

## Lecture 13,14: JUC; UC Signatures and Authentication

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## 1 Review of Last Week and Outline for This Week

Last week we showed how to realize  $F_{ZK}$  in the  $F_{COM}$ -hybrid model. We also showed how to realize any “standard” functionality in several settings, including:

- the  $F_{auth}$ - hybrid model for semi-honest adversaries,
- the  $(F_{auth}, F_{crs})$ - model for Byzantine adversaries,
- the  $F_{auth}$  hybrid model for Byzantine adversaries with an honest majority.

This week we will look at the motivation, formulation, proof and applications of Universal Composition with Joint State (JUC). We will develop a UC formulation for signature schemes and show its equivalence with CMA-security. Finally, we will look at how to achieve authenticated communication.

One question which arose from last week’s lectures was whether we really needed extremely long common reference strings (CRS) to realize  $F_{COM}$ . Indeed, a naive use of CRS would entail:

- A copy of  $F_{CRS}$  per copy of  $F_{COM}$ .
- $O(k)$  copies of  $F_{COM}$  per copy of  $F_{ZK}$ .
- $O(r)$  copies of  $F_{ZK}$  per copy of  $F_{CP}$  to complete a protocol with  $r$  rounds.
- $O(n)$  copies of  $F_{CP}$  for  $n$  parties.

One question is whether it is actually necessary to use so many copies of the CRS. We could consider a single copy of  $F_{MCOM}$  which uses a single CRS. This would not take advantage of the UC theorem, since we’d have to analyze the security of the entire protocol, including copies of  $F_{ZK}$  and  $F_{CP}$  as a single, monolithic unit. As it stands, the UC theorem is not equipped to handle protocols sharing some joint state.

Another example problem where protocols share some joint state are multiple key-exchange protocols based on the same signature scheme. That is, multiple key-exchange protocol instances using the same global parameters, such as public keys.

## 2 Formalization of Multi-Instance Composition

Taking a more abstract view of the problem, suppose we have:

- A protocol  $Q$  in the  $F$ -hybrid model that uses multiple copies of some functionality  $F$ . For example,  $F$  could be  $F_{COM}$  and  $Q$  the Blum ZK protocol for Hamiltonicity.

- A protocol  $P$  that realizes (in a single instance) multiple independent copies of  $F$ . For example,  $P$  could realize  $F_{MCOM}$ .

We want to know whether we can compose these two protocols and maintain security. To do so, we will now formalize a multi-instance extension of ideal functionalities as follows. Let  $F$  be an ideal functionality. Then a multi-session extension of  $F$ , denoted  $FF$  proceeds as follows:

- $FF$  runs multiple copies of  $F$ . Each copy of  $F$  has its own session identifier  $ssid$ .
- $FF$  expects all of its inputs to be of the form  $(sid, ssid, \dots)$ , where  $sid$  is the session id of  $FF$ .
- An incoming message with a given  $ssid$   $s$  is routed to the copy of  $F$  whose  $ssid$  is  $s$ . If no such copy of  $F$  exists,  $FF$  will invoke a new  $F$  and assign it the  $ssid$   $s$ .
- Whenever a copy of  $F$  generates some output  $(ssid, msg)$ , then  $FF$  will simply add its  $sid$  to that output and forward  $(sid, ssid, m)$  to the intended recipient.

Consider the following two examples:

- $F_{MCOM}$ :
  1. Upon receiving  $(sid, cid, C, V, \text{"commit"}, x)$  from  $(sid, C)$  do:
    - (a) Record  $(cid, x)$ .
    - (b) Output  $(sid, cid, C, V, \text{"receipt"})$  to  $(sid, V)$ .
    - (c) Send  $(sid, cid, C, V, \text{"receipt"})$  to  $S$ .
  2. Upon receiving  $(sid, cid, \text{"open"})$  from  $(sid, C)$  do:
    - (a) Output  $(sid, cid, x)$  to  $(sid, V)$ .
    - (b) Send  $(sid, cid, x)$  to  $S$ .

