Abstract

Random multithreaded NSP task graphs cannot be modelled in SP-based parallel programming models. In this report we present a new algorithm for converting NSP DAGs to SP DAGs. Our analysis establishes an upper bound of a factor of 2, on the increase in critical path length ($W_\infty$). However, the total work ($W_1$) remains the same. We also evaluate an implementation of our algorithm and perform subsequent analysis with real-world models of task graphs and random task graphs which provide encouraging results as compared to previous research. Specifically, we prove the correctness and analyze computational complexity of our algorithm followed by a heuristic based variant for further performance enhancements.

1 Introduction

A parallel multithreaded computation, represented as a non-series parallel DAG, presents inherent difficulties in performing cost estimations, analysis, scheduling and structured programming[1]. Also, most of the current multithreaded programming models like cilk[2] are based on series-parallel DAG modeling of the computations. However, most of the real world problems that are modeled for parallel system solutions are non-series-parallel DAGs. This exposes a lacuna which necessitates the development of a mechanism for an efficient translation technique from the NSP model to an SP model. The SP model enables the analysis and execution of such programs in the underlying multithreaded environment while preserving the original semantics of the computation. However, it should be noted that we mainly intend to address the conversion of NSP DAG representing the task nodes of a computation to an SP DAG. Further representation is required, if it is to be modeled in a particular multi-threaded computational model like cilk[2][3].

The remainder of the report is organized as follows. Section 2 introduces some terms and concepts used by the algorithm. Section 3 presents our approach and strategy, in addition to the proof of concepts used by our algorithm. Section 4 presents the high-level algorithm. Section 5 presents the proof of correctness for the algorithm, while Section 6 discusses the complexity analysis. Section 7 presents a qualitative assessment of the algorithm. Section 8 outlines the empirical analysis followed by some concluding remarks.

2 Some terms and concepts:

Here we outline some basic definitions that are used in the subsequent sections of the report.

Definition 1 Directed acyclic graph (DAG)

A graph $G$ is a pair $(V, E)$, where $V$ is a set of vertices, and $E$ is a set of edges between the vertices $E = (u,v)|u,v \in V$. The graph does not allow self-loops, adjacency is irreflexive, i.e. $E = (u,v)|(u,v \in V) \land (u \neq v)$ and has no path which starts and ends at the same vertex.

A DAG describes the structure of a parallel program. The nodes of the DAG represent tasks that the processors must complete, and the edges represent the dependencies between the tasks. Thus, if there is an edge $(u,v) \in E$, then $v$ cannot be executed until after $u$ completes. In this case we say that $u$ is a parent of $v$. 
Definition 2 Series parallel DAG (SP DAG)
A series-parallel DAG $G' = (V, E)$ is a directed acyclic graph with two distinguished vertices, a source 's' and a sink 't'. A family of series parallel graphs are described using the following grammar:
A series parallel DAG $G'(V, E)$ is one of the following:
A single edge extending from s to t, that is, $V = \{s, t\}$ and $E = \{(s, t)\}$.
Two series parallel graphs $G_1$ and $G_2$ composed in parallel. The sources $s_1$ and $s_2$ of $G_1$ and $G_2$ are merged into a single source s, and the sinks $t_1$ and $t_2$ are merged into a single sink t.
Two series parallel graphs composed in series. The sink $t_1$ of $G_1$ and the source $s_2$ of $G_2$ are merged into a single node.

Definition 3 Non-series parallel DAG (NSP DAG)
A graph $G$ that cannot be reduced to a representation addressed by the SP DAG. [Definition 2].

Definition 4 Critical path ($W_\infty$)
The number of nodes in the longest chain in the DAG $G$.

Definition 5 Total Work ($W_1$)
The total number of nodes in the DAG $G$.

Definition 6 Level of a vertex in a DAG (LEVEL(v))
The longest path length from the root to the vertex $v$.

Definition 7 Predecessor(s) of a vertex (PREDECESSOR(v))
The vertices that precede $v$, in a topological ordering of the DAG.

Definition 8 Successors(s) of a vertex (SUCCESSOR(v))
The vertices that succeed $v$, in a topological ordering of the DAG.

Definition 9 Sync node
A vertex with indegree $\geq 2$.

Definition 10 Spawn node
A vertex with outdegree $\geq 2$.

Definition 11 Path
A path is a sequence of edges that defines a possible way through connected vertices from one vertex to another.

Definition 12 Spawn Set (SPAWN(v))
The set of immediate PREDECESSOR(v), from which $v$ was spawned. This identifies the nodes that sync from multiple-independent execution paths.

Definition 13 Disjoint vertices
Two vertices $v_i$ and $v_j$ are disjoint if:
$$SPAWN(v_i) \cap SPAWN(v_j) = \emptyset$$

Definition 14 Disjoint sync node (DSN)
A node is a disjoint sync node if any of its parents are disjoint vertices.

