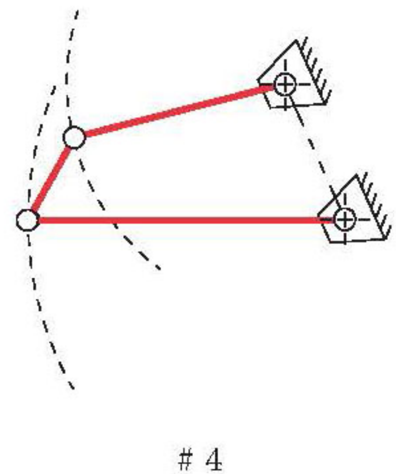
Kinematic inversions



(*a*) Two non-distinct crank-rocker inversions (GCRR)

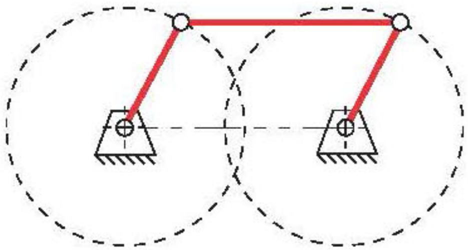


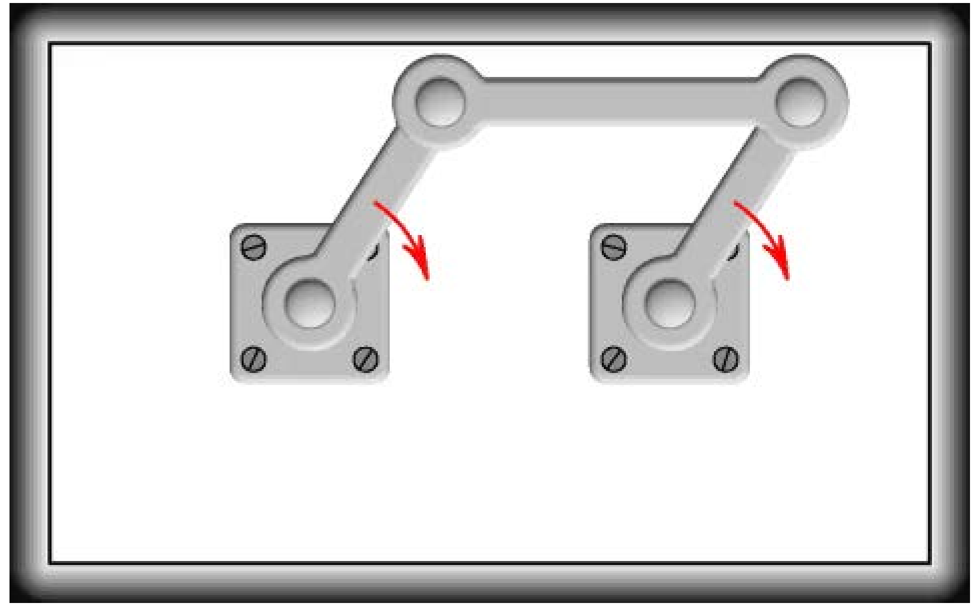
(*b*) Double-crank inversion (GCCC)
(drag link mechanism)



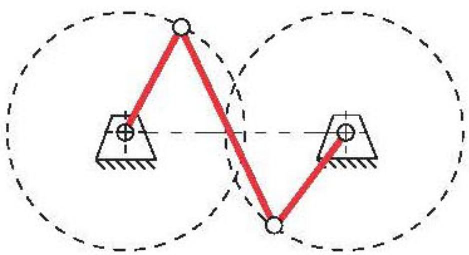
(*c*) Double-rocker inversion (GRCR)
(coupler rotates)

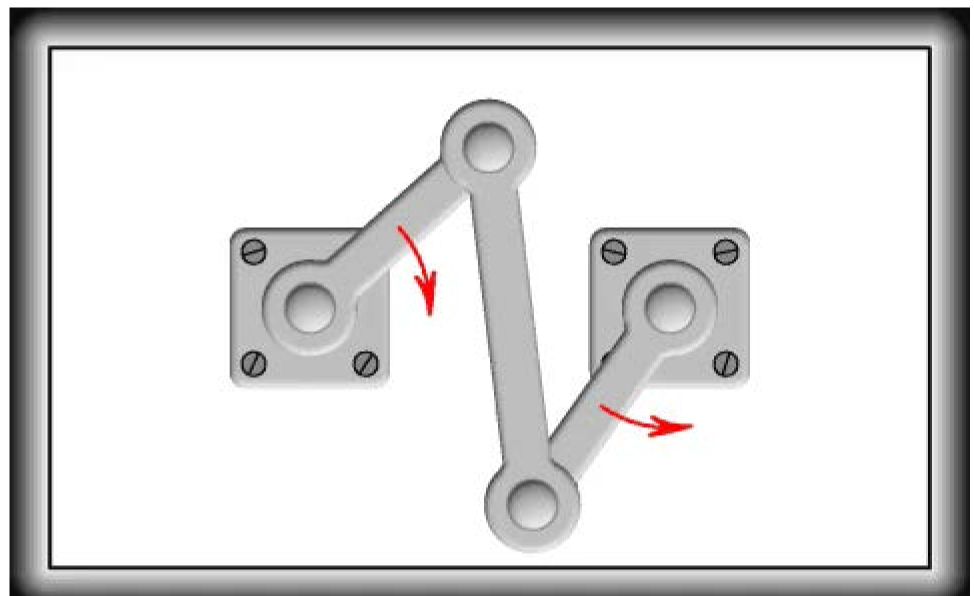
Four-bar linkage examples





Parallel 4-bar





Anti-parallel 4-bar

Gruebler's equation

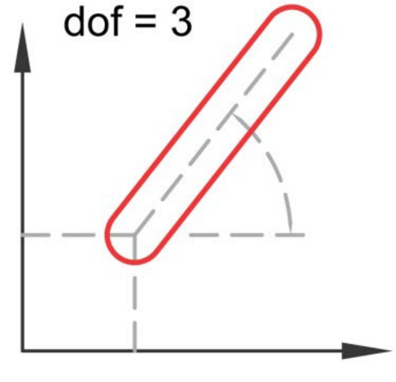
N = Number of Links (including ground link)

P = Number of Joints (pivot connections between links)

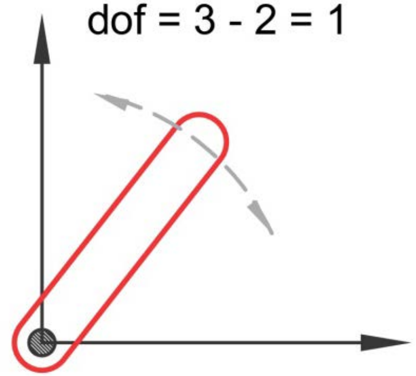
- Each link has 3 degrees of freedom
- Each pivot subtracts 2 degree of freedom