This  $F_{mcom}$  functionality is essentially the same as  $FF_{com}$ .

- $FF_{CRS}$  (with distribution  $D$ ):
  1. Upon receiving  $(sid, ssid, pid, \text{"crs"})$  from  $(sid, pid)$  do:
    - (a) If there is a recorded pair  $(ssid, v)$  then output  $v$  to  $(sid, pid)$  and send  $(ssid, pid, v)$  to the adversary.
    - (b) Else, choose a value  $v$  from  $D$ , record  $(ssid, v)$  and continue as in step 1.(a).

### 3 Universal Composition with Joint State (JUC)

The composition operation on protocols with joint state starts with:

- A protocol  $Q$  in the  $F$ -hybrid model (that may run multiple copies of  $F$ ).
- A protocol  $P$  that securely realizes  $FF$ .

Construct the composed protocol, denoted  $Q^{[P]}$ :

- At the first activation of  $Q^{[P]}$ , each party invokes a copy of  $P$  with some fixed  $sid$ , e.g.  $sid = 0$ .
- Whenever protocol  $Q$  calls a copy of  $F$  with input  $(sid = (s, x))$ ,  $Q^{[P]}$  calls  $P$  with input  $(sid = 0, ssid = (s, x))$ .
- Each output  $(0, s, y)$  of  $P$  is treated as an output  $(s, y)$  coming from the copy of  $F$  with  $sid = s$ .

**Theorem 1 (JUC: UC with Joint State)** *Let  $Q$  be a protocol in the  $F$ -hybrid model, and let  $P$  be a protocol that securely realizes  $FF$ . Then protocol  $Q^{[P]}$  emulates protocol  $Q$ .*

In other words, for any adversary  $A$  there exists an adversary  $H$  that for any environment  $Z$  we have

$$EXEC_{Q,H,Z}^F \approx EXEC_{Q^{[P]},A,Z}$$

**Corollary 1** *If  $Q$  securely realizes some ideal functionality  $G$ , then so does protocol  $Q^{[P]}$ .*

### 3.1 Application of JUC

One application of the JUC theorem is in the construction of [CLOS]. In this setting,  $F$  is  $F_{COM}$  and  $FF$  is  $F_{MCOM}$ :

- Can write and realize each functionality (e.g. ZK, C&P, general compiler) as a single instance,
- Can use the UC theorem to obtain a composed protocol  $Q$  in the  $F_{COM}$ -hybrid model. Protocol  $Q$  uses many copies of  $F_{COM}$
- Can then use the JUC theorem to compose  $Q$  with a single copy of the protocol that realizes  $F_{MCOM}$ , thus using a single copy of the CRS.

### 3.2 Proof of the JUC Theorem

The general proof outline will be to define a protocol  $Q'$  that is secure in the  $FF$ -hybrid model and show:

1. Protocol  $Q^{[P]}$  is identical to  $Q'^P$ .
2. Protocol  $Q'^P$  emulates  $Q'$ .
3. Protocol  $Q'$  emulates  $Q$ .

The definition of  $Q'$  is fairly straightforward.  $Q'$  essentially runs  $Q$ , then prepends its own identifier to any messages sent by  $Q$ . More specifically, protocol  $Q'$  in the  $FF$ -hybrid model is identical to  $Q$  except:

- $Q'$  uses a single copy of  $FF$  with a fixed  $sid$   $s'$  (For instance  $s' = 0$  or  $s'$  is the  $sid$  of  $Q'$  with some fixed prefix.)

- Any input  $x$  sent by  $Q$  to copy  $s$  of  $F$  is replaced with a call  $(s', s, m)$  to  $FF$ .
- Any output  $(s', s, y)$  from  $FF$  is treated as an output  $y$  coming from copy  $s$  of  $F$ .

