Design Principles
Structure and Mechanism

Structure:
Force is resisted

Mechanism:
Force flows into movement
Design Principles
Structure and Mechanism

Possible Responses to Applied Force

Structural Resistance

Structural Deflection (elastic or Inelastic)

Kinematic deflection
Mechanism paradigms

Synthesize a motion path

Synthesize a form change
Definitions

- **Kinematics**: the study of the motion of bodies without reference to mass or force
- **Links**: considered as rigid bodies
- **Kinematic pair**: a connection between two bodies that imposes constraints on their relative movement. (also referred to as a mechanical joint)
- **Ground**: static point of reference
- **Degree of freedom (DOF)**: of a mechanical system is the number of independent parameters that define its configuration.
Links types

**Figure 2-2**

Links of different order
Kinematic pairs

- Revolute: 1 Degree of Freedom
- Prismatic: 1 Degree of Freedom
- Screw: 1 Degree of Freedom
- Cylindrical: 2 Degrees of Freedom
- Spherical: 3 Degrees of Freedom
- Planar: 3 Degrees of Freedom
the same as that running off the other. The band for the
alley $a$ is identical—coincident—with that for $b$, the corre-

\[
(C_a P_1)^2 - b - \frac{c}{2}; \quad (V_d) = a, d.
\]
Straight-line linkages (James Watt)
Straight-line linkages (Richard Roberts)
Straight-line linkages (Tchebicheff)
Peaucellier Linkage
Straight-line linkages (Peaucellier)
Straight-line linkages (Kempe)
4-bar linkage types

Drag-link

\[ s + l \nless\ than\ not\ equal\ to\ p + q \]
(continuous motion)

Crank-rocker

\[ s + l \nless\ than\ not\ equal\ to\ p + q \]
(continuous motion)

Double-rocker

\[ s + l \gt p + q \]
(no continuous motion)

Parallelogram linkage

\[ s + l \nless\ than\ not\ equal\ to\ p + q \]
(continuous motion)
Kinematic inversions

(a) Two non-distinct crank-rocker inversions (GCRR)

(b) Double-crank inversion (GCCC) (drag link mechanism)

(c) Double-rocker inversion (GRCR) (coupler rotates)
Four-bar linkage examples

Parallel 4-bar

Anti-parallel 4-bar
Gruebler’s equation

\[ N = \text{Number of Links (including ground link)} \]
\[ P = \text{Number of Joints (pivot connections between links)} \]

- Each link has 3 degrees of freedom
- Each pivot subtracts 2 degree of freedom

\[ \text{DOF} = 3(N-1) - 2P \]
Examples

N = 3
P = 3
DOF = 3(3-1)-(2×3) = 0

N = 4
P = 4
DOF = 3(4-1)-(2×4) = 1
Examples

(a) Fivebar linkage—2 DOF

N = 5
P = 5
DOF = 2

(b) Geared fivebar linkage—1 DOF

Geared connection removes one degree of freedom
DOF = 1
Examples

\[
\begin{align*}
N &= 6 \\
P &= 7 \\
\text{DOF} &= 3 \times (6-1) - (2\times7) = 1
\end{align*}
\]

\[
\begin{align*}
N &= 8 \\
P &= 10 \\
\text{DOF} &= 3 \times (8-1) - (2\times10) = 1
\end{align*}
\]
Relation of DOF to special geometries

Agrees with Gruebler’s equation (*doesn’t move*)

Doesn’t agree with Gruebler’s equation (*moves*)

\[ \text{N} = 5 \]
\[ \text{P} = 6 \]
\[ \text{DOF} = 3 \times (5-1) - (2 \times 6) = 0 \]
Graph of linkages

DOF = 3(N-1) - 2P
Scissor Linkages
Scissor mechanisms
Historic examples of scissor mechanisms

Emilio Pinero
Historic examples of scissor mechanisms

Emilio Pinero
Examples of scissor mechanisms

Sergio Pellegrino

Felix Escrig
Curvature of scissor mechanisms

Off-center connection point \(\Rightarrow\) structures of variable curvature
Scissor Types