$$DOF = 3\,(N-1) - 2P$$

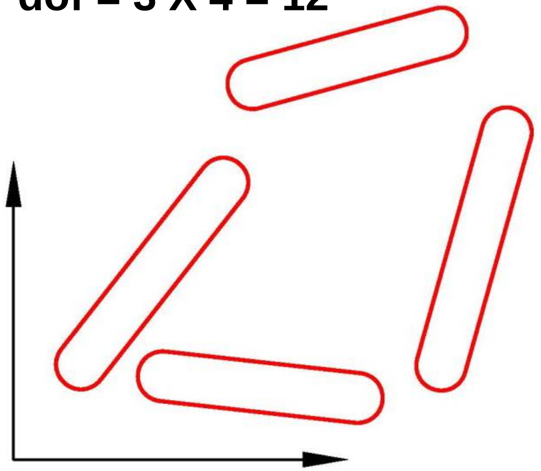
dof = 3



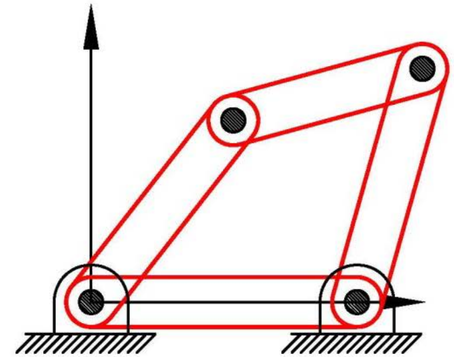
dof = 3 - 2 = 1



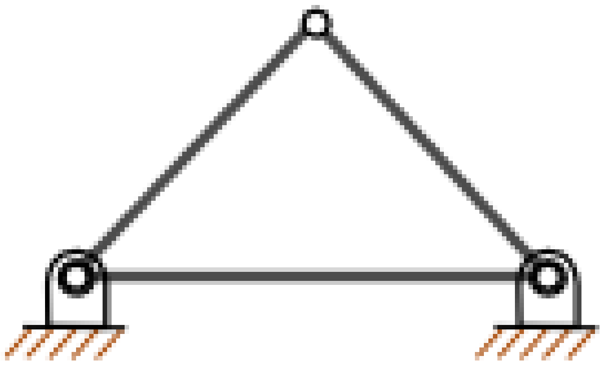
dof = 3 X 4 = 12



dof = 3 X (4-1) – (2 X 4) = 1



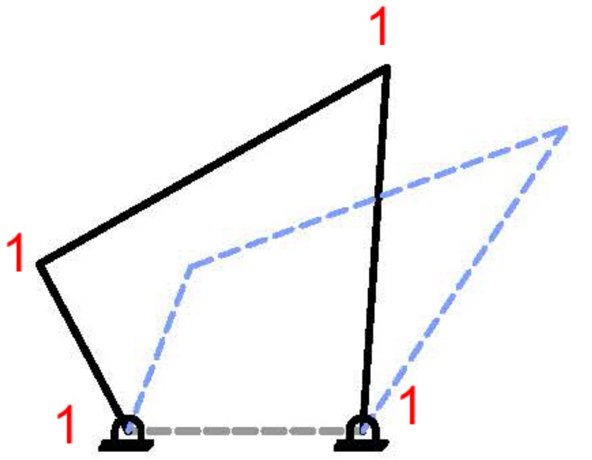
Examples



N = 3

P = 3

DOF = 3X(3-1)-(2X3)=0

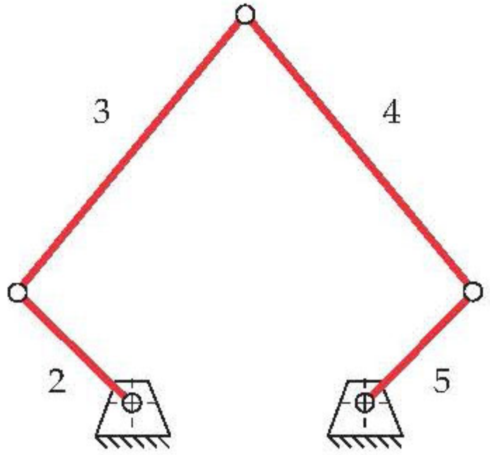


N = 4

P = 4

DOF = 3X(4-1)-(2X4)=1

Examples

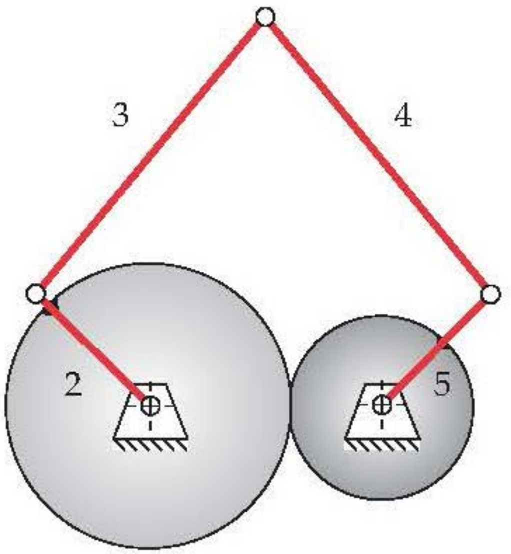


(*a*) Fivebar linkage—2 *DOF*

N = 5

P = 5

DOF = 2

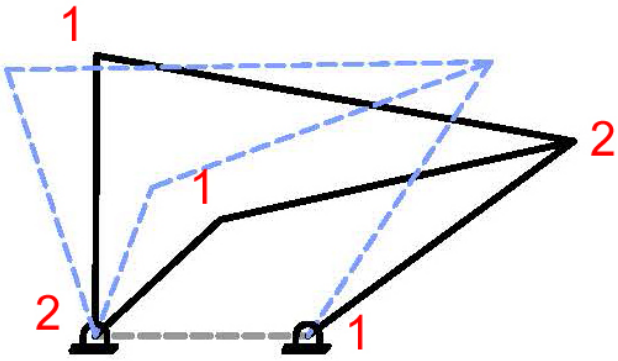


(*b*) Geared fivebar linkage—1 *DOF*

Geared connection removes one degree of freedom

DOF = 1

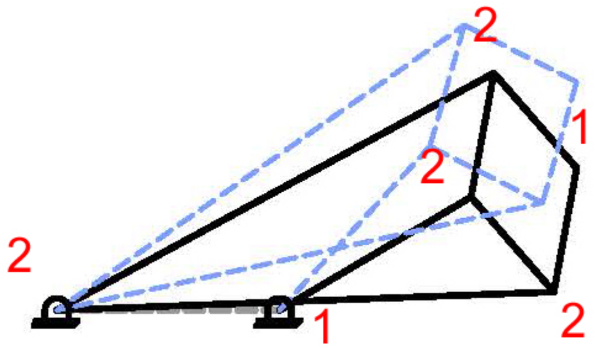
Examples



N = 6

P = 7

DOF = 3X(6-1)-(2X7)=1



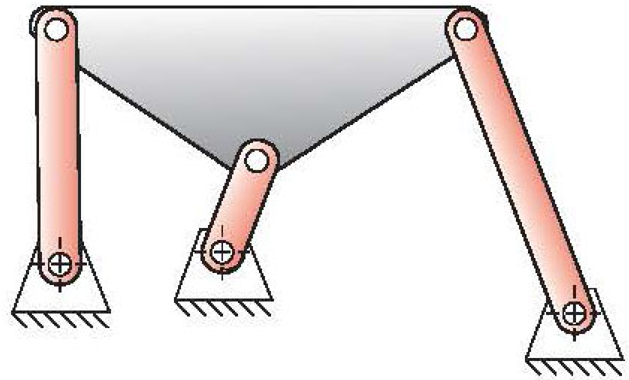
N = 8

P = 10

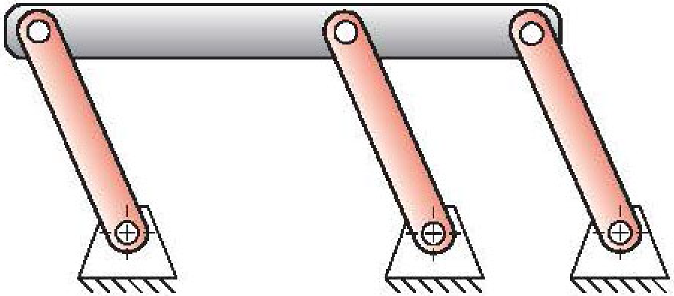
DOF = 3X(8-1)-(2X10)=1

Relation of DOF to special geometries

*Agrees with Gruebler's equation (**doesn't move**)*