By the UC theorem, protocol  $Q'^P$  emulates  $Q'$ . However,  $Q'^P$  is just another way of describing  $Q^{[P]}$ , thus  $Q^{[P]}$  emulates  $Q'$ . It remains to show that  $Q'$  emulates  $Q$ .

Let  $A'$  be an adversary interacting with  $Q'$  in the  $FF$ -hybrid model. Construct an adversary  $A$  that interacts with  $Q$  in the  $F$ -hybrid model. It can be verified that  $EXEC_{Q',A',Z}^{FF} = EXEC_{Q,A,Z}^F$  for all  $Z$ .

Adversary  $A$  runs  $A'$ :

- Messages sent by  $A'$  to parties running  $Q'$  are forwarded to the actual parties running  $Q$ .
- Messages from the parties running  $Q$  are forwarded to  $A'$ .
- For each message  $(s', s, m)$  sent by  $A'$  to  $FF$ ,  $A$  sends the message  $(s, m)$  to copy  $s$  of  $F$ .
- Whenever  $A$  gets a message  $m$  from a copy of  $F$  with  $sid = s$ , it forwards message  $(s', s, m)$  from  $FF$  to  $A'$ .
- Whenever  $A'$  corrupts a party,  $A$  corrupts the same party and reports the obtained information to  $A$ .

## 4 General Protocols in CRS model

Assume we had a protocol that realizes  $FF_{CRS}$  in the  $F_{CRS}$ -hybrid model, using only a single instance of  $F_{CRS}$ . Then it would suffice to construct protocols where each instance uses its own copy of  $F_{CRS}$ . For instance, realizing  $F_{COM}$  would be sufficient; we wouldn't need  $F_{MCOM}$ .

Several results from [CR03] are relevant to this setting:

- Any protocol that realizes  $FF_{CRS}$  in the  $F_{CRS}$ -hybrid model using only a single copy of  $F_{CRS}$  must be interactive, i.e. each party should send at least one message for each generation of a new common string.
- Using the Blum 3-move coin-tossing protocol, we can realize  $FF_{CRS}$  in the  $F_{MCOM}$ -hybrid model using only a single copy of  $F_{CRS}$ .
- Using protocol UCC, we get the desired results. (But could not get rid of the protocol UCC.)

## 5 Applications of JUC to Signature-Based Protocols

Another case where multiple protocol instances use the same subroutine are cases of protocols based on signature schemes. For example, signature-based message authentication,

key-exchange, or Byzantine Agreement. In all of these cases, protocols use long-term signature keys for multiple protocol sessions. That is, signature keys can be considered joint state shared among protocols.

Our goal is to define and analyze signature-based protocols for a single session (i.e. single session-key) and then use JUC to deduce that the multi-session interaction (using a single long-term signature module) is secure. To do that, we first need to be able to formalize the signature mechanism as an ideal functionality.

## 5.1 Digital Signatures as Ideal Functionalities

Digital signatures are typically thought of as a tool within protocols, rather than a “protocol” by itself. Still, it is useful and instructive to treat digital signatures as a protocol with a specified ideal functionality. Potential benefits include modularity of analysis, re-asserting the adequacy of existing security notions, and providing a bridge to formal analysis of protocols.

The question is how exactly to formalize digital signatures. There are two main approaches. One is to define signatures as a stand-alone primitive, as in [Can01, CK02, CR03, BH03, Can03]. The other is to define signatures as a part of a more complex functionality that also provides other services, as in [BPW03]. We’ll focus on the stand-alone approach because it is more modular.

## 5.2 Attempt 1

The following is our first attempt at defining an ideal functionality for a digital signature scheme:

1. On input  $(sid, \text{“Keygen”})$  from party  $(sid, S)$ , register party  $(sid, S)$  as the signer.
2. On input  $(sid, \text{“sign”}, m)$  from  $(sid, S)$  record  $m$ .
3. On input  $(sid, \text{“verify”}, m)$  from any party, return  $(sid, \text{“yes”}/\text{“no”})$  according to whether  $m$  has been recorded.