Parallel / symmetric

No curvature

Parallel / asymmetric

Variable curvature

Angulated

Constant curvature
Angulated scissors provides invariant angle during deployment.
Scissor mechanism: demonstration

Parallel / Symmetric

[Diagram of a scissor mechanism showing parallel and symmetric movement]
Scissor mechanism: demonstration

Off-center connection
Scissor mechanism: demonstration

Angulated
Angulated link: geometric construction

1. 2 straight links
2. Redraw links as 2 triangles
3. Rotate triangles to angle of curvature
4. Redraw angulated links
Angulated link: geometric construction
Tong linkage

Hinged rhombs

Tong linkage
Hinged rhombs – transforming between configurations

straight

arc

circle

ellipse
Arc - geometric construction
Circle - geometric construction
Ring linkages
Ring linkages
Ellipse - geometric construction
Unequal rhombs
Unequal rhombs
Constructing expanding polygons

Perpendicular bisectors
Irregular polygon – geometric construction

All rhombs are similar
(same angles, different sizes)
Degrees of freedom of a tong linkage

Number of pivots for a tong linkage:
P = \frac{3N}{2} - 2
DOF = 3 \times (N-1) - 2P
= 3N - 3 - (3N - 4) = 1

Number of pivots for a closed tong linkage:
P = \frac{3N}{2}
DOF = 3 \times (N-1) - 2P
= 3N - 3 - 3N = -3
Spatial interpretation of Gruebler’s equation

$DOF = 3(N-1) - 2P$

$N = 8$
$P = 12$
$DOF = -3$
Unequal rhombs with crossing connection
Unequal rhombs with crossing connection
Polygon linkages with fixed centers
circular linkage with fixed center (four spokes)
Circular linkages with fixed center
Construction for off-center fixed point
Polygon linkages – off-center fixed point
Polygon linkages – off-center fixed point

folded

expanded
Polygon linkages – off-center fixed point
Polygon linkages – off-center fixed point
Ring linkages
Degrees of freedom (Graver formulation)

Each point (pivot) has 2 degrees of freedom
Each link subtracts 1 degree of freedom
\( J = \) Number of joints (2D points in the plane)
\( R = \) Number of links (not including ground link)

\[
\text{DOF} = 2J - R
\]
Examples

$$\text{DOF} = 2J - R$$

\[\begin{align*}
J &= 2 \\
R &= 3 \\
\text{DOF} &= (2 \times 2) - (3 \times 1) = 1
\end{align*}\]

\[\begin{align*}
J &= 3 \\
R &= 5 \\
\text{DOF} &= (2 \times 3) - (5 \times 1) = 1
\end{align*}\]

\[\begin{align*}
J &= 4 \\
R &= 7 \\
\text{DOF} &= (2 \times 4) - (7 \times 1) = 1
\end{align*}\]
Geometric Folding Algorithms

Linkages, Origami, & Polyhedra

Erik D. Demaine & Joseph O'Rourke

Coming soon in Japanese
Configuration Space

configuration → point

$\theta_2$

Figure 2.1
Rigidity

Figure 4.1
Rigidity Depends on Configuration

Figure 4.2

rigid flexible
**Generic Rigidity**

generically rigid

Rigid realizations

(a) Flexible realizations

(b) Rigid realizations

generically flexible

Flexible realizations
Generic Rigidity

generically rigid

generically flexible

rarely flexible

rarely rigid

Figure 4.4
Laman’s Theorem [1970]

- Generically rigid in 2D if and only if you can remove some extra bars to produce a minimal graph with
  - \(2J - 3\) bars total, and
  - at most \(2k - 3\) bars between every subset of \(k\) joints
Laman’s Theorem [1970]

- Generically rigid in 2D if and only if you can remove some extra bars to produce a minimal graph with
  - $2J - 3$ bars total, and
  - at most $2k - 3$ bars between every subset of $k$ joints

![Graph Example]