*Doesn't agree with Gruebler's equation (**moves**)*



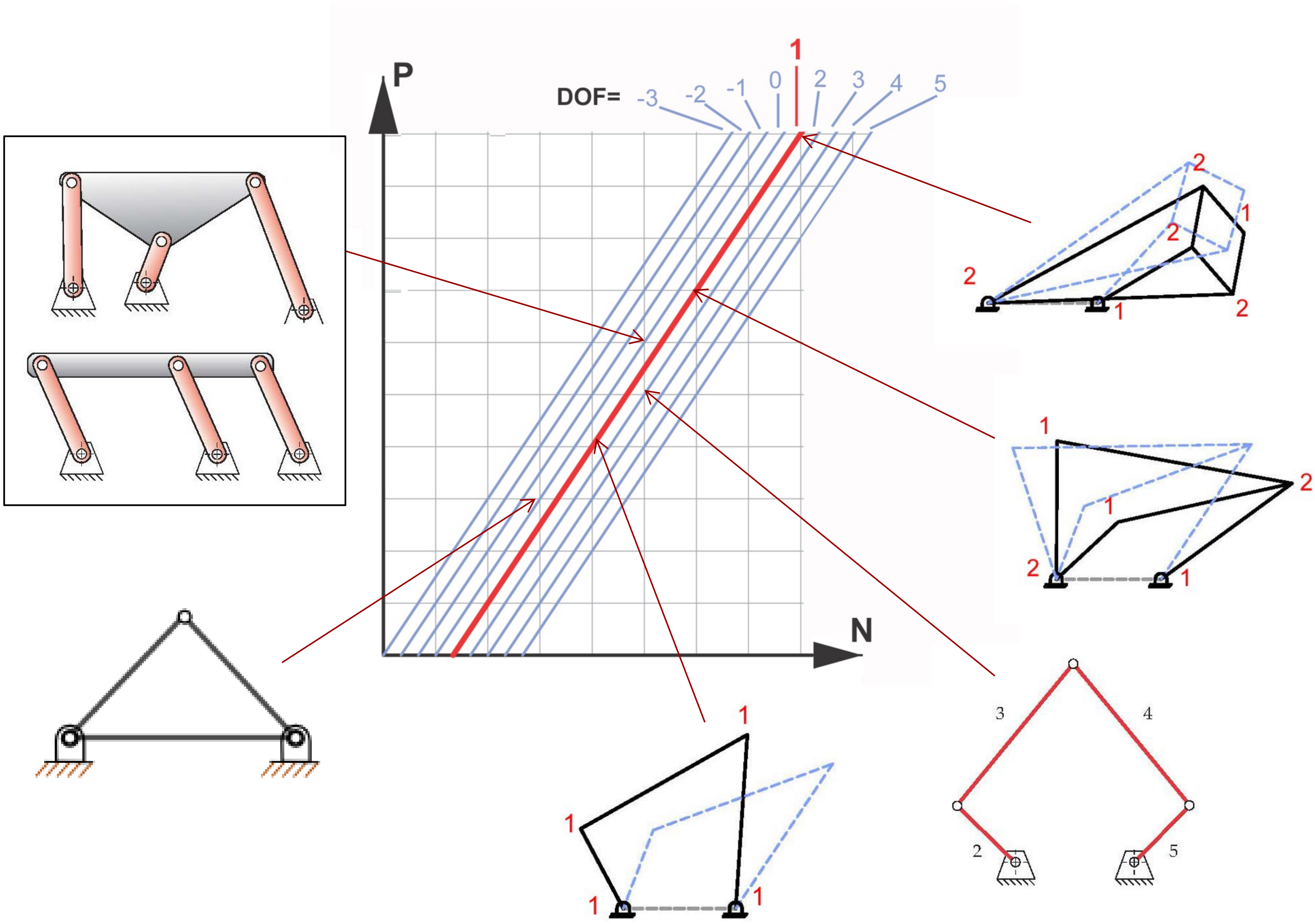
N = 5

P = 6

DOF = 3X(5-1)-(2X6) **= 0**

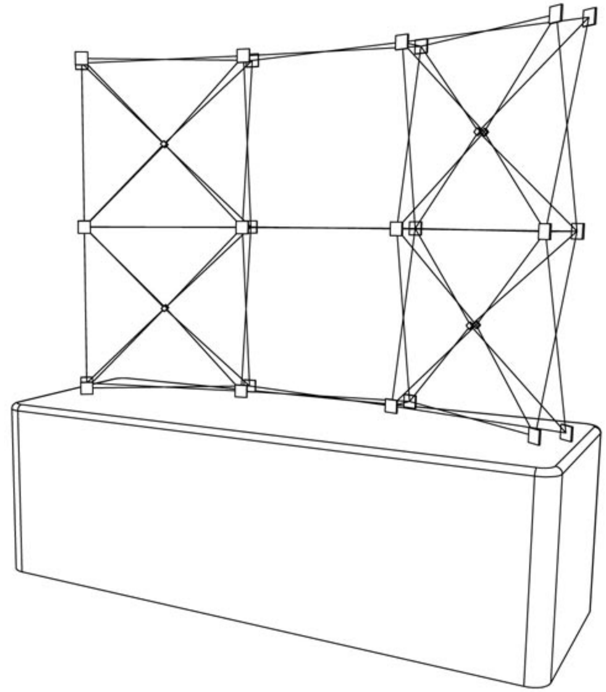
Graph of linkages

DOF=3(N-1)-2P



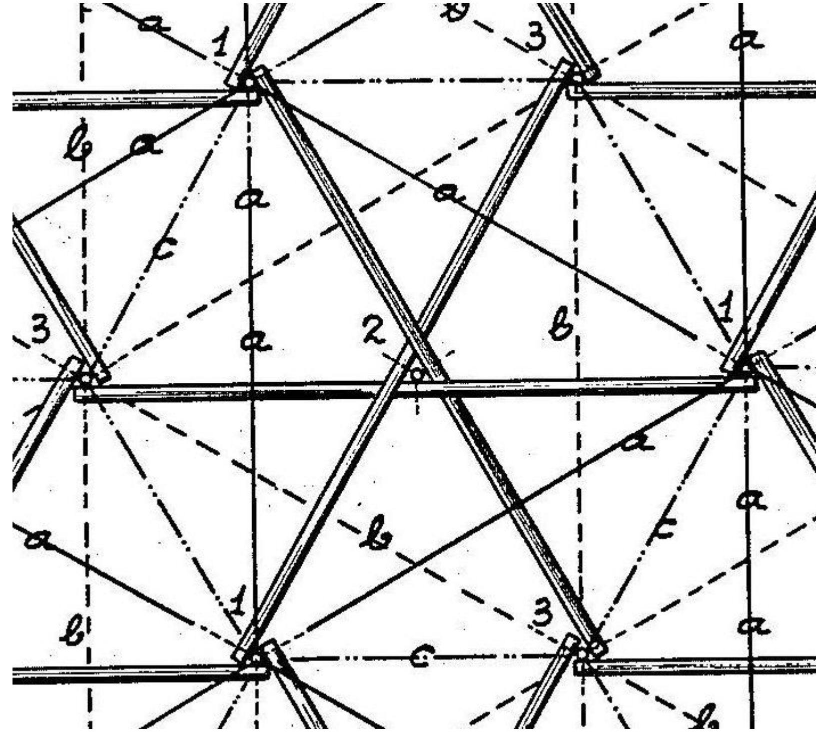
Scissor Linkages

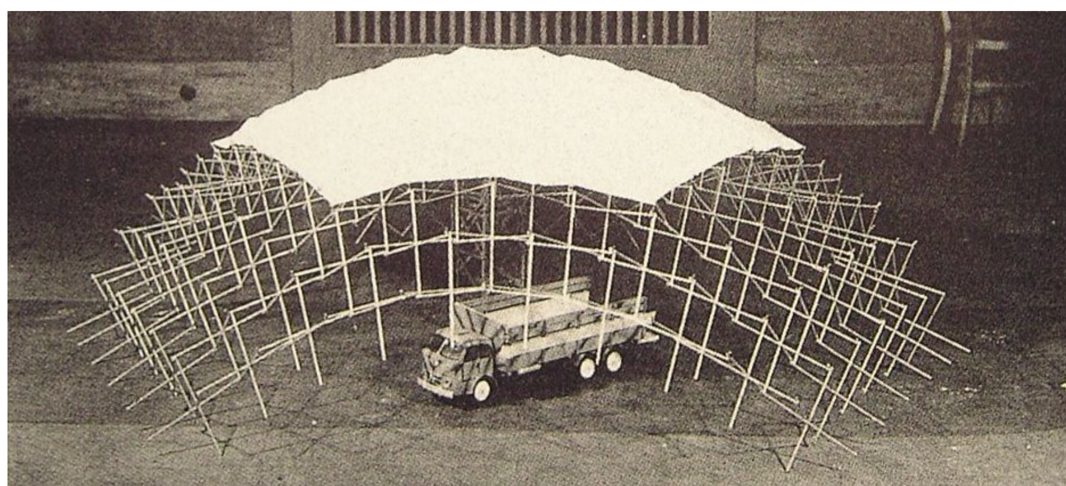
Scissor mechanisms



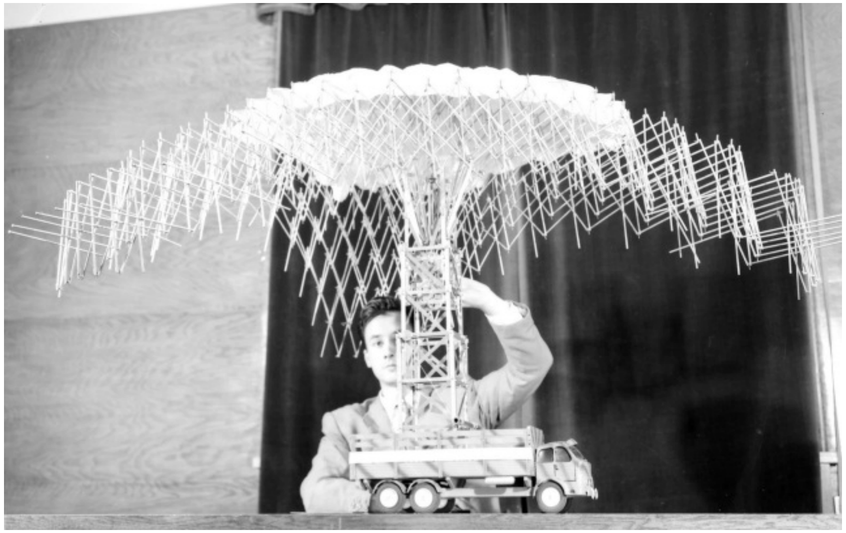
Historic examples of scissor mechanisms

Emilio Pinero



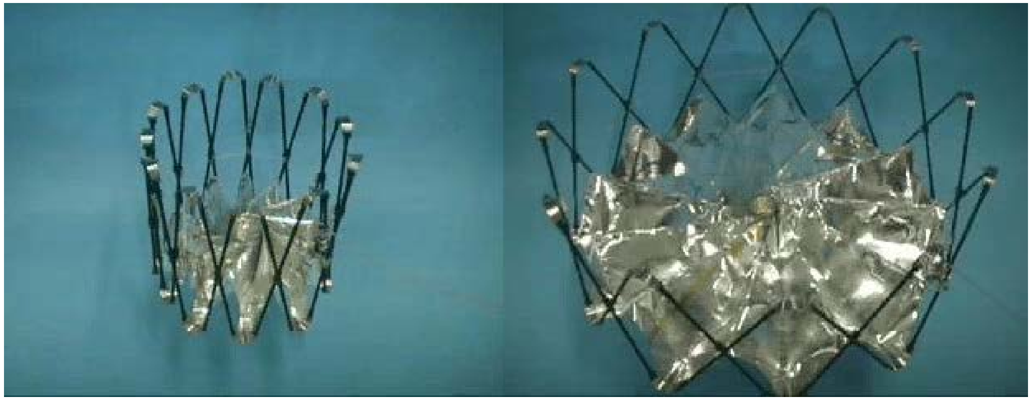


Historic examples of scissor mechanisms



Emilio Pinero

Examples of scissor mechanisms

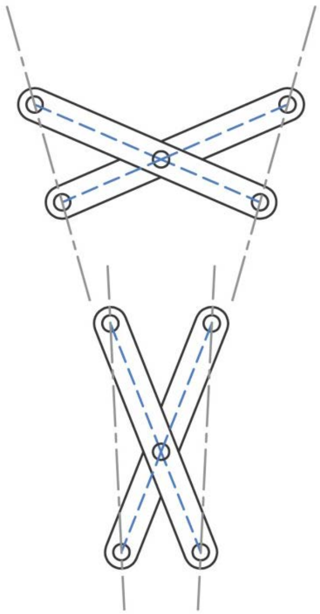


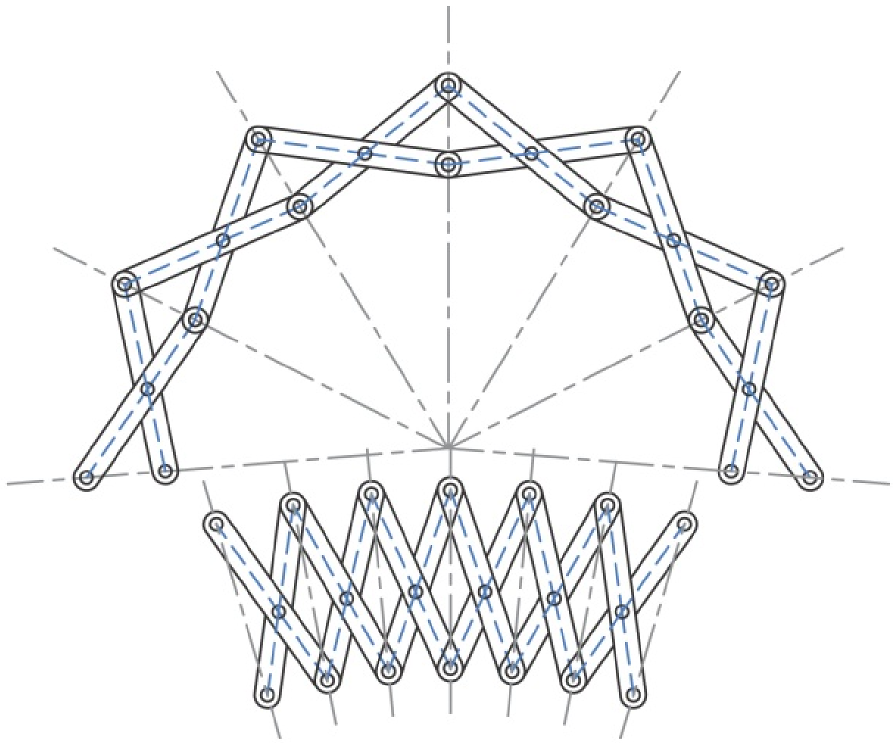
Sergio Pellegrino

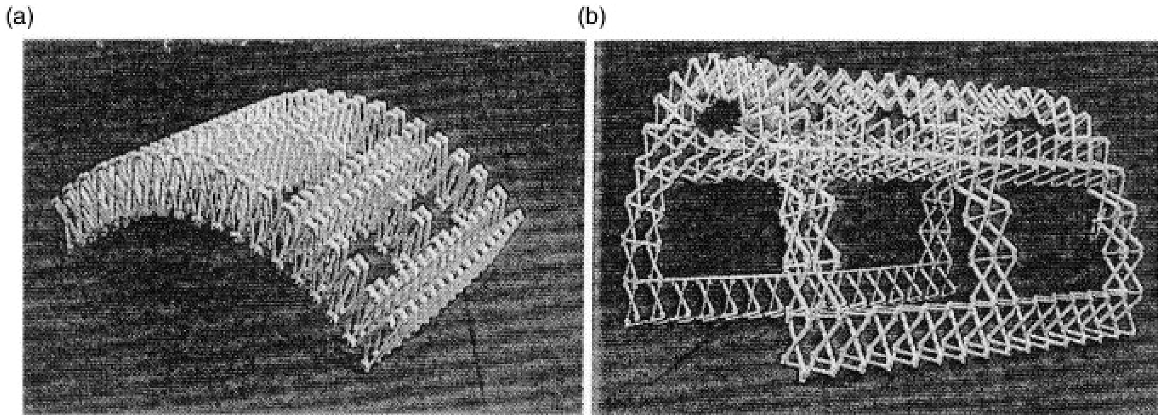
Felix Escrig

Curvature of scissor mechanisms

Off-center connection point => structures of variable curvature

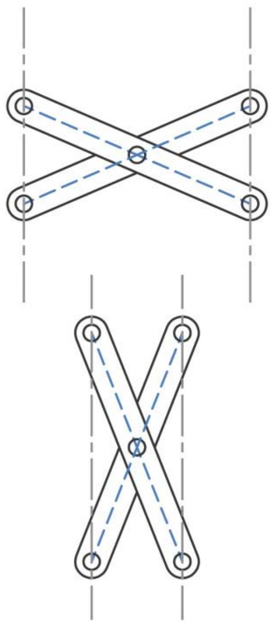




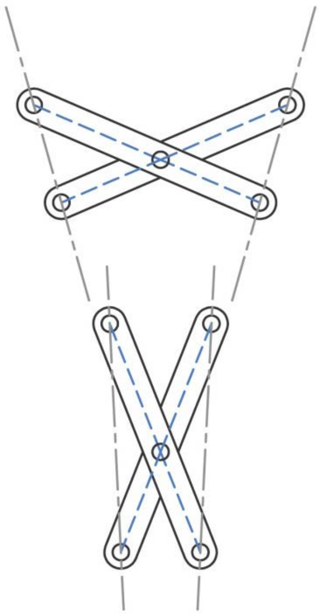


Scissor Types

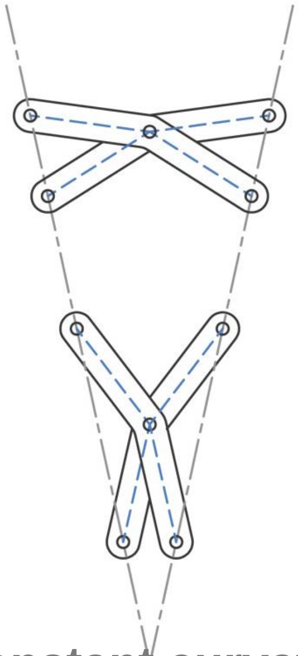
Parallel / symmetric