This definition is too idealized. Any protocol that realizes this functionality will have to take care of transmitting signatures and public keys between parties. This would not meet our intuition of a “signature scheme”.

## 5.3 Attempt 2

The following protocol attempts to resolve the problem by specifically creating a signature key and a signature string:

1. On input  $(sid, \text{"Keygen"})$  from party  $(sid, S)$ , register party  $(sid, S)$  as the signer, AND return to party  $(sid, S)$  a public key  $v$  selected at random.
2. On input  $(sid, \text{"sign"}, m)$  from  $(sid, S)$  return a random signature  $s$  and record  $(m, s, v)$ .
3. On input  $(sid, \text{"verify"}, m, s, v')$  from any party, return  $(sid, \text{"yes"/"no"})$  according to whether  $(m, s, v')$  has been recorded.

It looks as though the functionality is no longer too ideal. In particular since the signature string and verification key are part of the interface, communicating the values from one party to another becomes the job of the calling protocol, rather than the job of the signature scheme. But this formulations still has several problems. For one thing, the public key in the ideal functionality is a random number. Many real-world systems do not do this (and we consider them secure). The scheme is also very deterministic. Specifically, the ONLY accepted signature is the one that was generated by  $(sid, S)$  as the triplet  $(m, s, v)$ . For some applications, we want digital signature schemes that have multiple valid signatures for a given message. Finally, the ideal scheme is such that even an invalid signer can't pick a 'weak' signing key.

So in all, we have now made the scheme TOO restrictive. On to our next attempt.

### 5.4 Attempt 3

We will once again attempt to resolve the previous problems:

1. On input  $(sid, \text{"Keygen"})$  from party  $(sid, S)$ , register party  $(sid, S)$  as the signer, forward  $(sid, S)$  to  $A$  (the simulator/adversary), obtain a public key  $v$  from  $A$  and output  $v$  to  $(sid, S)$ .
2. On input  $(sid, \text{"sign"}, m)$  from  $(sid, S)$ , forward  $(sid, m)$  to  $A$ , obtain a "signature"  $s$  from  $A$ , output  $s$  to  $(sid, S)$ , and record  $(m, s, v)$ .
3. On input  $(sid, \text{"verify"}, m, s, v')$  from any party, return  $(sid, f)$  where:
  - If  $(m, s, v')$  is recorded then  $f = 1$ .
  - If  $S$  is uncorrupted and  $(m, s^*, v')$  is unrecorded for any  $s^*$ , then  $f = 0$ .
  - Else forward  $(m, s, v')$  to  $A$  and obtain  $f$  from  $A$ .

First note that the second assignment of  $f$  (where  $S$  is uncorrupted) ensures that phony signatures can't be generated by  $A$ , but allows for more than one valid signature for a signed message.

It almost seems as if we've reached a good definition, except that there is still one glaring problem. Now that we have allowed  $A$  the power to decide whether a signature is valid or

not when the signer is corrupt, we have also allowed  $A$  to change his mind (for the exact same signature, he can say “yes” once and “no” at another point). This is not a desirable feature for a signature scheme.

### 5.5 Attempt 4

We will make a minor change to the functionality, forcing the system to stick to its decision once it is made:

1. On input  $(sid, \text{“Keygen”})$  from party  $(sid, S)$ , register party  $(sid, S)$  as the signer, forward  $(sid, S)$  to  $A$  (the simulator/adversary), obtain a public key  $v$  from  $A$  and output  $v$  to  $(sid, S)$ .
2. On input  $(sid, \text{“sign”}, m)$  from  $(sid, S)$ , forward  $(sid, m)$  to  $A$ , obtain a “signature”  $s$  from  $A$ , output  $s$  to  $(sid, S)$ , and record  $(m, s, v, 1)$  unless a prior record  $(m, s, v, 0)$  exists, in which case output an error message.
3. On input  $(sid, \text{“verify”}, m, s, v')$  from any party, return  $(sid, f)$  where:
  - If  $(m, s, v', b)$  is recorded then  $f = b$ .
  - If  $S$  is uncorrupted and  $(m, s^*, v', 1)$  is unrecorded for any  $s^*$ , then  $f = 0$ .
  - Else forward  $(m, s, v')$  to  $A$  and obtain  $f$  from  $A$ . Record  $(m, s, v', f)$

