General Layout Strategy

Tree of meshes (not mesh of trees)

\[ T(n) : \]

\[ N = n^2 \log n \]

\[ \text{Area:} \]

\[ S(n) = 2S(n/2) + n \]

\[ G(n) = \Theta(n^2 \log^2 n) \]
Fold and squash:

\[ n^2 \times \log n \text{ layers} \rightarrow \Theta(n^2 \log^2 n) \text{ area.} \]

**Truncated TOM:** \( \text{TOM}(n, k) \) - top \( k \) levels.

Area = \( \Theta(n^2 k^2) \)

**Decomposition Trees**

T is a \( (w_1, w_2, \ldots, w_r) \) decomposition tree for \( G=(V, E) \):

1. Vertices in \( V \) mapped to leaves of \( T \).
2. Edges in \( E \) run through links of \( T \).
3. #edges leaving subtree rooted at depth \( i \) is \( \leq w_i \).

For \( 1 \leq \alpha \leq 2 \), \( G \) has a \( (w, \alpha) \) decomp tree if it has a \( (w, 1, w/\alpha, w/\alpha^2, \ldots, 0(1)) \) decomp tree.

A decomp tree is balanced if all subgraphs at the same depth have same # vertices to within 1.
Layout strategy
1. Start with $(w, \sqrt{w})$ decom tree.
2. Balance the decom tree.
3. Embed the balanced tree in trunc TDM.
4. Use trunc TDM layout to yield $O(w^2g^2n)$ area layout.

Balancing decom trees
Warm-up: Necklace with black and white pearls.
How many cuts to divide into 2 sets, each with half the pearls of each color?

Lemma. Consider any 2 strings composed of an even # of black pearls and an even # of white pearls. By making at most 2 cuts, the pearls can be partitioned into 2 sets, each containing 2 strings, such that each set has 1/2 the pearls of each color.

Pf. (Continuity arg.)
Lemma. Let $T$ be a CBT drawn with $n$ leaves on a straight line, and consider any set $S$ of $k$ consecutive leaves of $T$. Then, $S$ can be a forest $F$ of complete binary subtrees of $T$ if

1. $S$ is a leaf of $F$.
2. At most 2 trees of $F$ have any given height.
3. Depth of largest tree in $F$ is $\leq \log k$.

Proof. \( F \) be forest of maximal CBT's whose leaves lie only in $S$. (1) & (3) follow. Use induction to prove (2).

Then, let $G$ be a graph on $n$ vertices that has a $\{w_1, w_2, \ldots, w_r\}$ decomp tree $T$. Then, $G$ has a $\{w_1', w_2', \ldots, w_{l(G')}\}$ balanced decomp tree $T'$, where

$$
\omega'_i = \frac{1}{4} \sum_{k=1}^{r} \omega_k.
$$

Proof. Color leaves of $T$; 1 = node of $G$, 0 = empty.

Recursively split $T$'s $w_i$ leaves evenly. Each stage has $S$ strings of consecutive leaves from $T$, each of which has $\leq 2$ CBT's of a given height.
Total # wires leaving a string
< sum of wires leaving each of its children.
\[ w_i < 4 \sum_{k=i}^{\infty} w_k. \]

Corollary: A graph with a \((w, \alpha)\) decamp tree, \(\alpha\) const,
has an \((O(w), \alpha)\) balanced decamp tree.

Proof: Sum is geometric:
\[ w_i = 4 \frac{w}{\alpha} \sum_{k=i}^{\infty} \frac{w}{\alpha^{k-1}} \]
\[ \leq 4 \frac{w}{\alpha^{\alpha-1}} \left( \frac{\alpha}{\alpha-1} \right) \]

Graph has \((4w\alpha/(\alpha-1), \alpha)\) decamp tree.

Next week: Embed in trinc TOM->layout.
Area-universal networks.

«Exam issues>>