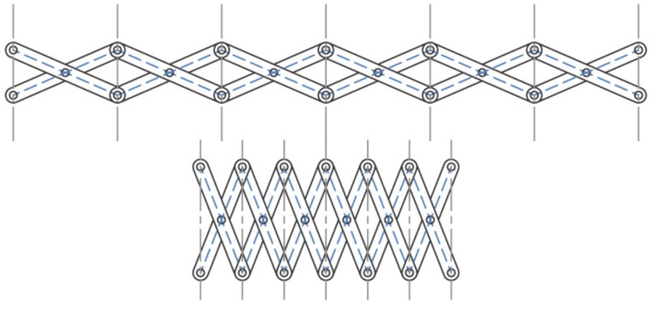
Parallel / asymmetric



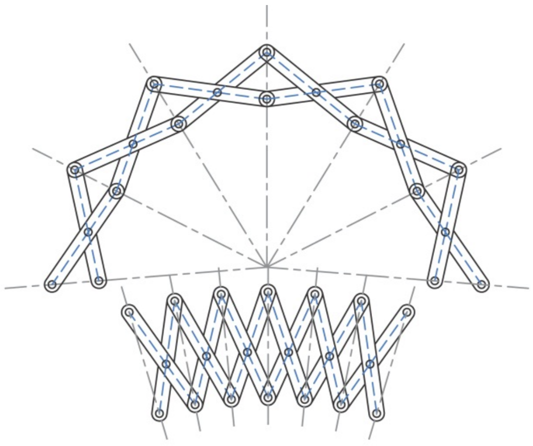
Angulated



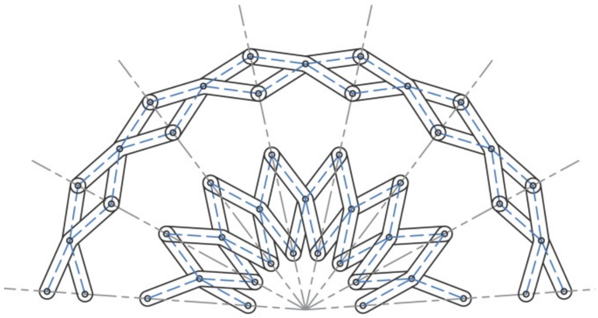
No curvature



Variable curvature

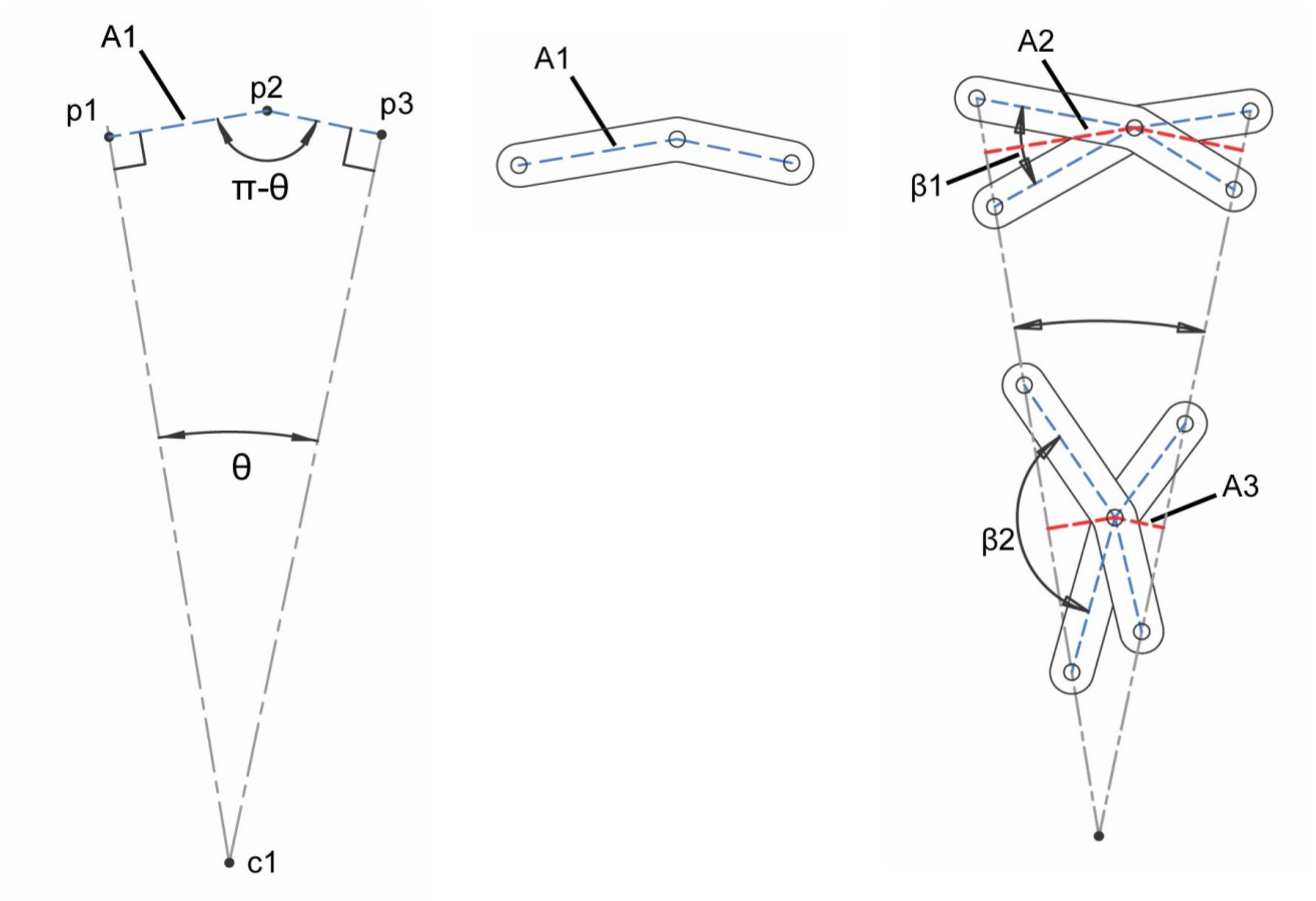


Constant curvature



Angulated scissors

Provides invariant angle during deployment

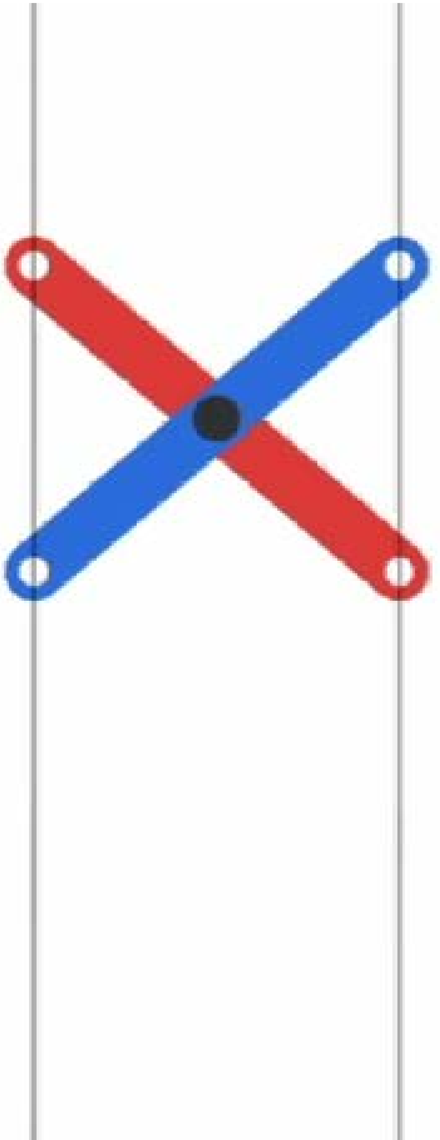


Scissor mechanism: demonstration

Parallel / Symmetric

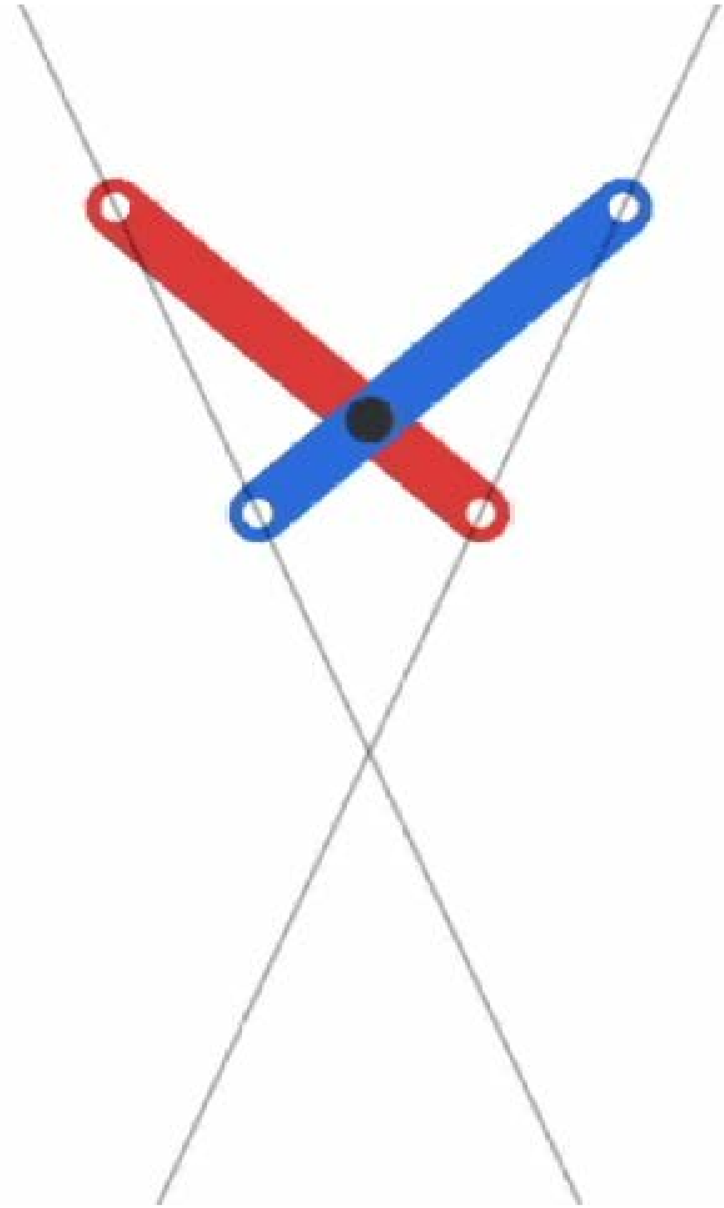
Scissor mechanism: demonstration

Off-center connection

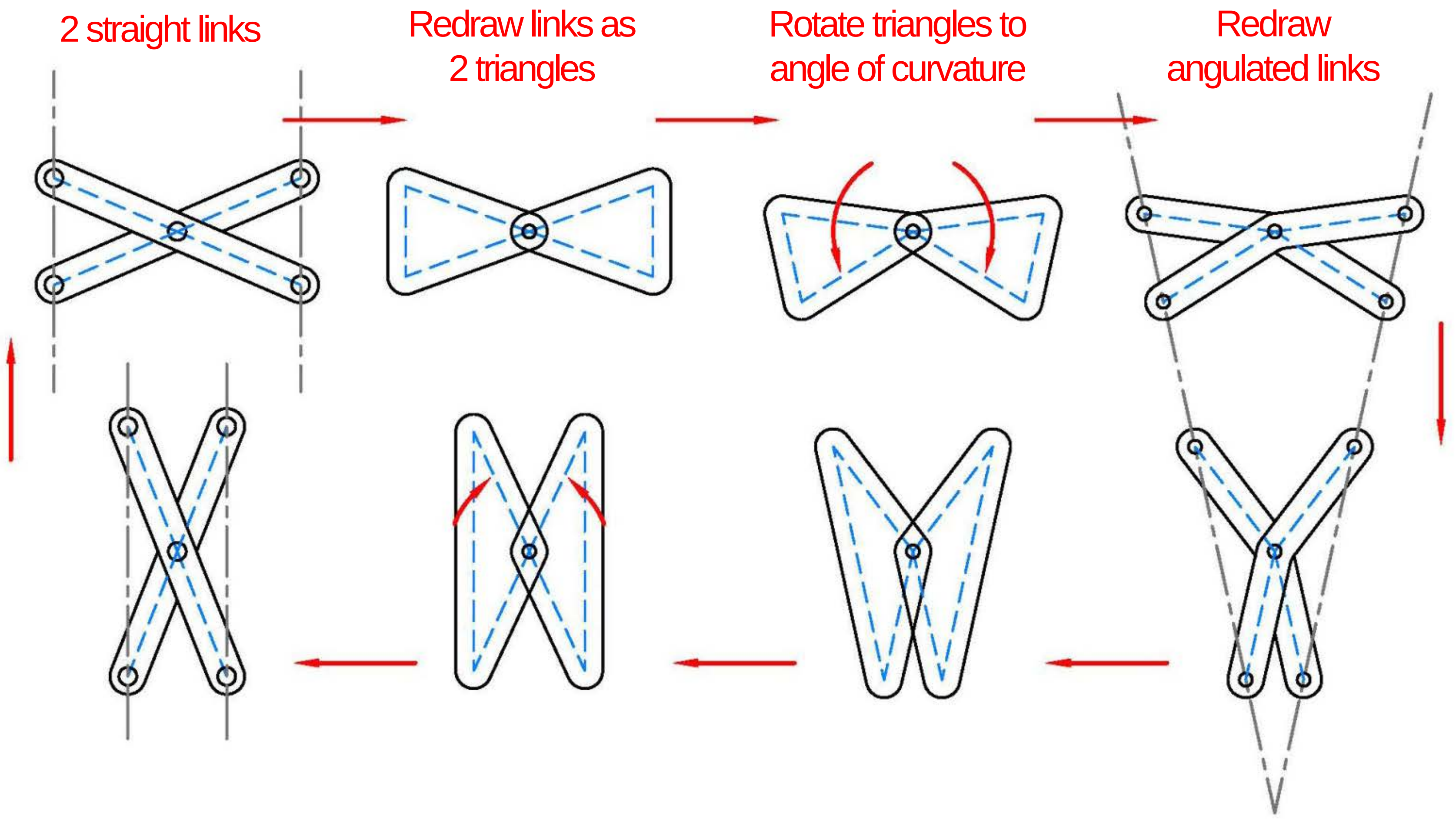


Scissor mechanism: demonstration

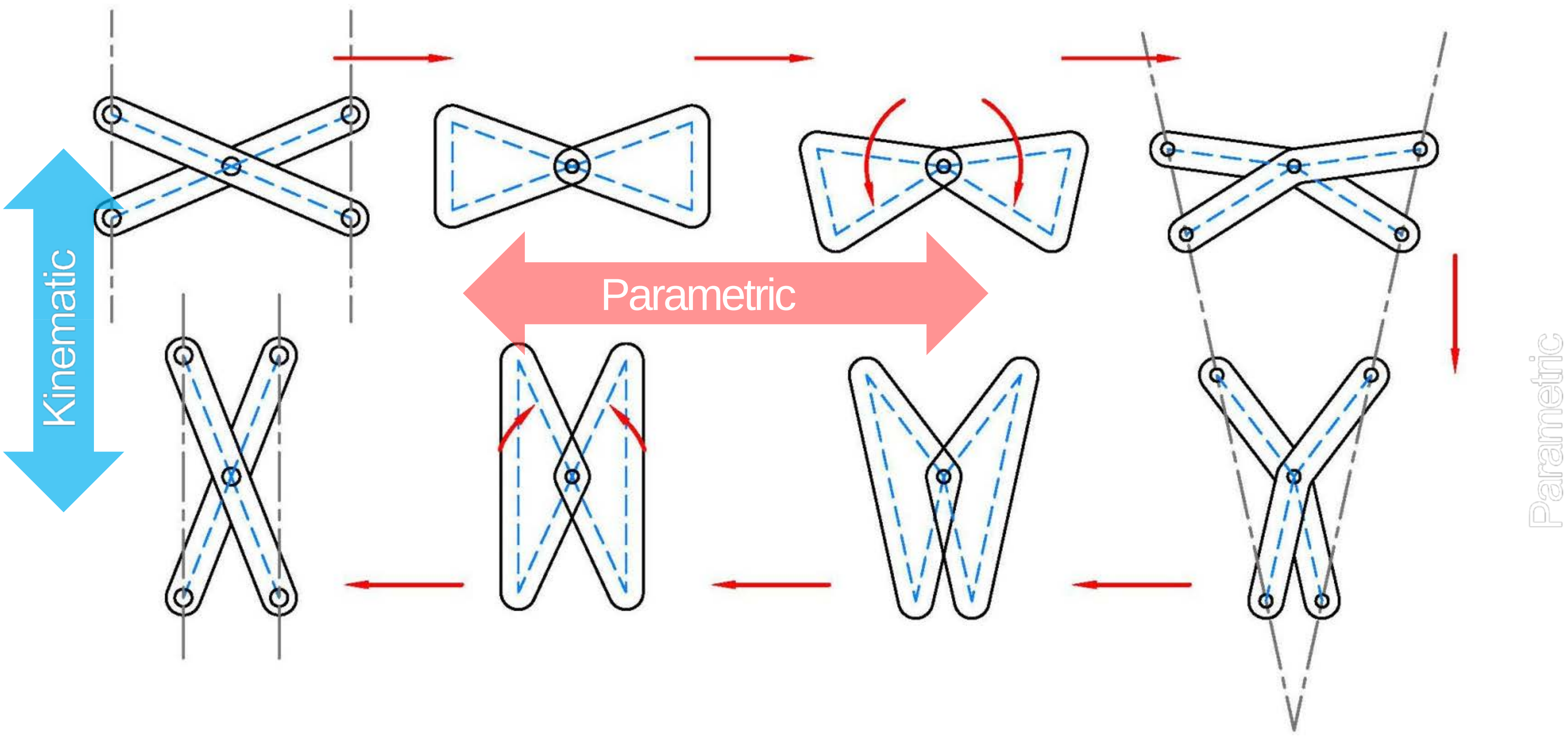
Angulated



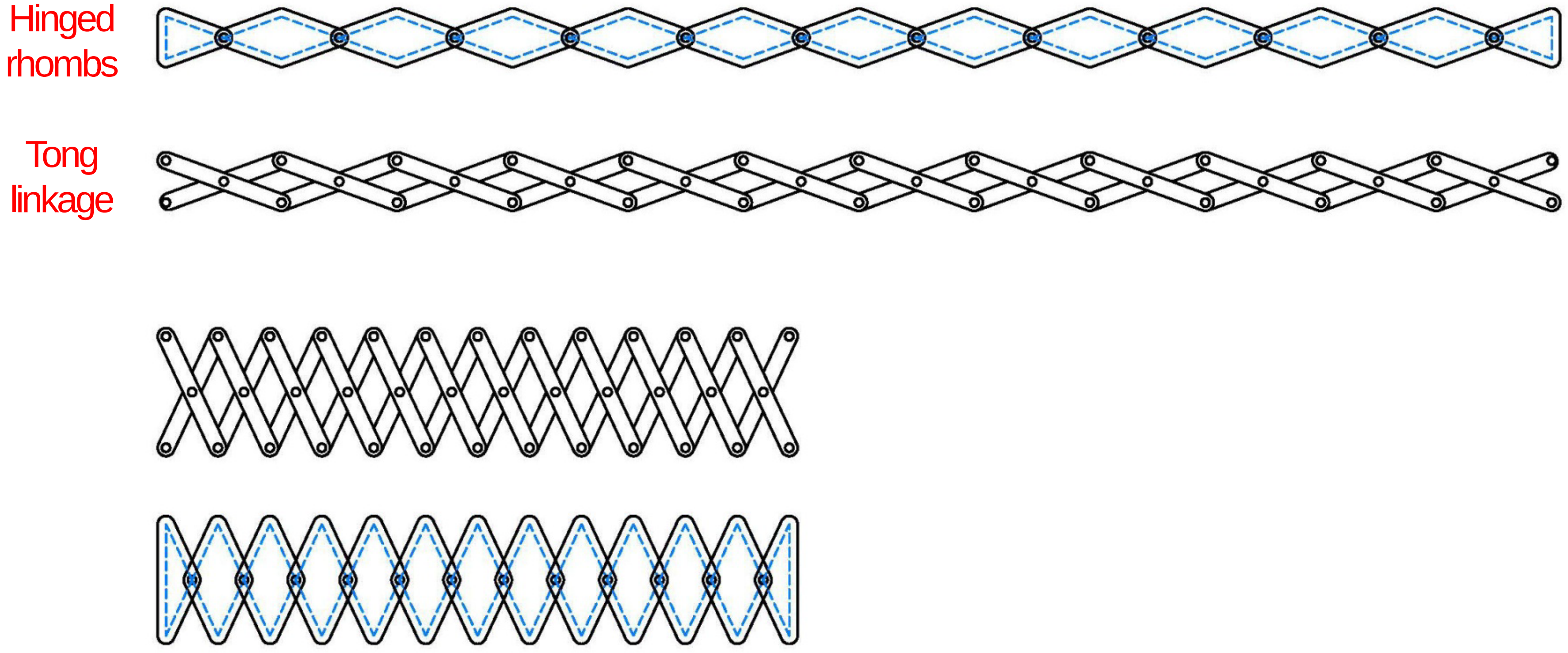
Angulated link: geometric construction



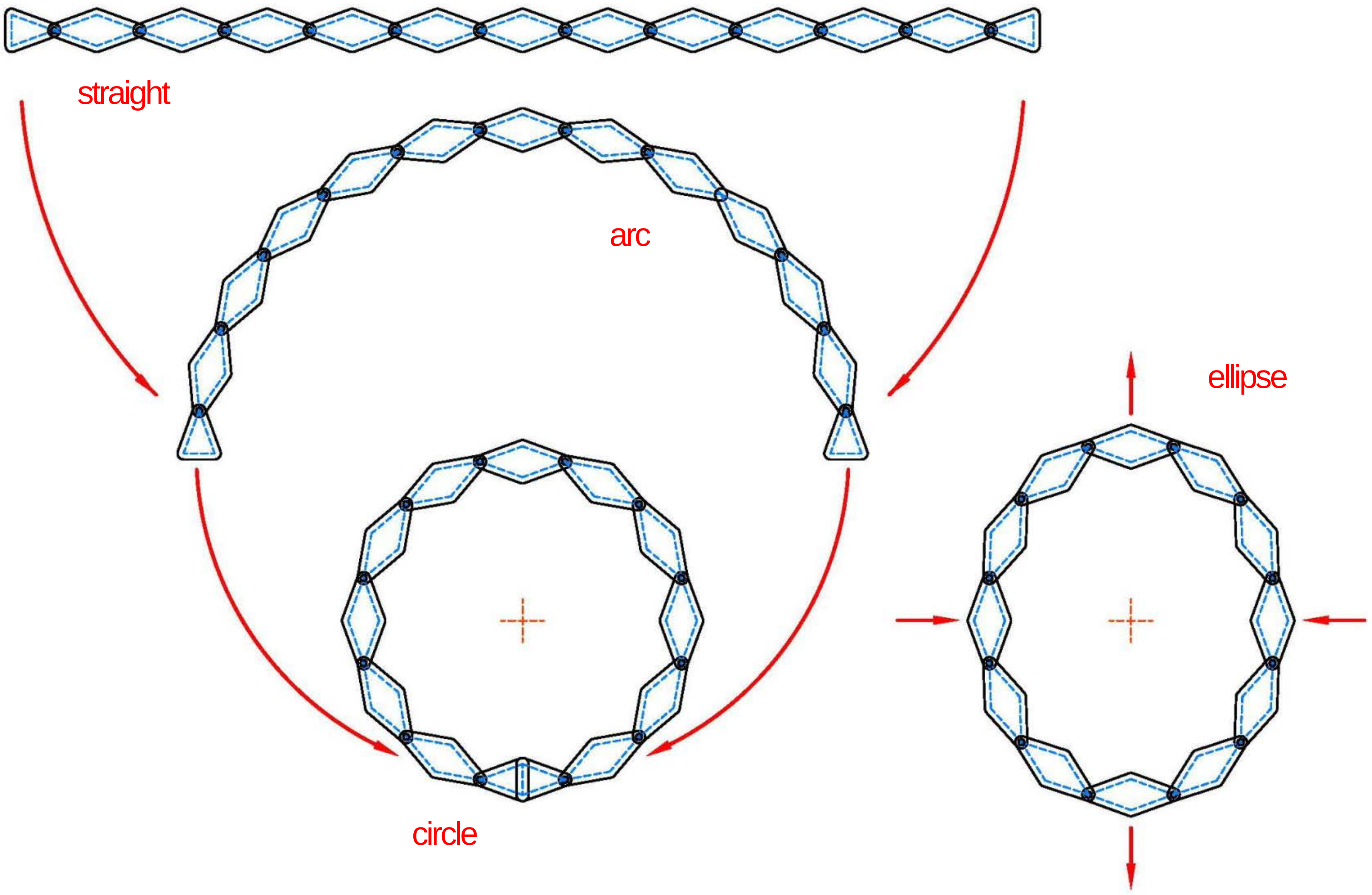
Angulated link: geometric construction



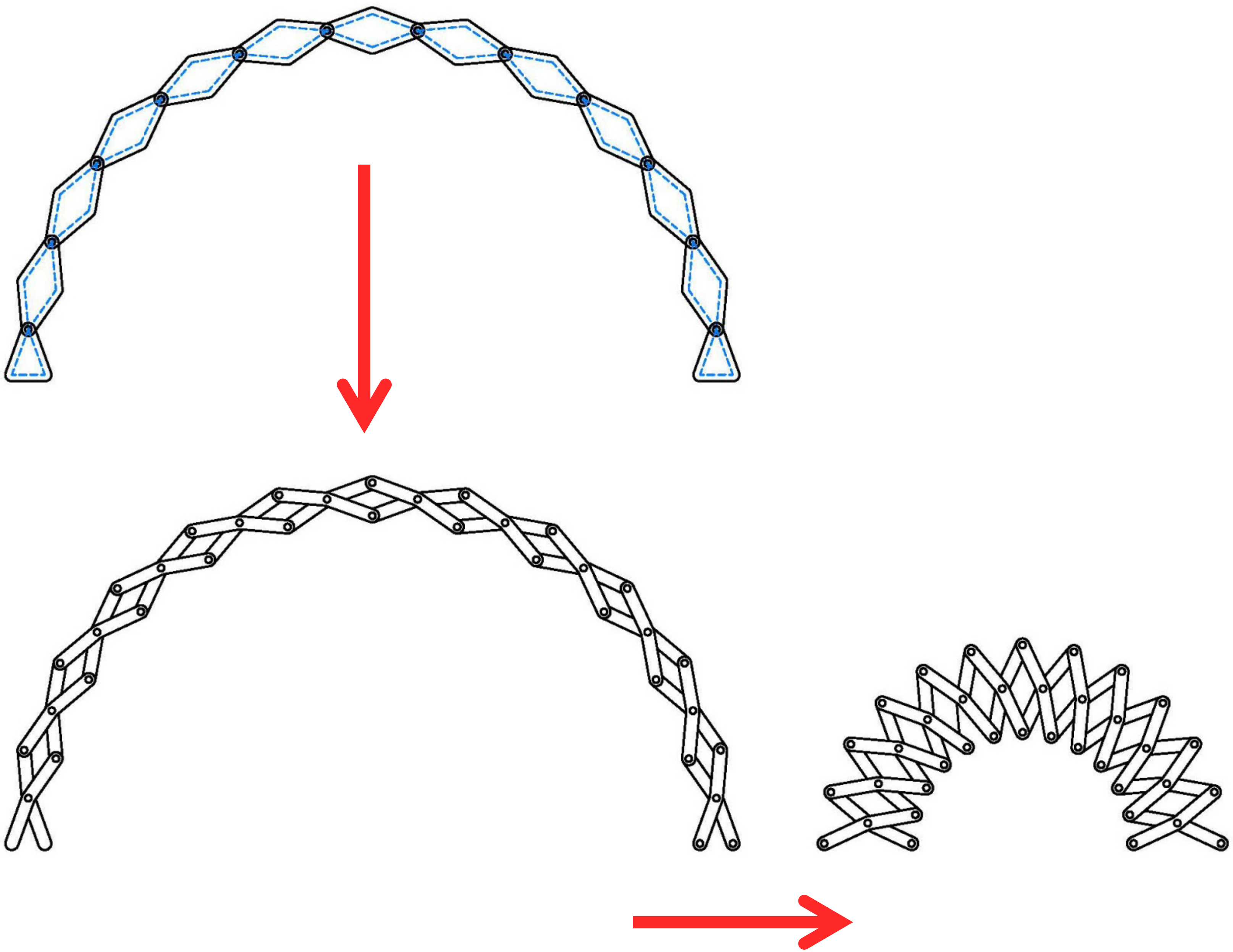
Tong linkage



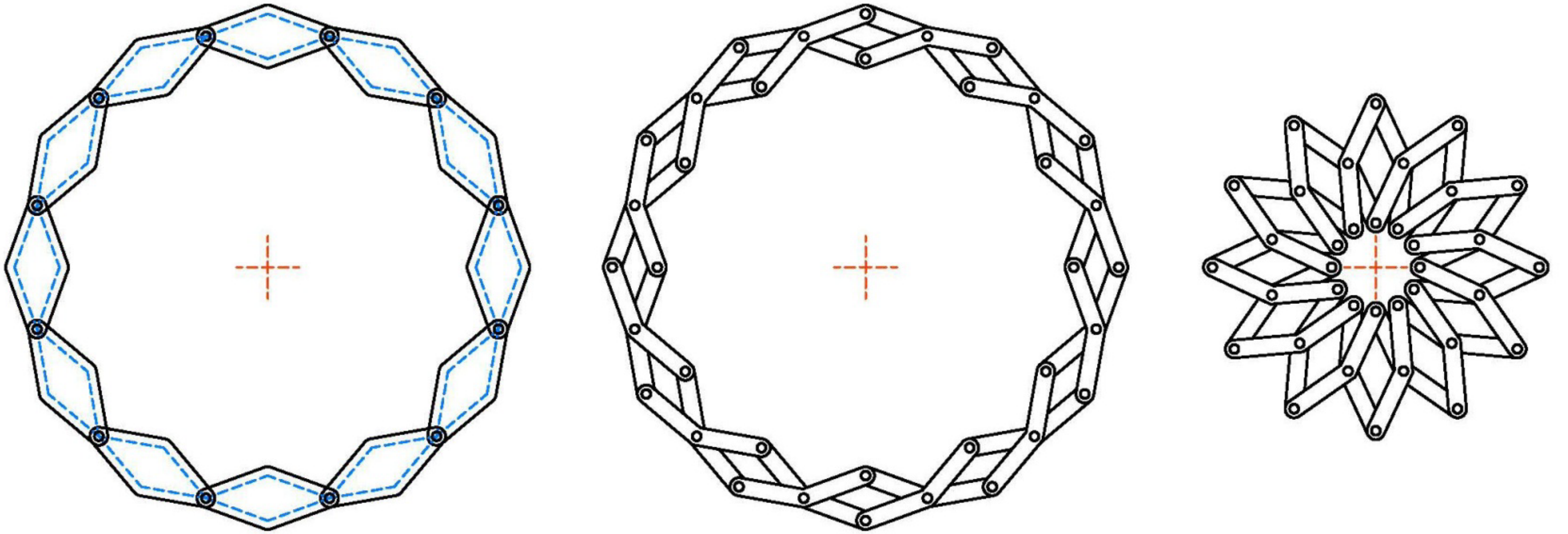
Hinged rhombs – transforming between configurations