Sadly, although we’re almost there, there’s still one more minor change we need to make. The problem with the above protocol is that it doesn’t actually predefine a fixed signer. In a signature scheme, the signer of a document is a known value, but in the above, other than claiming to be the role of the signer, nothing forces it. It isn’t clear that only one signer is allowed.

### 5.6 $F_{sig}$

We will fix this problem by encoding who the signer is within the session ID. That way, for every signature session, the ‘signer’ is well-known to all parties, even if he hasn’t signed anything yet. This will yield our ideal signature functionality:

$$F_{sig}$$

1. On input  $(sid, \text{“Keygen”})$  from party  $(sid, S)$ , verify  $sid = (sid', S)$  otherwise, ignore input. Forward  $(sid, S)$  to  $A$ , obtain a public key  $v$  from  $A$  and output  $v$  to  $(sid, S)$ . This stage may only be done once.
2. On input  $(sid, \text{“sign”}, m)$  from  $(sid, S)$ , where  $sid = (S, sid')$  forward  $(sid, m)$  to  $A$ , obtain a “signature”  $s$  from  $A$ , output  $s$  to  $(sid, S)$ , and record  $(m, s, v, 1)$  unless a prior record  $(m, s, v, 0)$  exists (in which case output an error message).
3. On input  $(sid, \text{“verify”}, m, s, v')$  from any party, return  $(sid, f)$  where:
  - If  $(m, s, v', b)$  is recorded then  $f = b$ .
  - If  $S$  is uncorrupted and  $(m, s^*, v', 1)$  is unrecorded for any  $s^*$ , then  $f = 0$ .
  - Else forward  $(m, s, v')$  to  $A$  and obtain  $f$  from  $A$ . Record  $(m, s, v', f)$

And voila, we have a working ideal functionality. This is the functionality we will attempt to achieve. However, it should be noted that this functionality relays all messages and signatures to the adversary, which means whatever protocol achieves it is not a secret or anonymous signature scheme.

## 6 Realizing $F_{sig}$

It turns out that it is fairly easy to make a real-world protocol that can securely realize  $F_{sig}$ . Given any signature scheme  $H = (GEN, SIG, VER)$ , we construct the following protocol  $P_H$ :

- When invoked with  $(sid, \text{“Keygen”})$  and  $pid = S$  ensure that  $sid = (S, sid')$ . Now run  $(p, v) \leftarrow GEN(k)$ . Return  $v$  to the caller (and keep  $p$  private).
- When invoked with  $(sid, \text{“sign”}, m)$  compute  $s \leftarrow SIG(p, m)$  and return  $s$ . We allow  $SIG$  to maintain state between activations.
- When invoked with  $(sid, \text{“Verify”}, m, s, v')$  return  $VER(m, s, v')$

We show that this scheme is indeed secure:

**Theorem 2** *A scheme  $H$  is existentially unforgeable against chosen message attacks if and only if the protocol  $P_H$  securely realizes  $F_{sig}$ .*

Recall the definition of security we will use:

**Definition 1** *a scheme  $H = (Gen, Sig, Ver)$  is existentially unforgeable against chosen message attacks (is EU-CMA) if it has the following three properties:*

**Completeness** : *For all adversaries  $F$ :*

$$Pr_{(p,v) \leftarrow Gen(); m \leftarrow F(v)}[Ver(m, Sig(p, m), v) = 1] \sim 1$$