Arc - geometric construction



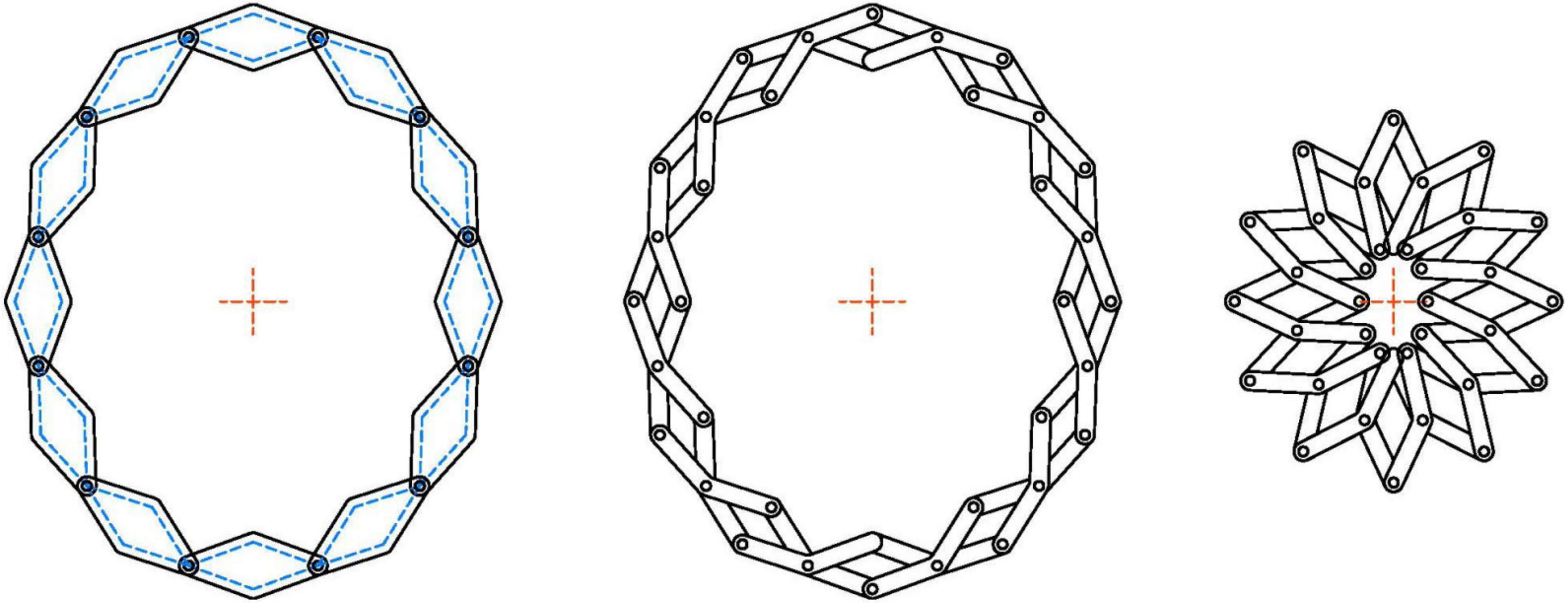
Circle - geometric construction



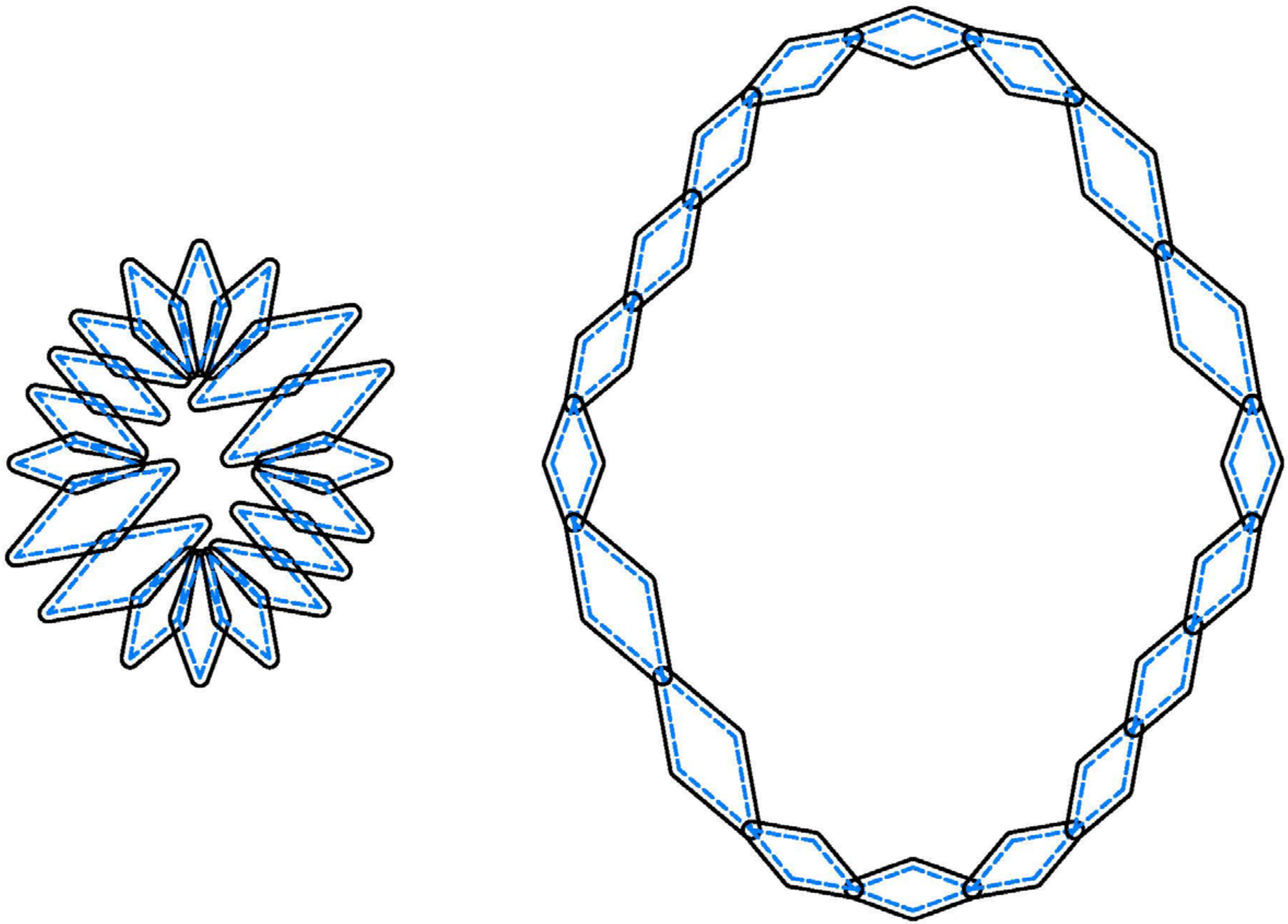
Ring linkages

Ring linkages

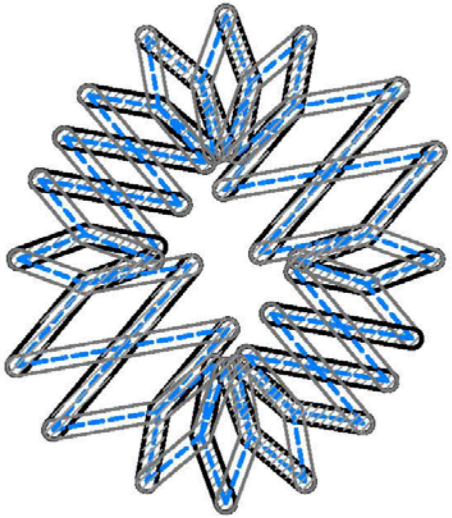
Ellipse - geometric construction

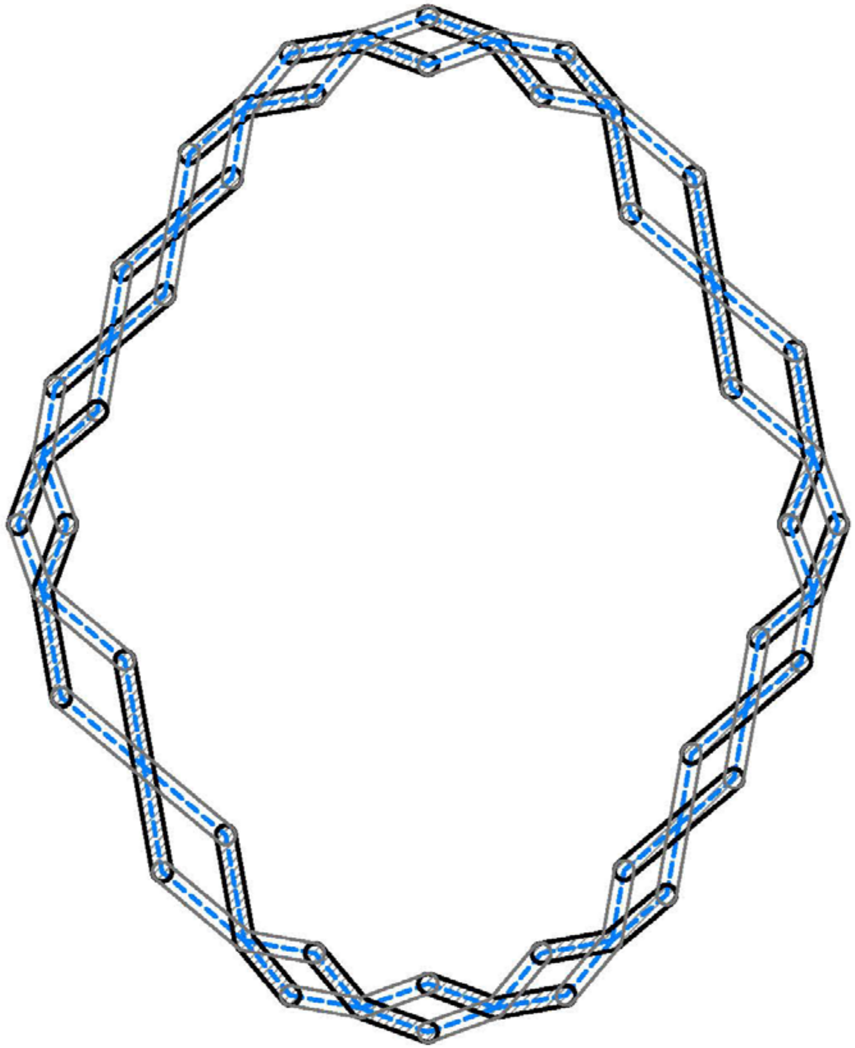


Unequal rhombs

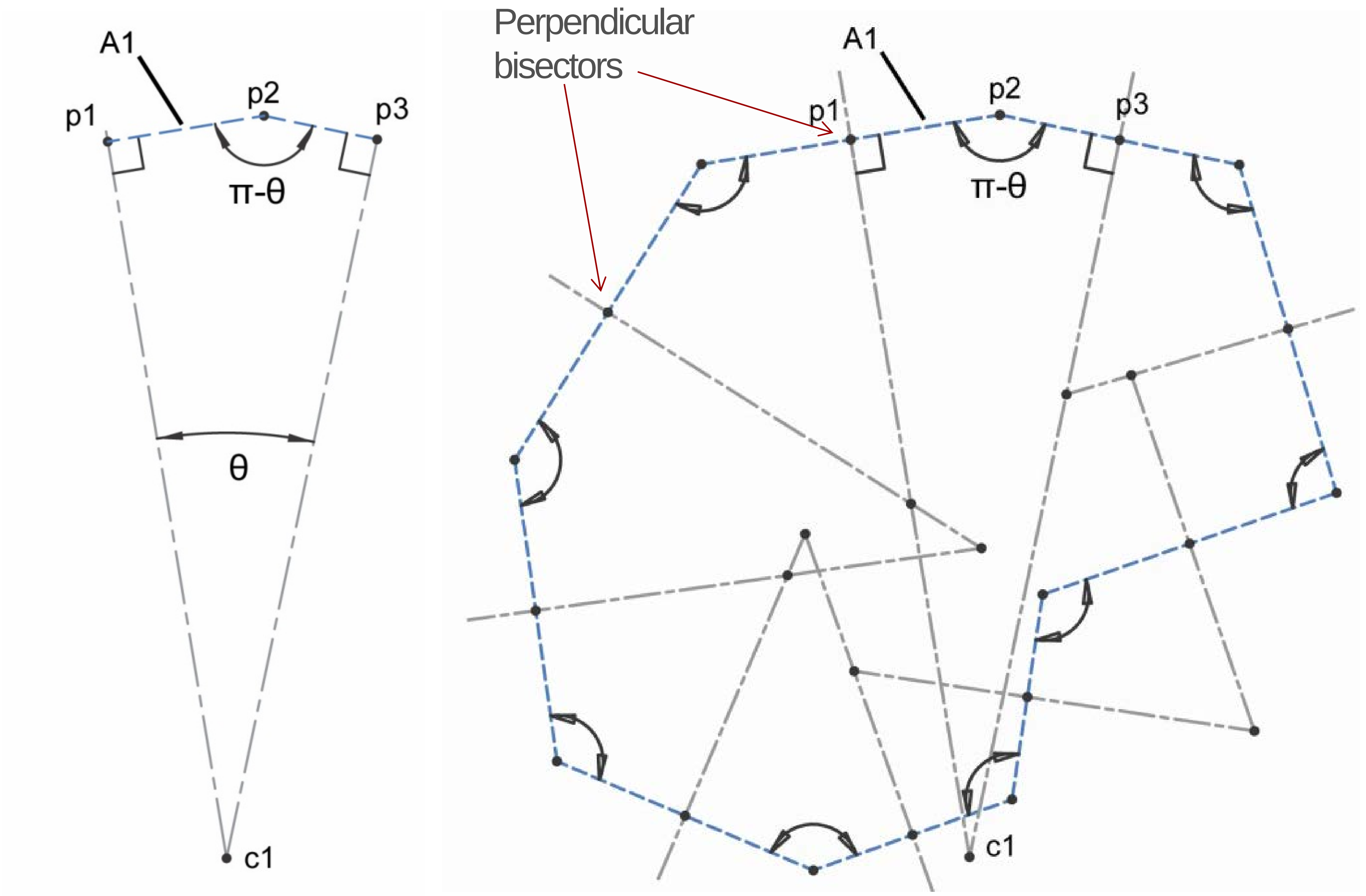


Unequal rhombs

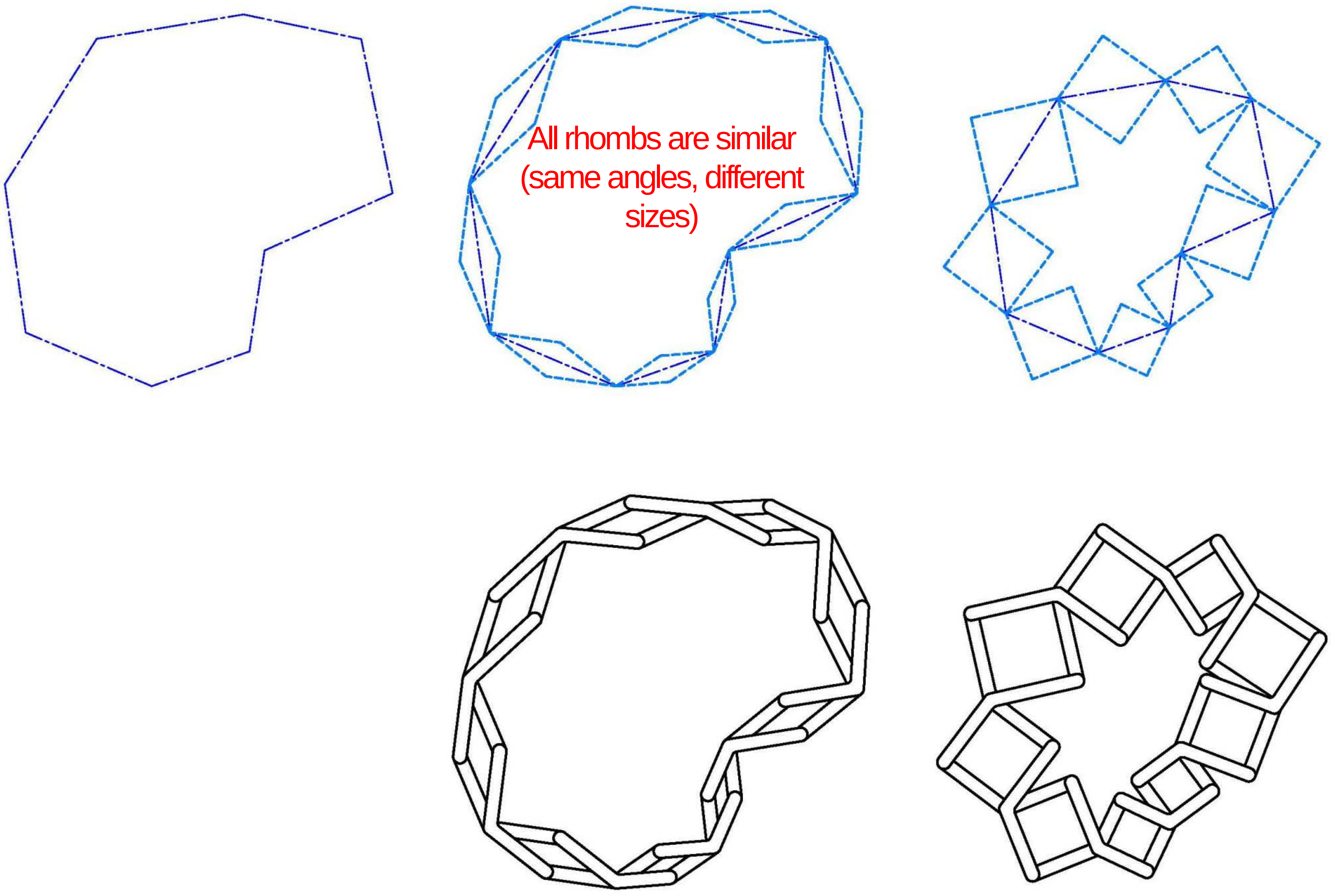




Constructing expanding polygons



Irregular polygon – geometric construction



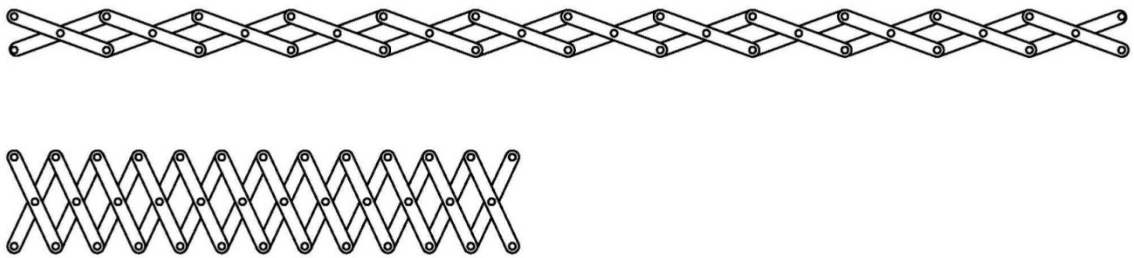
Degrees of freedom of a tong linkage

Number of pivots for a tong linkage:

P = 3N/2 -2

DOF = 3 X (N-1) – 2P

= 3N – 3 – (3N – 4) **= 1**

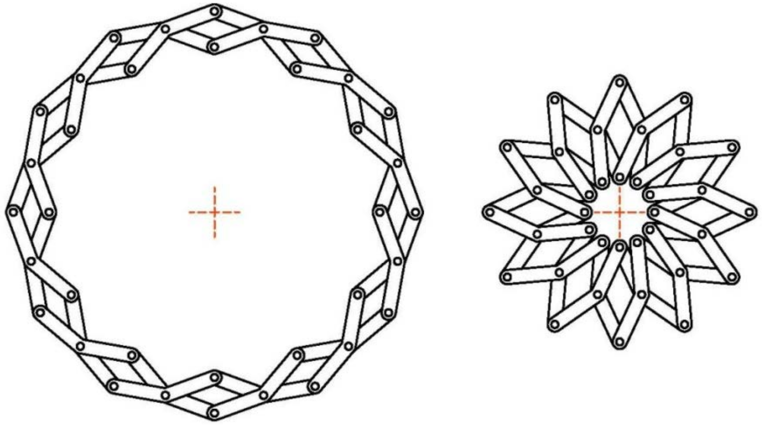


Number of pivots for a closed tong linkage:

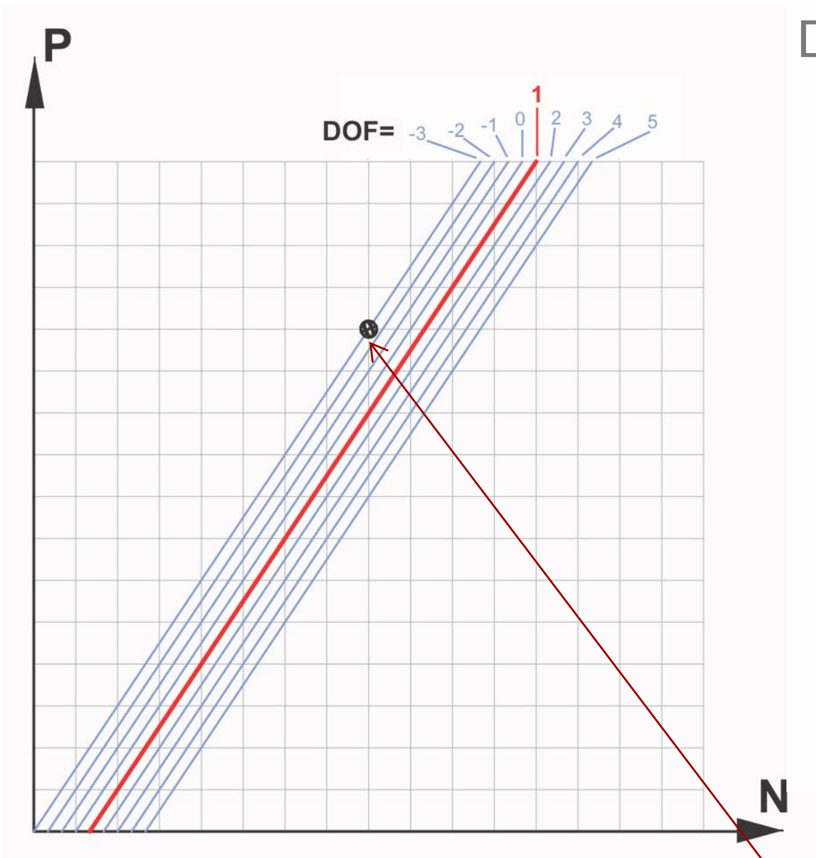
P = 3N/2

DOF = 3 X (N-1) – 2P

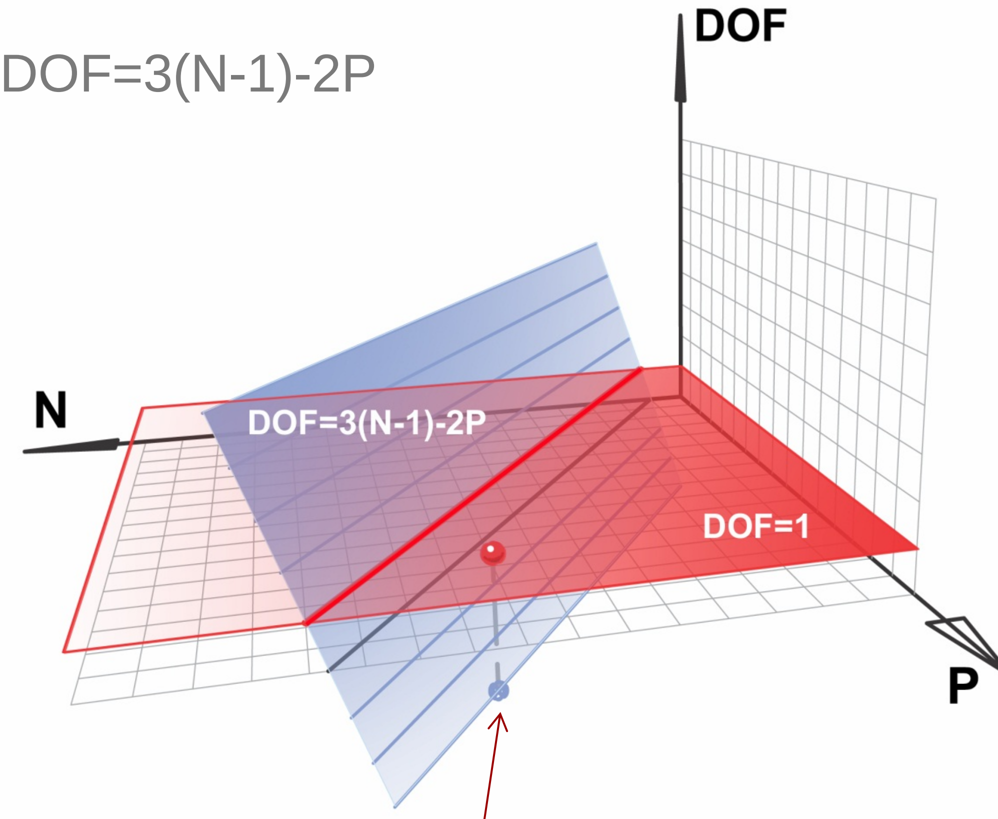
= 3N – 3 – 3N = **- 3**

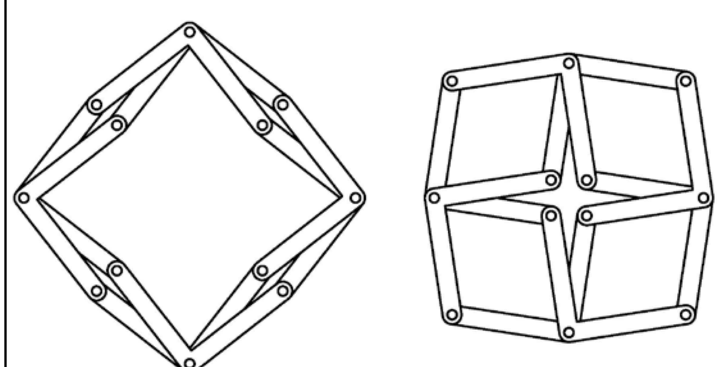


Spatial interpretation of Gruebler's equation



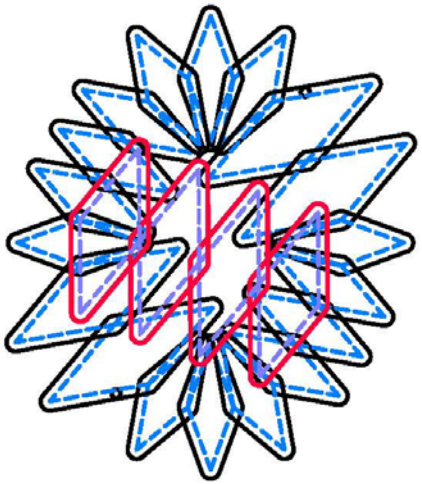
$$DOF=3(N-1)-2P$$

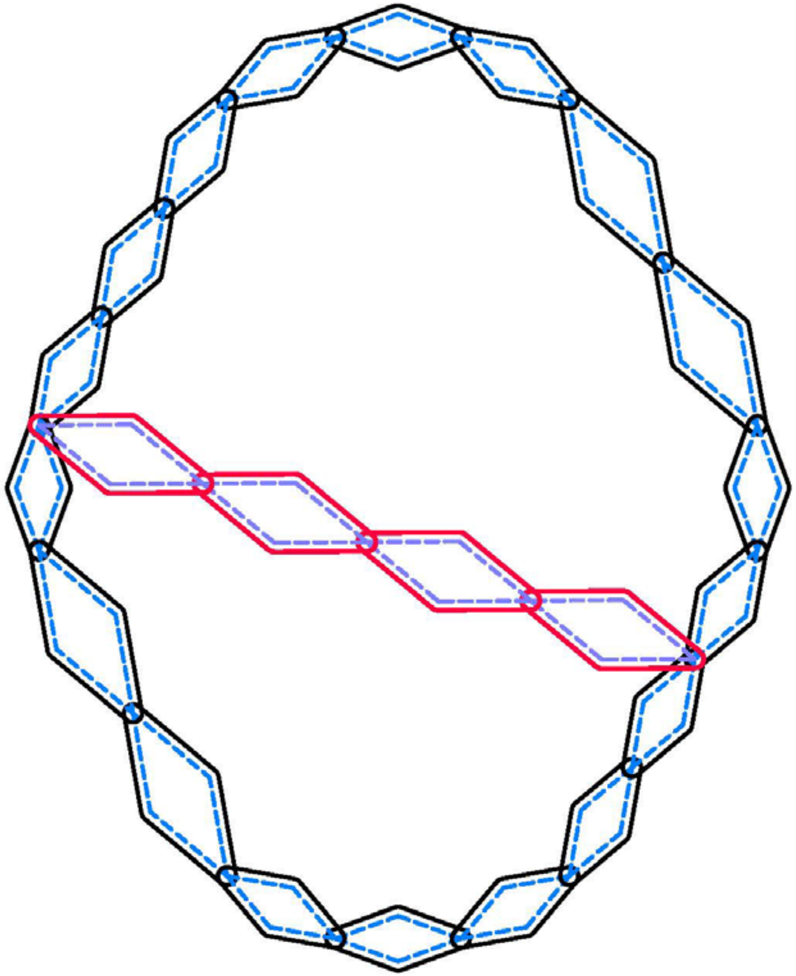




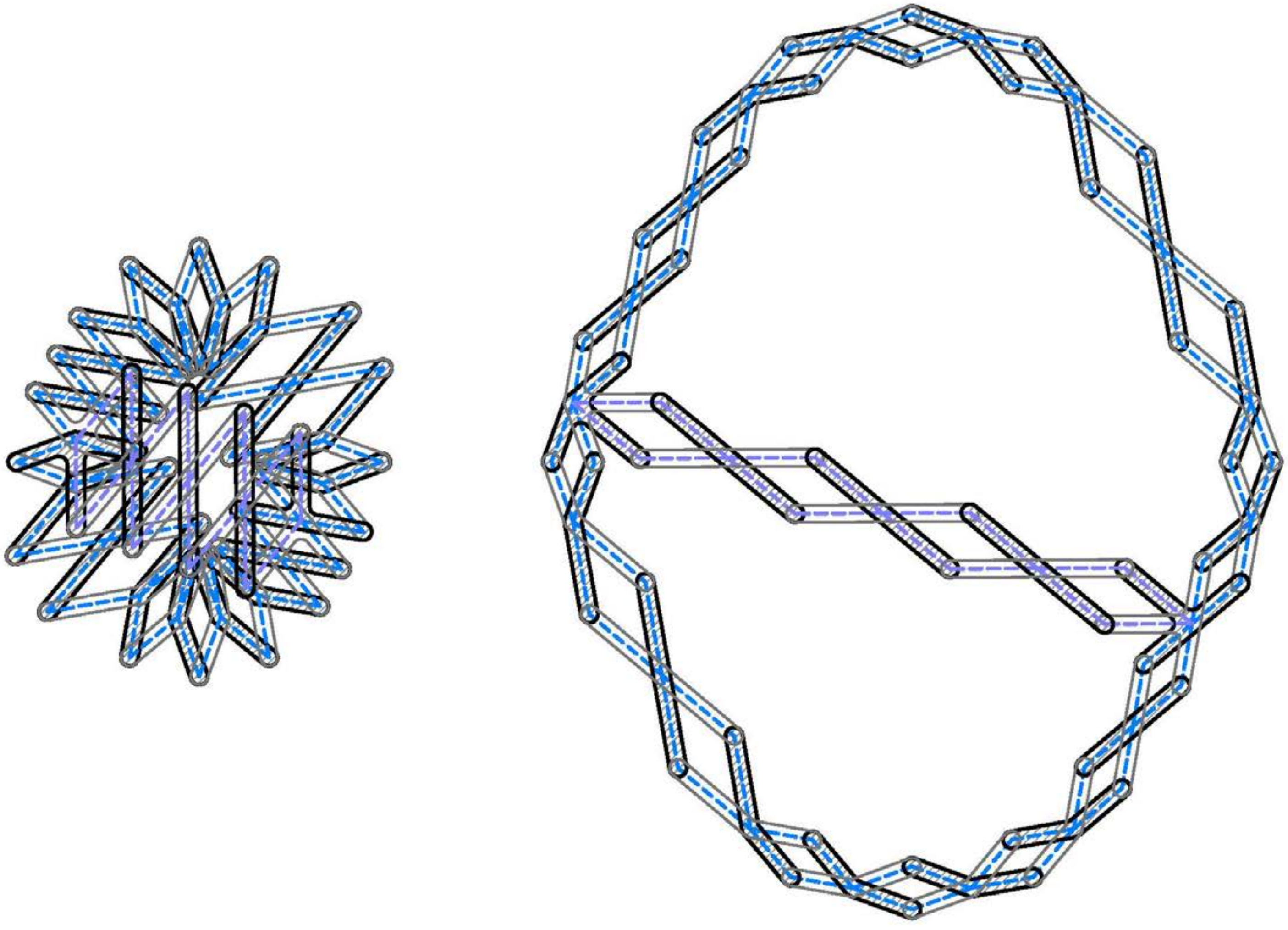
$N = 8$
$P = 12$
$DOF = -3$

Unequal rhombs with crossing connection

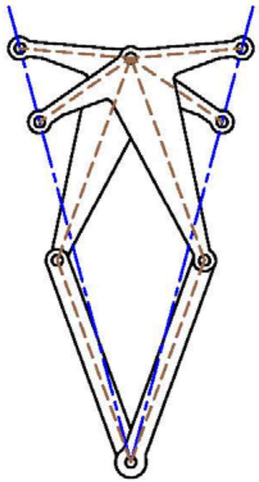


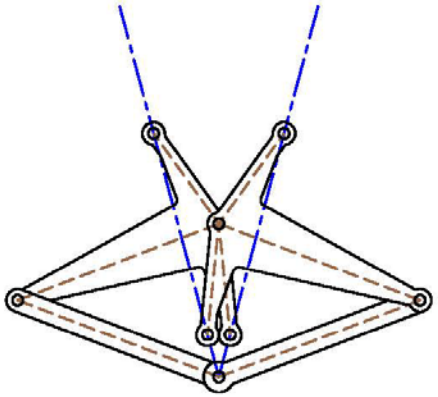


Unequal rhombs with crossing connection

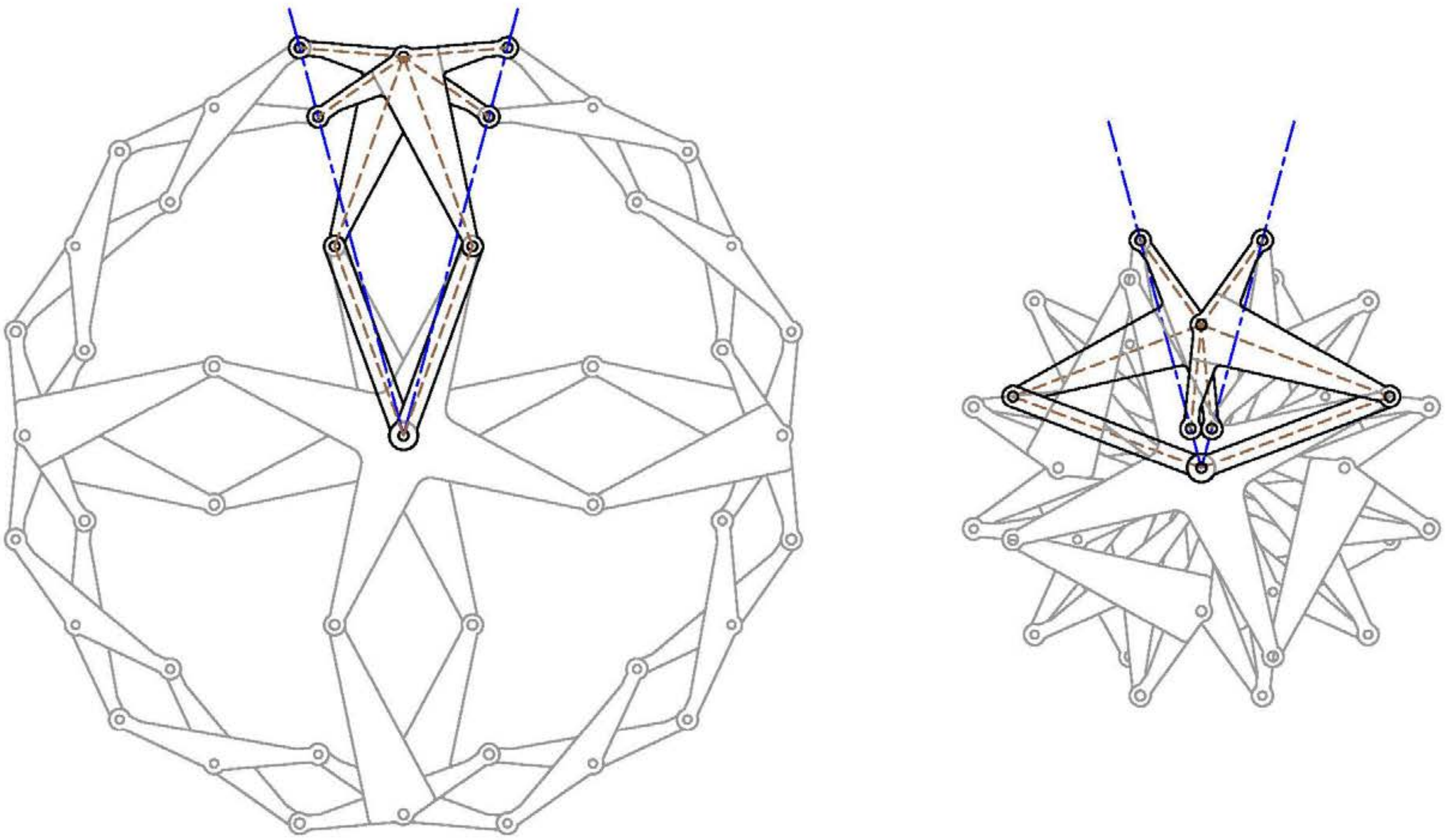


Polygon linkages with fixed centers

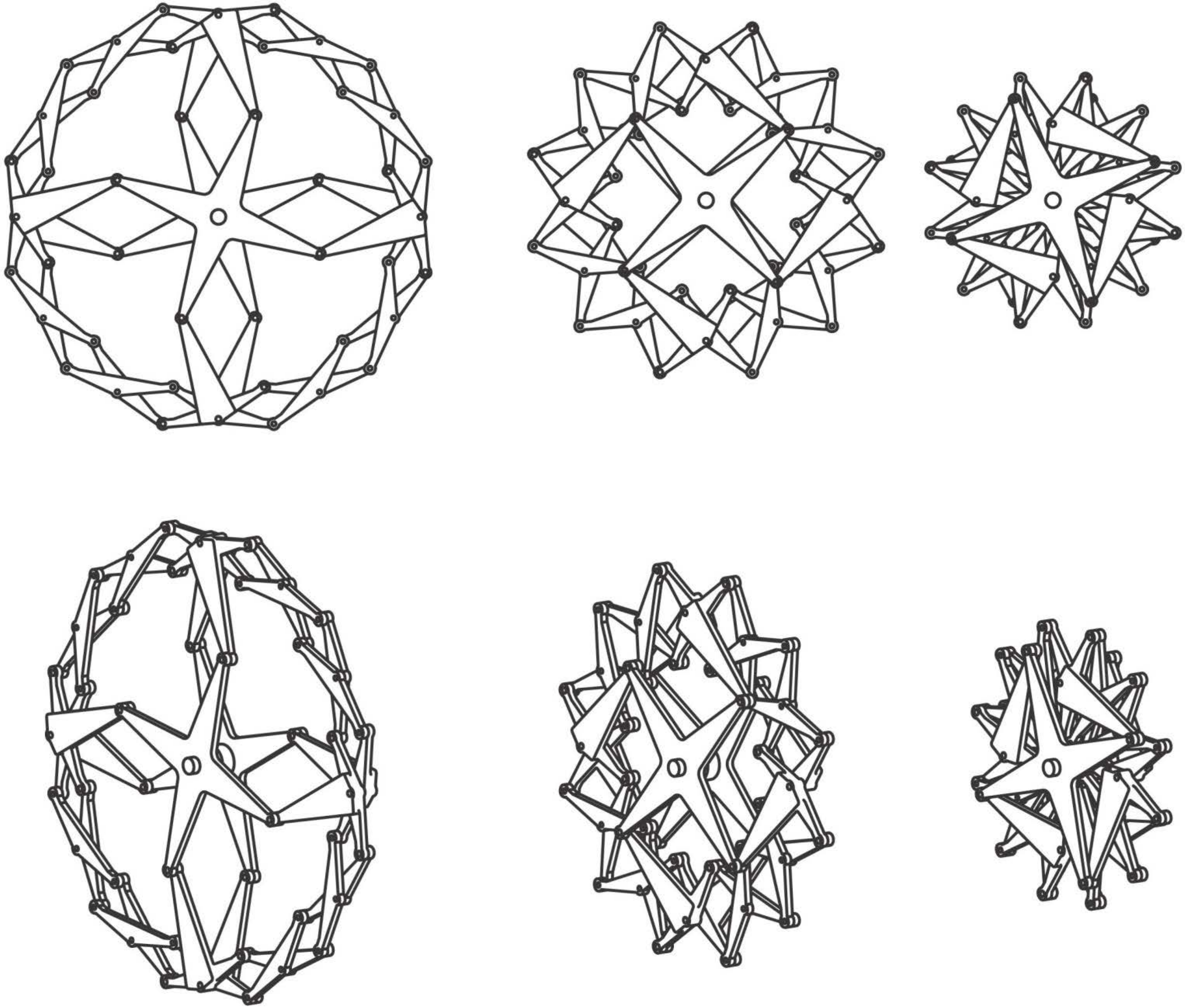




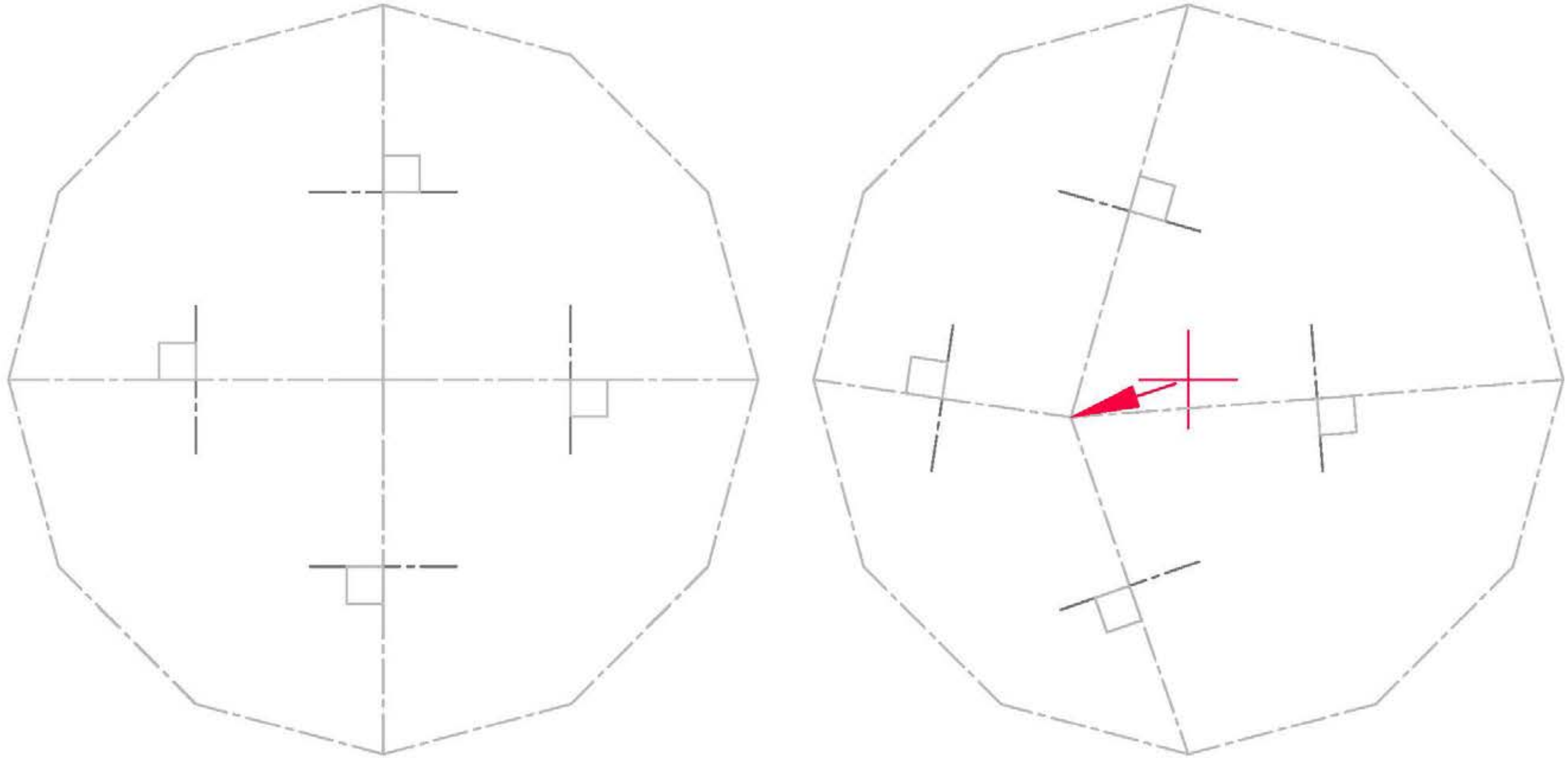
circular linkage with fixed center (four spokes)



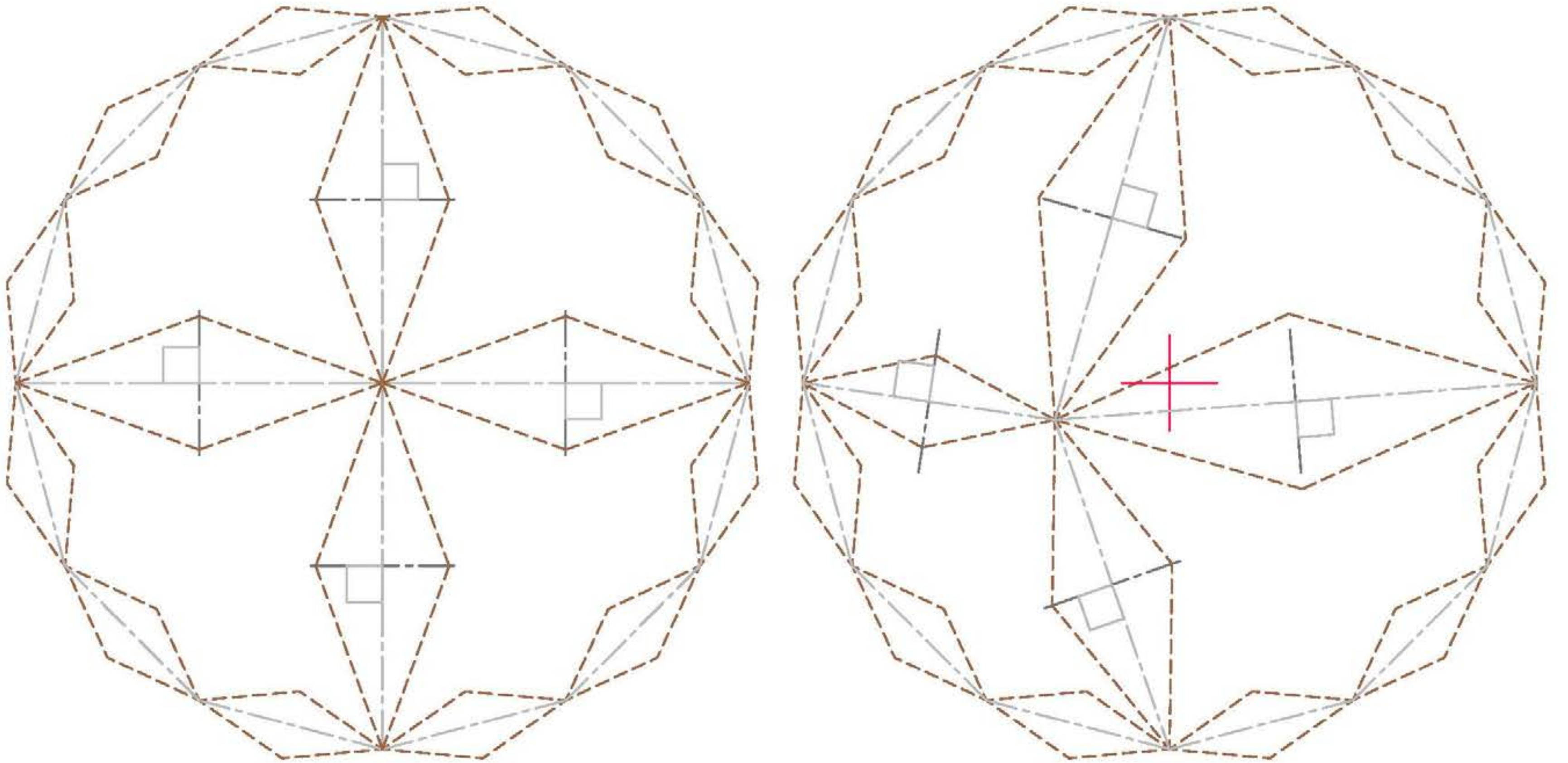
Circular linkages with fixed center



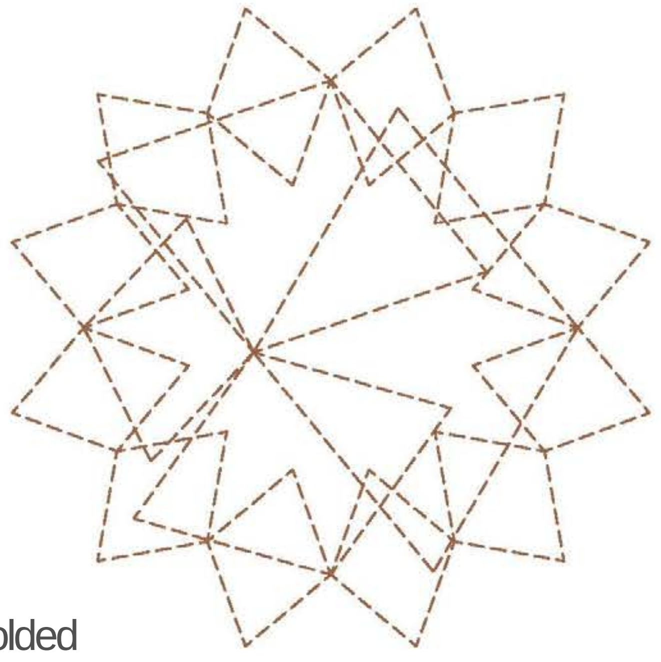
Construction for off-center fixed point



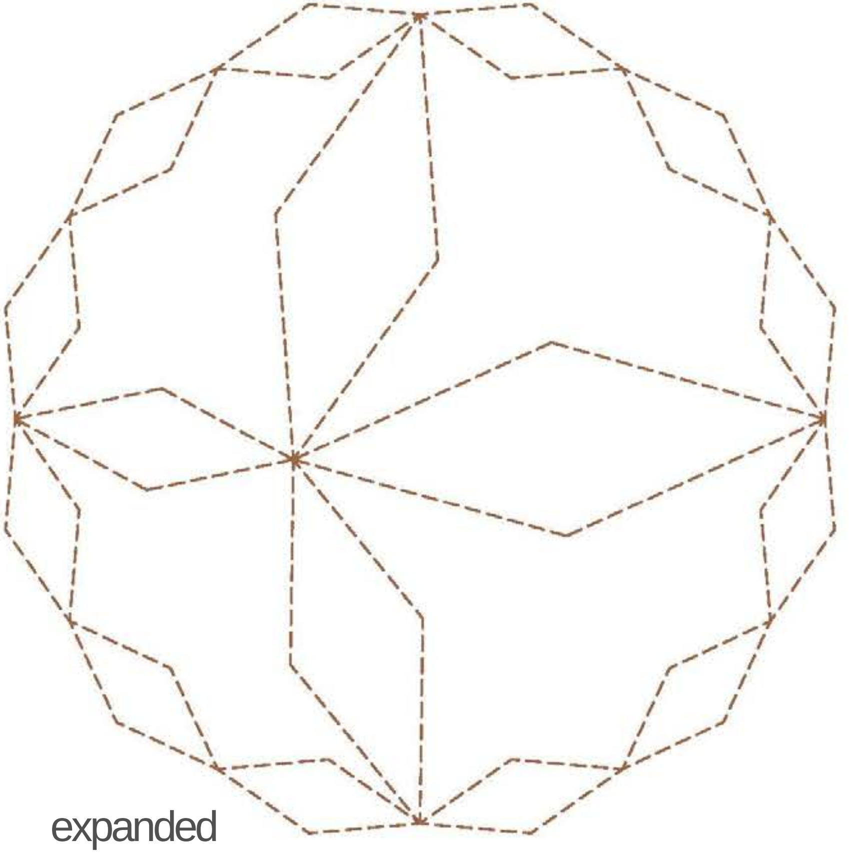
Polygon linkages – off-center fixed point