**Consistency** : For all  $(m, s, v)$ :

$$\text{Variance}[\text{Ver}(m, s, v)] \sim 0$$

**Unforgeability** : Over all  $(p, v) \leftarrow \text{Gen}(k)$ ,  $(m^*, s^*) \leftarrow F^{\text{SIG}(p,*)}(v)$  where  $m^*$  is not one that  $F$  has asked for a signature of:

$$\text{Pr}_{\text{above}}[\text{Ver}(m^*, s^*, v) = 1] \sim 0$$

*To clarify the above (it is intended to be an interactive system), we allow the adversary to know the public key, and make polynomially many queries to a signing machine. After receiving many signatures, he attempts to generate a valid new signature for a new message. Our scheme is unforgeable if his chances of doing well are negligible.*

### 6.1 Proof that $(P_H \text{ realizes } F_{\text{sig}}) \longleftrightarrow (H \text{ is EU-CMA secure})$ .

- $P_H$  realizes  $F_{\text{sig}} \longrightarrow H$  is EU-CMA secure.

For this to be true means we can show that  $H$  has the three properties of EU-CMA. We will therefore go over each property and prove it.

**Completeness** : To prove this, we will set up a contradiction. Assume that  $H$  is not complete. That means that there exists some adversary for  $H$  (we call it  $F$ ) that can find a message  $m$  whose signature doesn't verify correctly. So from that, we can build an environment  $Z$  and an adversary  $A$  where the environment instructs the signer to sign  $m$  and then verify the signature. In the ideal model, that message still verifies correctly (verification is independent of  $H$ ). In the hybrid model, it will not, and so the environment can distinguish. This leads us to conclude that  $P_H$  doesn't securely realize  $F_{\text{sig}}$  which is a contradiction.

**Consistency** : To prove, we will use the same trick. Assume that  $H$  is not consistent, so the result of the verification function varies. By a similar trick to the above, we construct an environment that calls for several verifications in a row of the same message and signature. In the ideal, we always get the same result, while in the real, we will flip-flop. The environment can distinguish, which violates the assumption of secure realization of  $F_{\text{sig}}$ .

**Unforgeability** : Assume there exists a forger  $G$  for  $H$ . We will once again show how the environment can distinguish the ideal from the real (and form a contradiction). As normal behavior, the environment invokes a signer/verifier/adversary triplet. Additionally, the environment runs the forger  $G$  internally. To get a key to give to  $G$ , the environment tells the signer to run keygen (yielding only a public key  $v$  which the environment feeds to  $G$ ).  $G$  now will ask for many messages  $m_i$  to be signed. For each such message, the environment will pass it to the uncorrupted signer to sign. It will get a signature  $s_i$  for each  $m_i$ , which it will hand over to  $G$ . Finally,  $G$  will generate a pair  $m^*, s^*$  which it claims to be a valid signature (and is indeed, since we claim  $G$  is a forger for  $H$ ). The environment will now ask the verifier to verify the signature  $s^*$ , and output the bit that the verifier does. Note that in the ideal environment, the final signature

is NOT a valid signature since it was never recorded as such, so it will always output 0. On the other hand, in the real-world environment, the final signature IS a valid signature with non-negligible probability, so the environment can distinguish (and we contradict).

- $H$  is **EU-CMA secure**  $\rightarrow P_H$  realizes  $F_{sig}$  .

We will again do this through contradiction. Assume  $P_H$  does not securely realize  $F_{sig}$ . That means that for any simulator  $S$  there is an environment ( $Z$ ) that can distinguish between an interaction with the ideal functionality  $F_{sig}$  and  $S$ , and the real protocol  $P_H$  and the dummy adversary. Specifically, we will look at the environment that can distinguish with respect to the following generic simulator  $S$ :

- When asked by  $F_{sig}$  to generate a key,  $S$  runs  $(p, v) \leftarrow Gen(k)$  and returns  $v$  (just as per the protocol).
- When asked by  $F_{sig}$  to generate a signature  $s$  on a message  $m$ ,  $S$  runs  $s \leftarrow Sig(p, m)$  and returns  $s$ .
- When asked by  $F_{sig}$  to verify a signature  $(m, s, v')$ ,  $S$  runs  $f \leftarrow Ver(m, s, v')$  and returns  $f$ .

As can be seen,  $S$  follows the exact decision behavior of the original protocol  $P_H$ .

Let  $\mathbb{B}$  be the event that in a run of  $Z$  and  $S$  in the ideal model, the signer never signed  $m$ , and yet still an  $(sid, \text{“Verify”}, m, s, v)$  activation answered with a 1. It can be verified that if event  $\mathbb{B}$  doesn't occur,  $Z$ 's view of the ideal and real executions are statistically close (these can be randomized signatures). We know that  $Z$  can distinguish, so  $\mathbb{B}$  must occur with non-negligible probability. Using this knowledge, we can now construct a forger  $G$  for  $H$  (thereby contradicting our assumption that  $H$  is EU-CMA secure).

Here is how  $G$  works. It first takes the code for  $Z$  and runs it internally:

- When  $Z$  attempts to start a signer with “KeyGen”,  $G$  doesn't start a signer, but instead feeds  $Z$  the output for the signer, that being the value  $v$  that  $G$  is trying to break.
- When  $Z$  attempts to get the signer to sign a message  $m$ ,  $G$  intercepts this and passes the  $m$  to its signing oracle. It will get a valid signature  $s$ , which it will hand back to  $Z$  (looking like the valid signature coming back from the signer).
- When  $Z$  attempts to verify a signature  $(m, s, v)$ ,  $G$  can check whether  $(m, s, v)$  is a forgery (this is event  $\mathbb{B}$ ). If it is, then it outputs  $(m, s, v)$ . Otherwise, it continues to run  $Z$ .
- If/When  $Z$  asks to corrupt the signer,  $G$  aborts.

We know that  $\mathbb{B}$  occurs with non-negligible probability, so  $G$  generates a valid forgery with non-negligible probability, which is a contradiction on the assumption that  $H$  is EU-CMA secure.

**Corollary 2** *From the above proof we see that  $P_H$  is adaptively secure iff it is non-adaptively secure*

Clearly if it is adaptively secure, it is non-adaptively secure (special case). To see the other direction, simply note  $G$  aborts on a “corruption” message from  $Z$  irrelevant of when it happens, and yet our contradiction still holds.

## 7 Authenticated Communication using $F_{sig}$

We wish to establish an authenticated communication network on top of an unauthenticated network. Using this, we can provide  $F_{auth}$  as an underlying functionality. We will follow this plan:

- We define a registry functionality  $F_{reg}$  which allows parties to “register” their public keys for others to securely retrieve.
- We realize  $F_{auth}$  in the  $(F_{reg}, F_{sig})$  – *hybrid* model by:
  - Creating a certification functionality  $F_{cert}$  that provides a binding between signatures and parties.
  - Realize  $F_{cert}$  in the  $(F_{reg}, F_{sig})$  – *hybrid* model.
  - Realize  $F_{auth}$  in the  $F_{cert}$  – *hybrid* model.
- We will then authenticate multiple messages using a single key pair by:
  - Defining  $FF_{cert}$
  - Realize  $FF_{cert}$  using many calls to  $F_{cert}$ .
  - Use the JUC theorem to combine.

### 7.1 The “Public Registry” Functionality $F_{reg}$

This functionality allows a party to register a value (nominally its public key) to a system in a way that allows any party to ask for that value:

$F_{reg}$

1. When receiving  $(sid, \text{“register”}, v)$  from party  $(sid, S)$ , verify that  $sid = (S, sid')$ , send  $(sid, S, v)$  to the adversary, and record  $(S, v)$ .
2. When receiving  $(sid, \text{“retrieve”}, S)$  from any party, return  $(sid, S, v)$  if there is a record  $(S, v)$  else return  $(sid, S, -)$ .