Polygon linkages – off-center fixed point

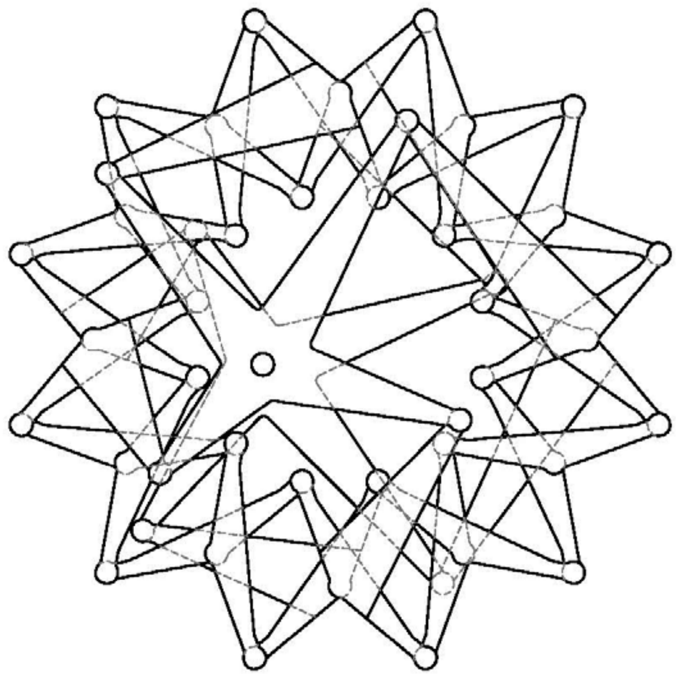


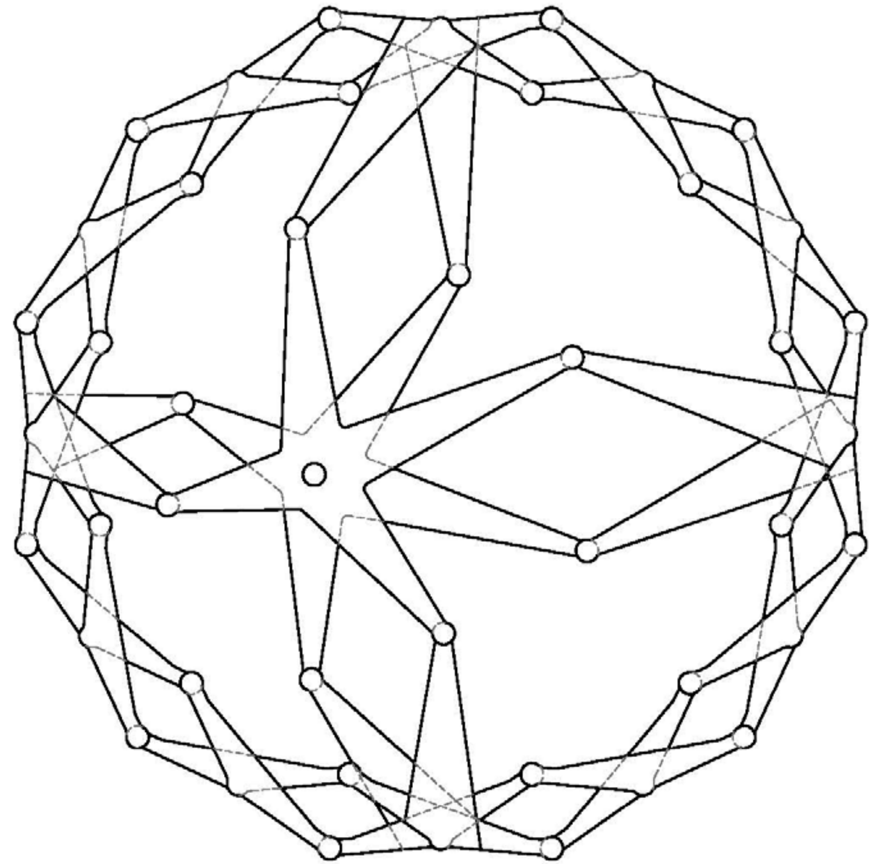
folded



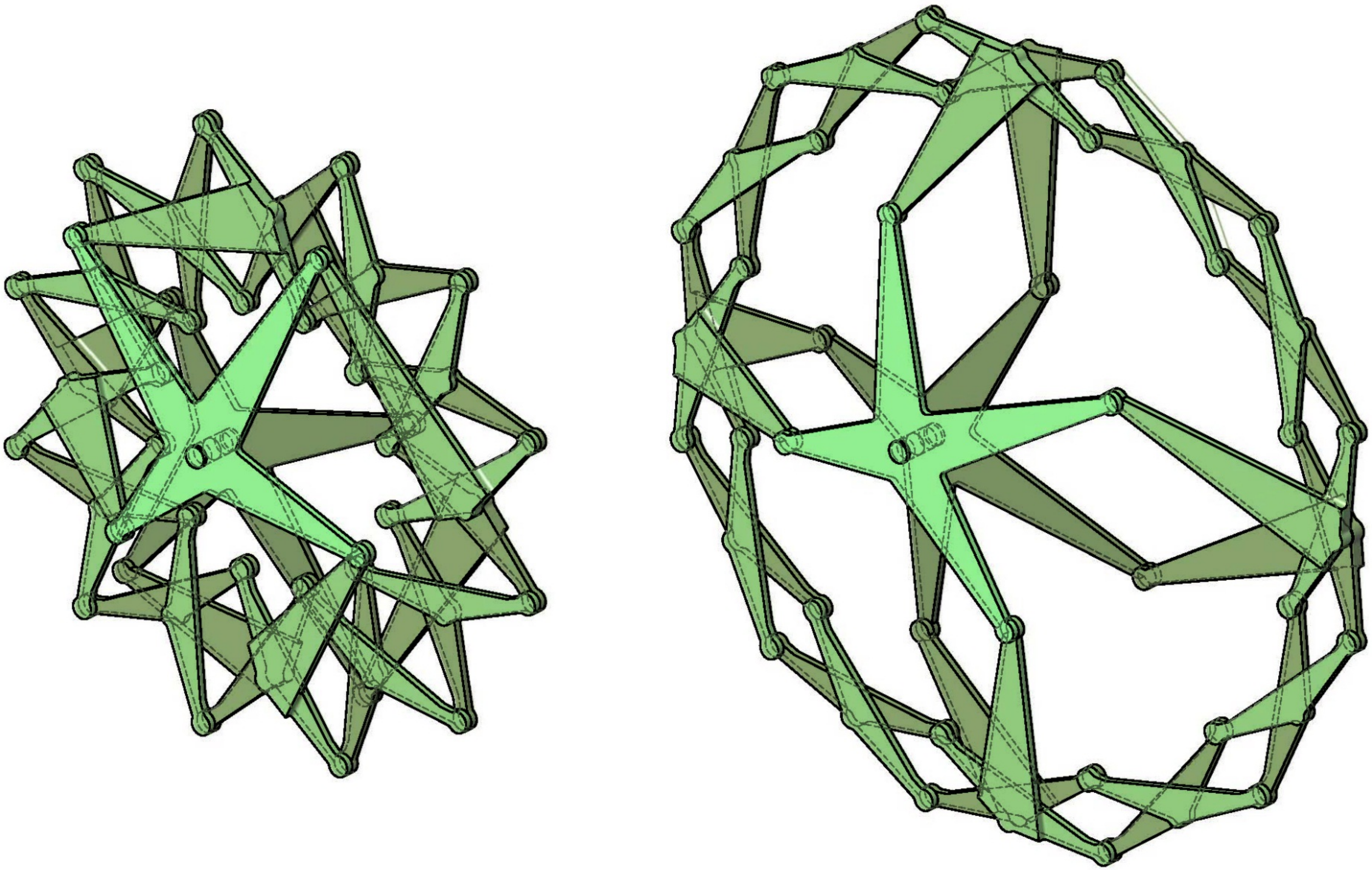
expanded

Polygon linkages – off-center fixed point

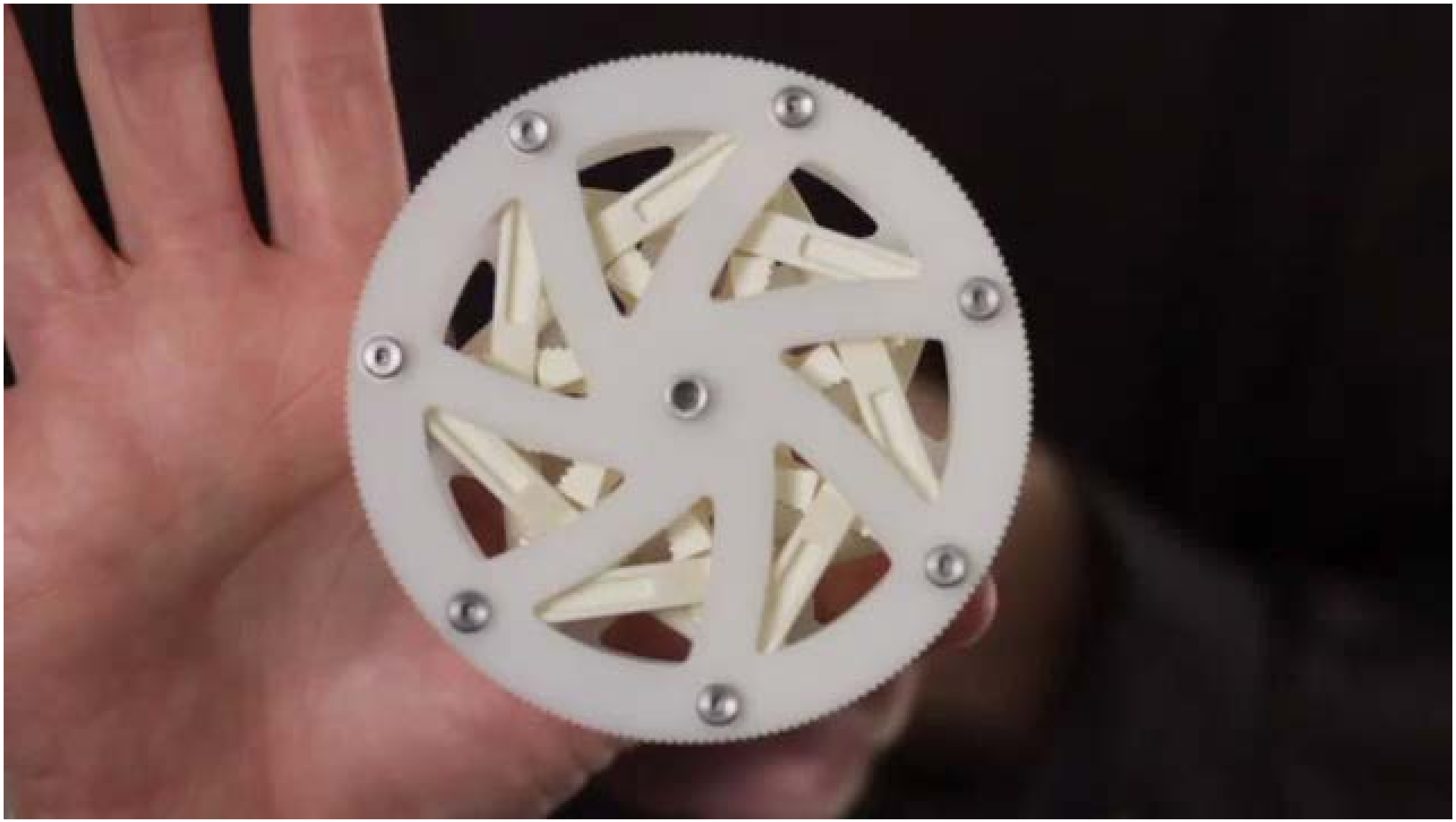




Polygon linkages – off-center fixed point



Ring linkages



Degrees of freedom (Graver formulation)

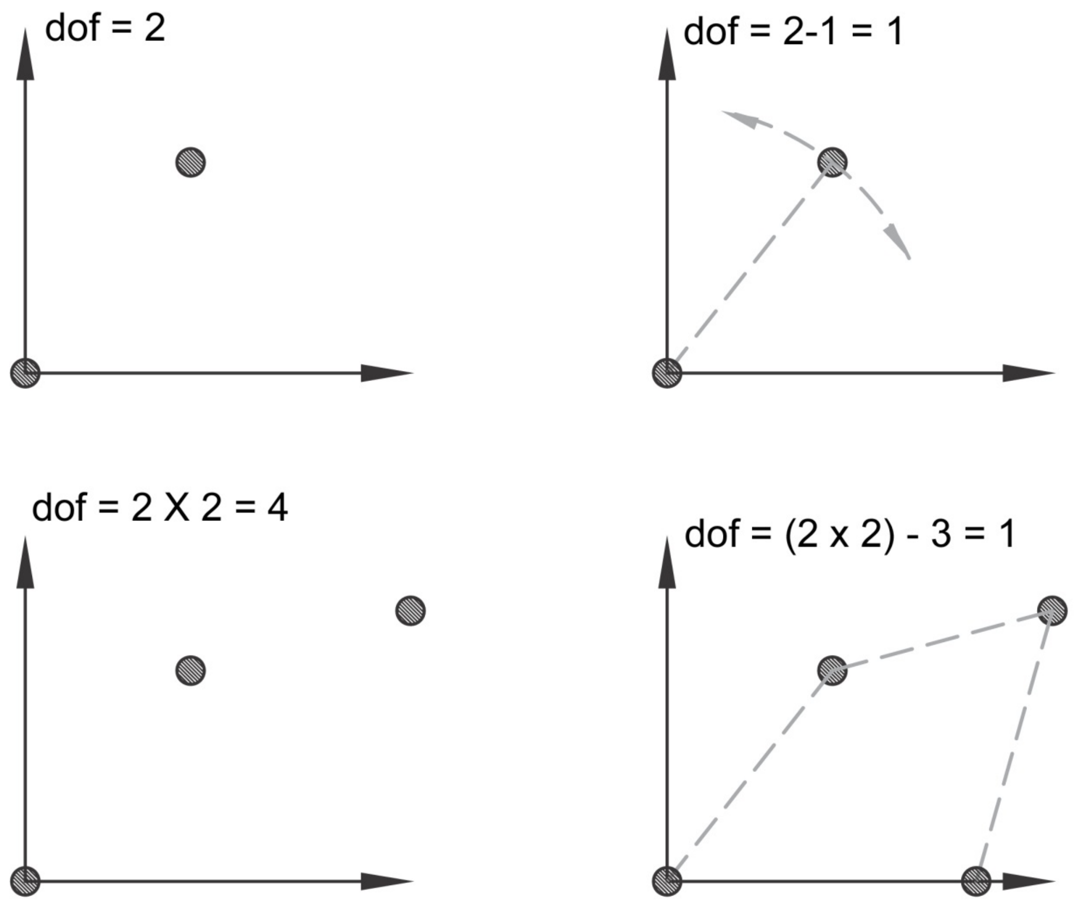
Each point (pivot) has 2 degrees of freedom

Each link subtracts 1 degree of freedom

J = Number of joints (2D points in the plane)

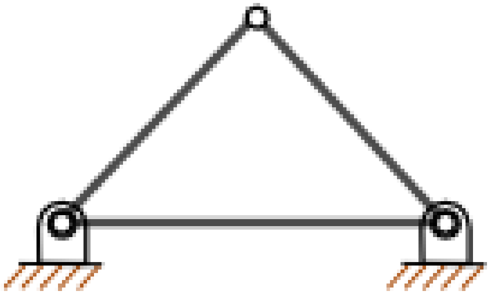
R = Number of links (not including ground link)

$$DOF = 2J - R$$



Examples

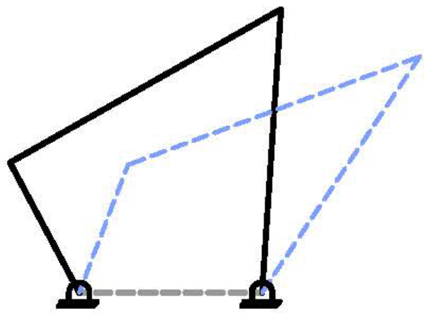
$$DOF = 2J - R$$



J = 1

R = 2

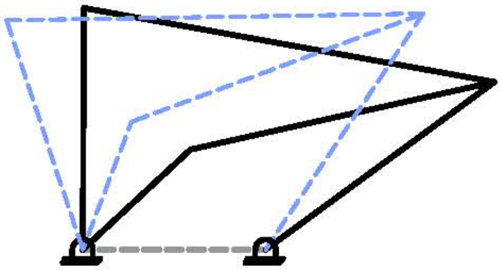
DOF = (2X1)-(1X2)=0



J = 2

R = 3

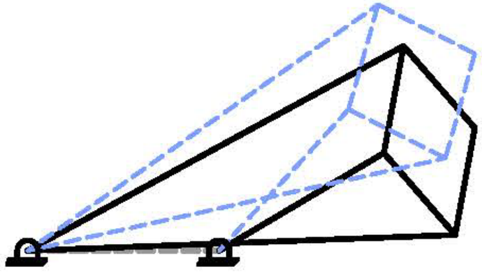
DOF = (2X2)-(3X1)=1



J = 3

R = 5

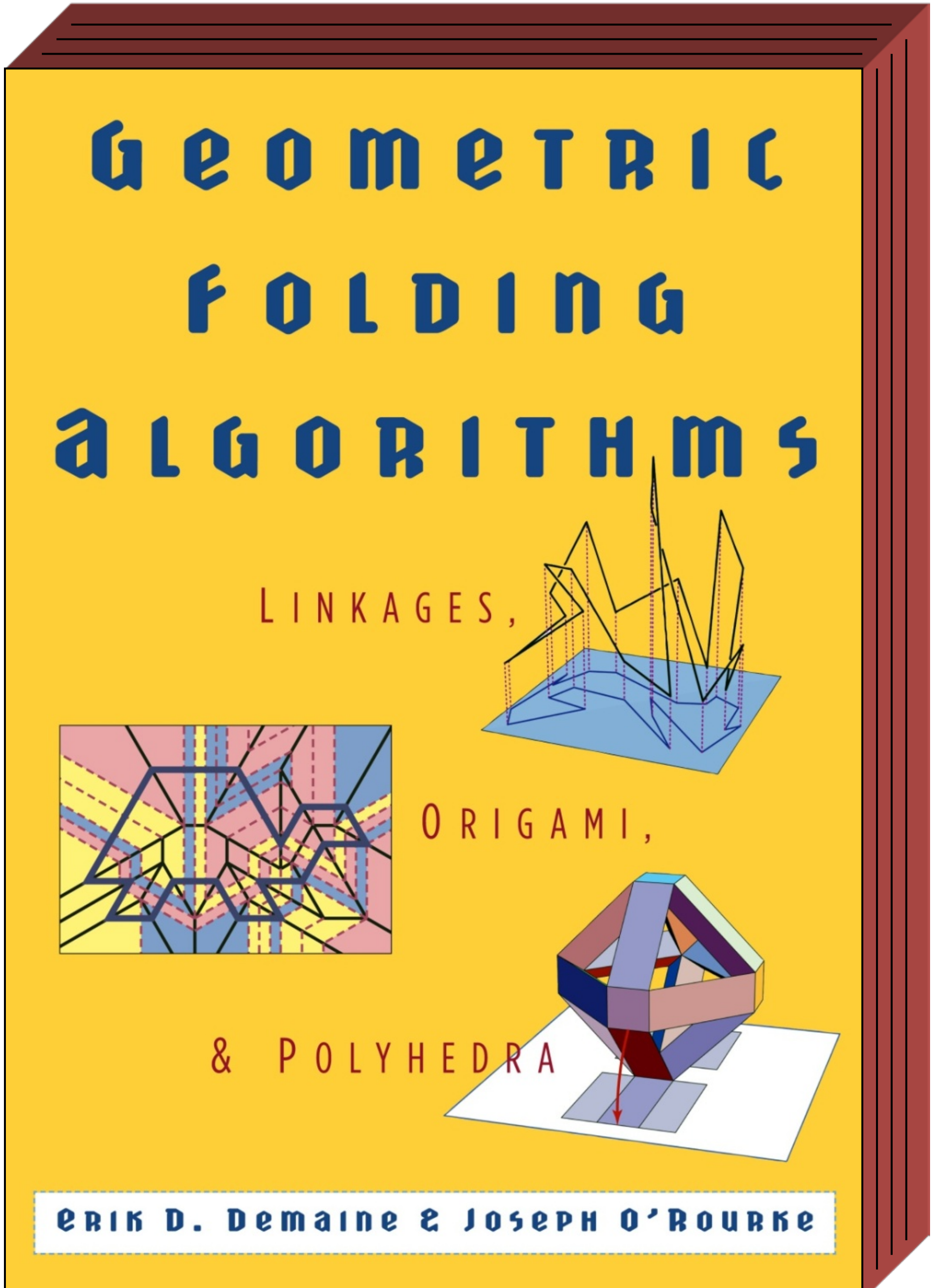
DOF = (2X3)-(5X1)=1



J = 4

R = 7

DOF = (2X4)-(7X1)=1


Geometric
Folding
Algorithms
Linkages,
Origami,
& Polyhedra
Erik D. Demaine & Joseph O'Rourke


幾何的な Geometric
Folding Algorithms
折りアルゴリズム
リンケージ, 折り紙, 多面体
エリック・D・ドメイン&ジョセフ・オルーク著
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上原隆平訳
Linkages,
Origami,
Polyhedra
近代科学社

Configuration Space

configuration ⟶ point

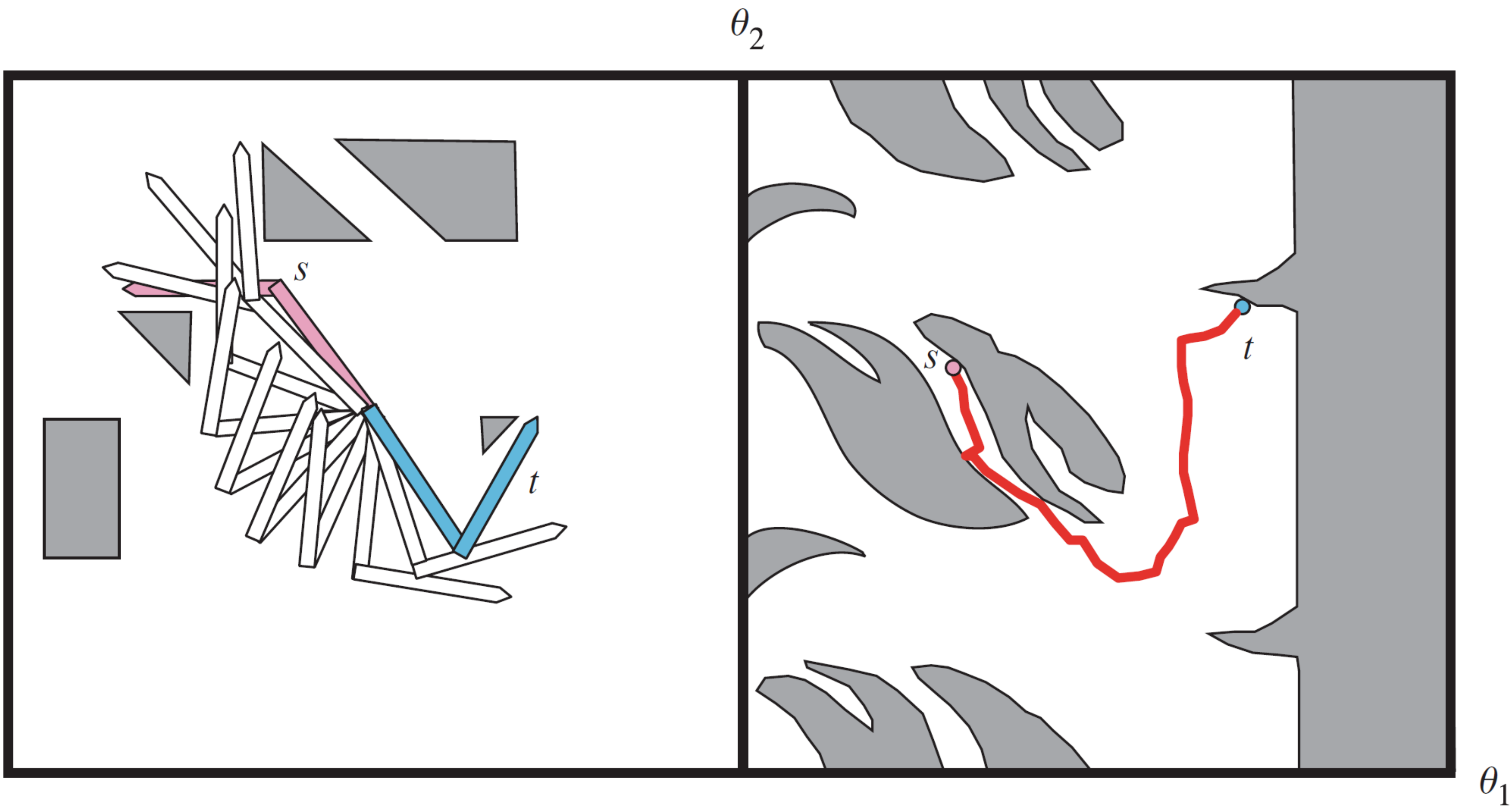
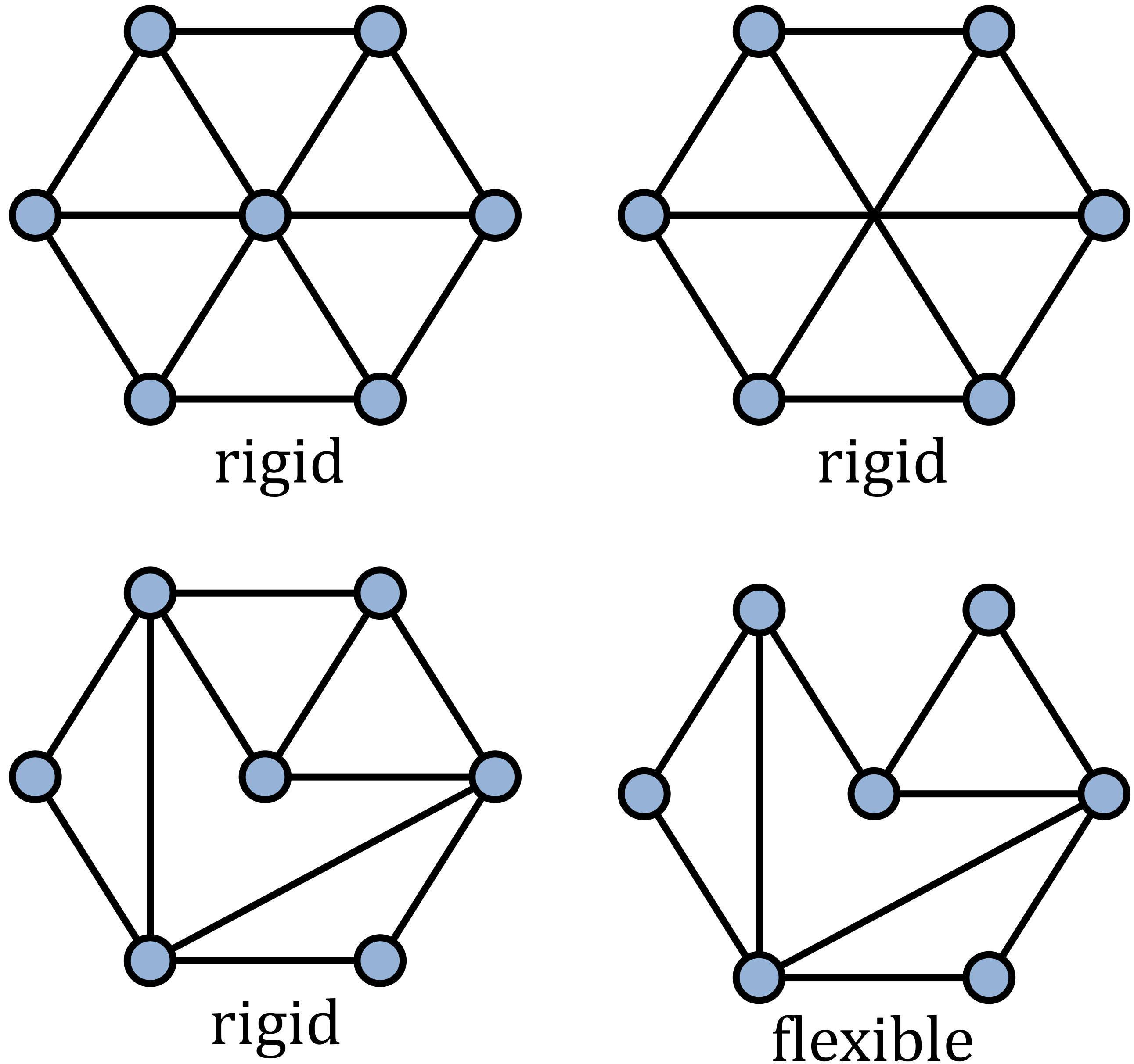


Figure 4.1

Rigidity



Rigidity Depends on Configuration

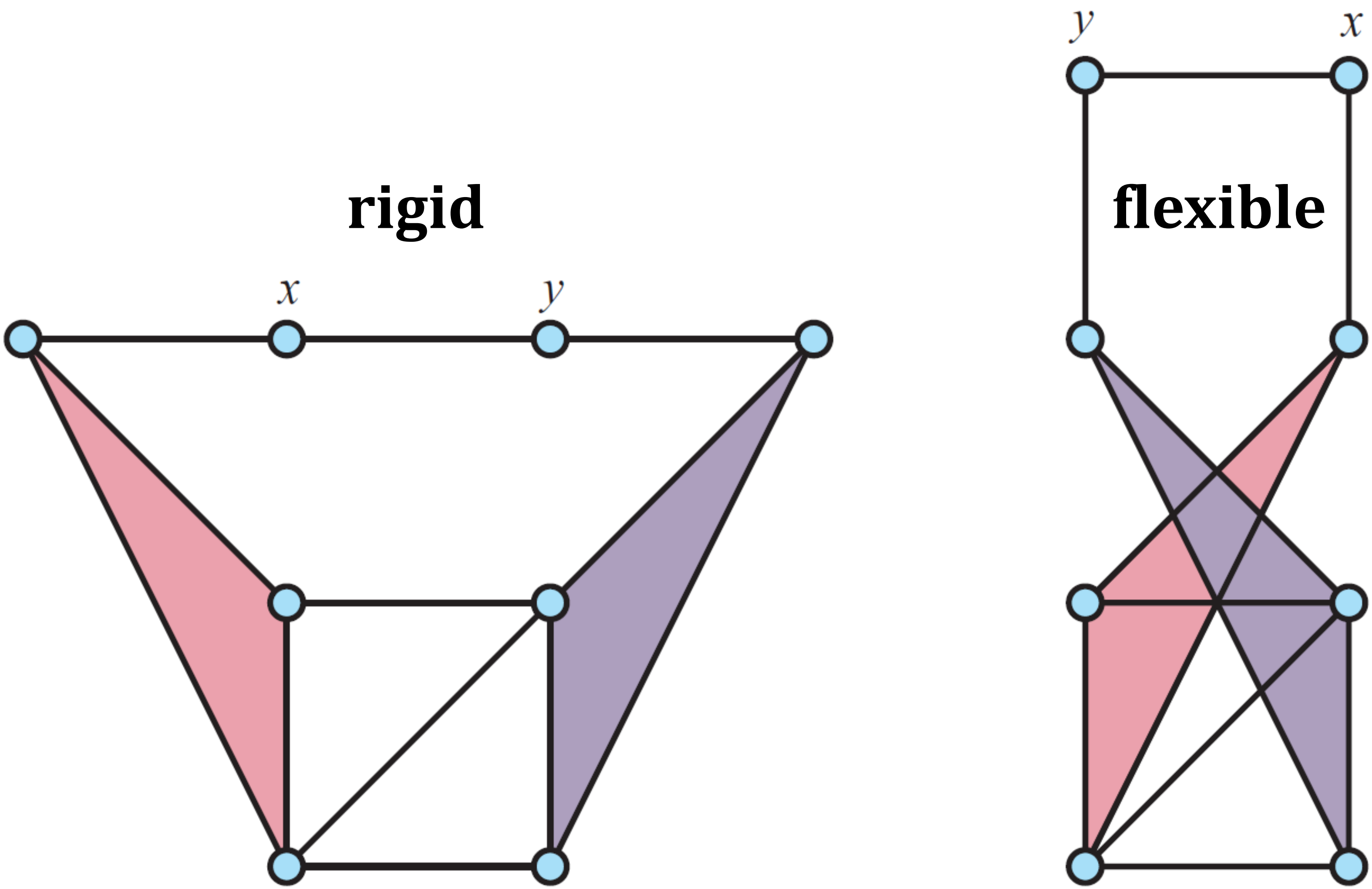


Figure 4.3

Generic Rigidity

generically rigid | **generically flexible**

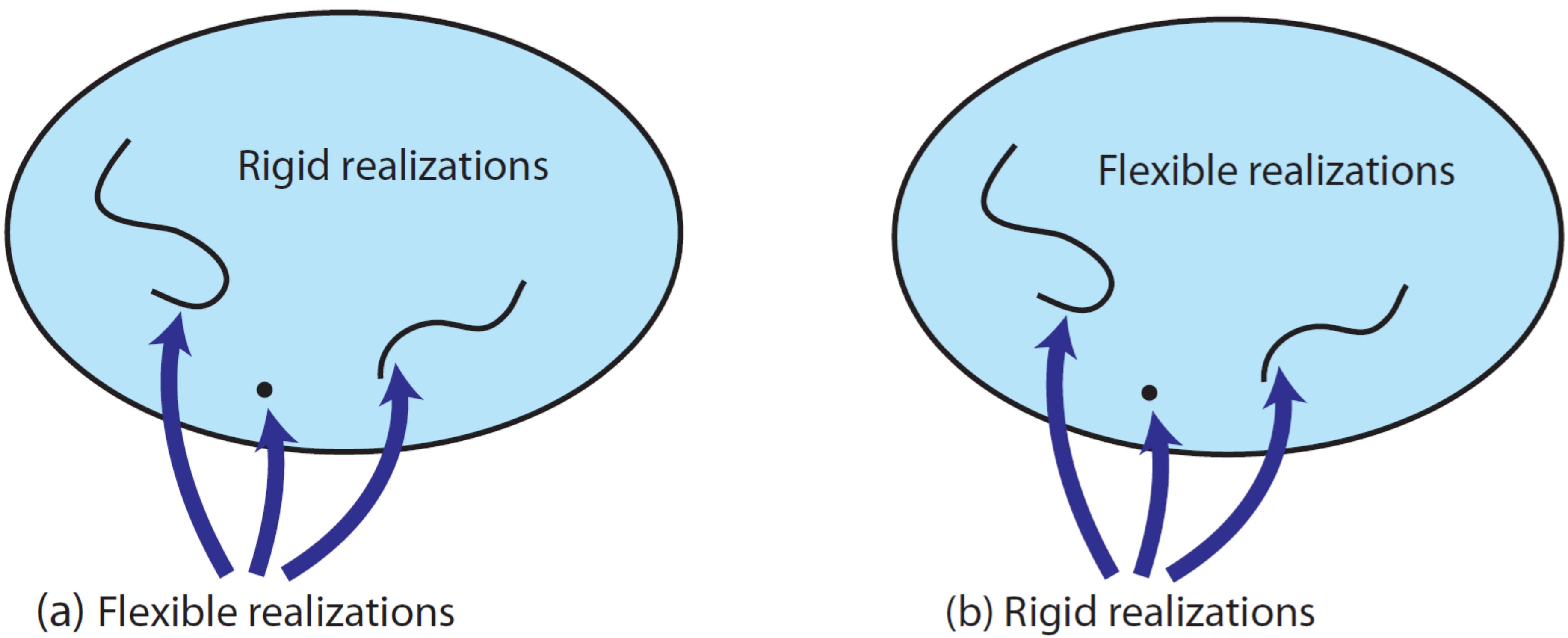
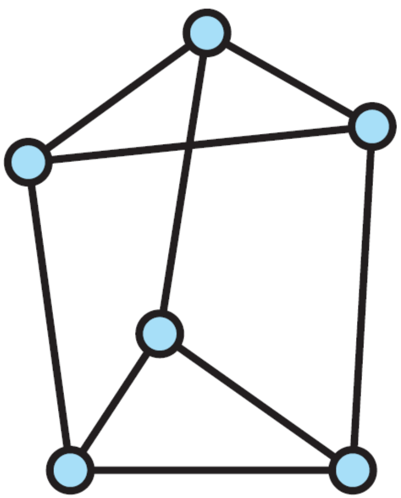
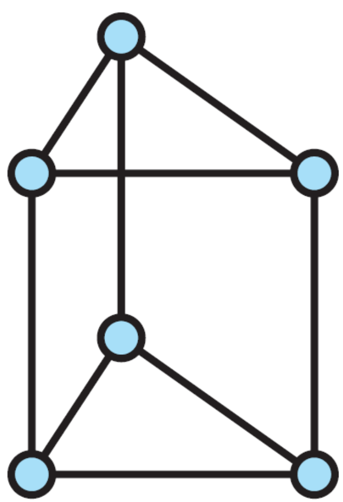


Figure 4.4

Generic Rigidity

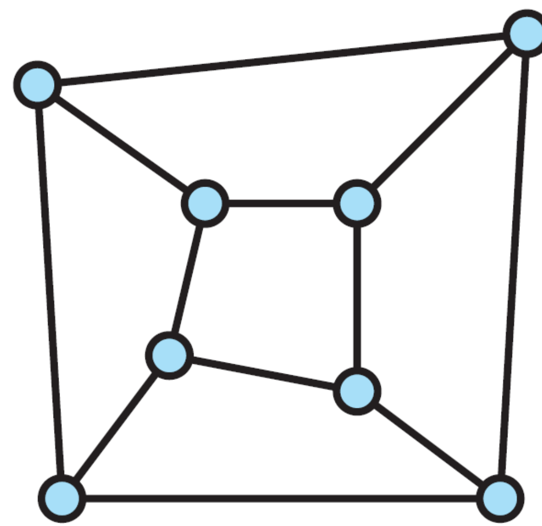
generically rigid

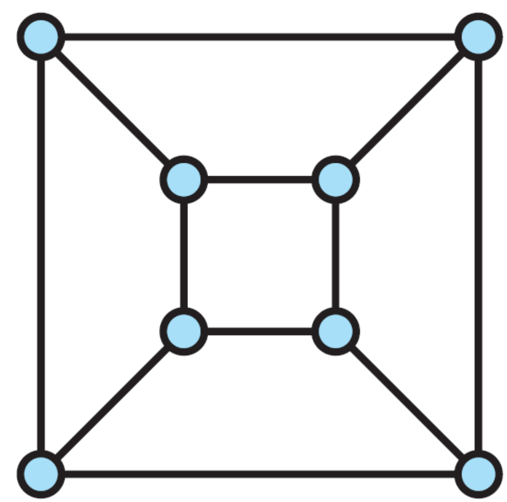




rarely
flexible

generically flexible

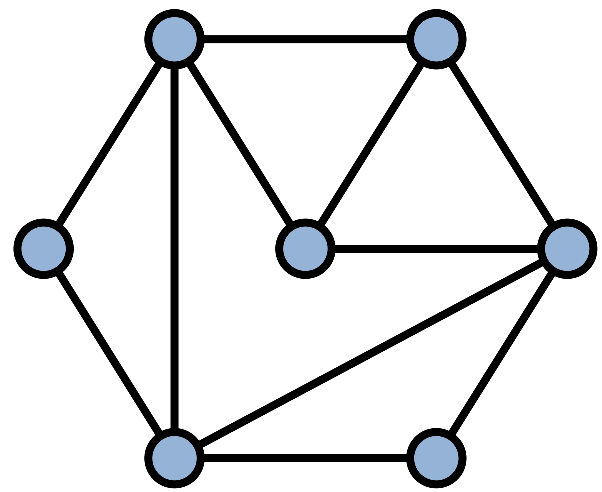




rarely
rigid

Laman's Theorem [1970]

- **Generically rigid in 2D** if and only if you can remove some extra bars to produce a **minimal graph** with
  - $2J - 3$ bars total, and
  - at most $2k - 3$ bars between every subset of k joints



Laman's Theorem [1970]

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