Note that there is no verification of the public key being anything (no proof of knowledge of  $p$  or the like). Additionally, the way our definition works, the registrant is part of the  $sid$ , so only one person can register their key per functionality.

## 7.2 The Certification Functionality $F_{cert}$

$F_{cert}$

1. On input  $(sid, \text{"sign"}, m)$  from  $(sid, S)$  where  $sid = (S, sid')$ , forward  $(sid, m)$  to  $A$ , obtain a “signature”  $s$  from  $A$ , output  $s$  to  $(sid, S)$  and record  $(m, s, 1)$ . Verify that no  $(m, s, 0)$  record exists.
2. On input  $(sid, \text{"verify"}, m, s)$  from any party, return  $(sid, f)$  where:
  - If  $(m, s, b)$  is recorded, then  $f = b$ .
  - If  $S$  is uncorrupted and  $(m, s^*, 1)$  is unrecorded for any  $s^*$ , then  $f = 0$ .
  - Else forward  $(m, s)$  to  $A$  and obtain  $f$  from  $A$ , recording  $(m, s, f)$

Note that  $F_{cert}$  is similar to  $F_{sig}$  except there is no more key generation, and instead it is all done with respect to the signers identity (which is encoded in the  $sid$ ).

## 7.3 Realizing $F_{cert}$ in the $(F_{reg}, f_{sig})$ – hybrid model

The following is the protocol that realizes the  $F_{cert}$  functionality:

1. At first activation, signer  $(sid, S)$  ensures that  $sid = (S, sid')$  and calls  $F_{sig}$  with  $(sid.0, \text{"keygen"})$ , obtains  $v$ , and calls  $F_{reg}$  with  $(sid.1, \text{"register"}, v)$ .
2. When activated with input  $(sid, \text{"sign"}, m)$ ,  $(sid, S)$  verifies  $sid = (S, sid')$  and calls  $F_{sig}$  with  $(sid.0, \text{"sign"}, m)$ , obtaining a signature  $s$  which it outputs.
3. When activated with input  $(sid, \text{"verify"}, m, s)$  where  $sid = (S, sid')$ , the activated party calls  $F_{reg}$  with  $(sid.1, \text{"retrieve"}, S)$  obtains the public key  $v$ , and calls  $F_{sig}$  with  $(sid.0, \text{"verify"}, m, s, v)$ . It outputs the result of that call.

The simulation is perfect for the above (all the smarts/computation are done in the underlying protocols). Similarly, the security is unconditional.

## 7.4 Realizing $F_{auth}$ in the $F_{cert}$ – hybrid model

The following is the authenticated message transmission functionality  $F_{auth}$ :

$F_{auth}$

1. Receive input  $(sid, S, R, m)$  from party  $(sid, S)$ .
2. Output  $(sid, S, R, m)$  to party  $(sid, R)$
3. Send  $(sid, S, R, m)$  to the adversary/simulator.
4. halt

And the protocol that achieves it:

1. When activated with input  $(sid, S, R, m)$ , party  $(sid, S)$  calls  $F_{cert}$  with  $(S.sid, \text{"sign"}, m.R)$ , obtains a signature  $s$ , and sends  $(sid, S, m, s)$  to  $(sid, R)$ .
2. When receiving message  $(sid, S, m, s)$ ,  $(sid, R)$  calls  $F_{cert}$  with  $(S.sid, \text{"verify"}, m.R, s)$ . If the result is 1, then output  $(sid, S, R, m)$ .

Again the simulation is perfect, and the security is unconditional. Also, although most real protocol use nonces as part of the message to ensure unique messages, this system uses  $sid$  values, which are supposed to be unique (indeed, a natural way to obtain unique  $sid$ 's is to exchange nonces and concatenate the nonces into the  $sid$ ).

Next lecture we will discuss how to achieve authenticate multiple messages using a single verification key (by using the JUC theorem).

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