Definition 15 Global-Level
The level of the most-recent disjoint sync node in a topological ordering of the DAG.
Definition 16
*C bit of a vertex*: Represents the resolved status of a vertex.

*R bit of a vertex*: Represents node(s) in the NSP DAG, which are the most recently processed vertices in the resultant SP DAG.

*D-bit of a supernode*: The D bit of a super node S is set when \( \exists \) vertices \( v_i, v_j \in S \), such that \( v_i, v_j \) are disjoint vertices.

Definition 17 *Depth(G)*
The level of the leaf vertex \( v \in G \).

### 3 Approach and Strategy

This section addresses the main concepts and strategies that are the cornerstones of our algorithm. Each conceptual idea is justified by its correctness and ability to perform a correct and efficient transformation.

#### 3.1 Breadth first task node visitation (BFT)

Our high level algorithm is based on the breadth first traversal of the task nodes in the NSP DAG. The objective is to identify non-interdependent task nodes that could be run in parallel. At each level, the algorithm resolves the NSP problems (Appendix A), if it exists. There is a subtle difference in the way our algorithm performs the breadth-first traversal of the graph as compared to the normal BFT. In the BFT we follow, a node is enqueued if and only if all its parents have been enqueued and processed in a previous iteration.

The following lemma shows that all nodes considered in an iteration of the breadth first traversal are in the same level.

**Lemma 1** Let \( v_1, v_2, v_3, \ldots v_m \) be the vertices considered in a particular iteration of the BFT.

Then, \( \forall i, j \) where \( i, j \in \{1 \ldots m\} \), Level\((v_i) = Level(v_j), i \neq j \)

**Proof** A breadth-first traversal of the DAG \( G \) is a level-wise traversal, which does not advance to the next level, Level\((j)\) unless and until all the vertices \( \{V\} \) of Level\((i) [i < j] \) are visited and colored. By the inherent method of breadth first traversal, the parents of a vertex are discovered and colored before their children are discovered and colored. Hence, all the nodes at a particular BFT are at same level, due to the way in which breadth-first traversal discovers the vertices.

Further the following Lemma proves that the precedence relation is maintained during each iteration of the BFT.

**Lemma 2** The resolution of a vertex \( v \) preserves the precedence relation.

**Proof** A vertex \( v \in G \) is resolved only if Parents\((v)\) have been colored and resolved. A vertex \( v \in V \) might have multiple paths \( p_1, p_2, \ldots p_n \). A vertex \( v \) is colored and resolved only if the vertex has been reached through the longest path \( p_i \). Thus, without loss of generality, if it has been reached through the longest path, all the other paths to vertex \( v \), would have been traversed [Lemma 1]. Thus, the resolution of a vertex, would never lead to a situation that would violate the precedence.
Now we would introduce another set of definitions.

**Definition 18  Vertex Coloring**

A vertex $v$ in the NSP DAG $G$ is $C$-colored if it has been traversed and resolved. Further a vertex is resolved if it is added to the resultant SP DAG $G'$. The resolved status is represented by a bit known as the C-bit, associated with each vertex of the NSP DAG. If the C-bit of $v$ is set, we call $v$ a colored vertex.

Further a vertex is $R$-colored (using a R-bit) if it has already been resolved and does not have any descendants up to the current processing level in $G'$. The R-coloring propagates down at each iteration to mark the last frontier of processing in $G'$, where all those parents of the newly R-colored vertices, are un-colored ($R = 0$).

This results in another definition.

**Definition 19 Augmented-In-Degree of a vertex**

The number of colored parents incident on the vertex $v$.

To achieve the proven objectives in an efficient manner, the **Vertex Coloring** approach was achieved using a bit vector implementation.

This is shown below in Figure: 1.

![Figure 1: Vertex coloring in two consecutive iterations of BFT](image)

**3.2 SuperNode concept**

As per the previous discussion, at each level, the NSP issues would be resolved iteratively. In order to identify and group non-interdependent task nodes at each level, the concept of **supernode** is proposed. Each supernode is an encapsulation of task nodes with the following properties.

- **There are no interdependencies between the task nodes in a supernode.**
  
  In Figure: 2, there are no interdependencies between nodes 2, 3, 4. Therefore they are grouped into one supernode. But in Figure: 3 since there is a dependency between nodes 3 and 4, node 4 would not be considered for the super node with node 3, even though it lies in the same depth as 3 from the root.

- **There is at least one common parent between every task node in a supernode.**
  
  Therefore nodes with disjoint parent sets would be in different task nodes. In Figure: 3, task nodes 6, 7 have common parents 1, 2, 3. Therefore they are grouped to a single super node. But the nodes 8, 9, 10 have common parents 4, 5. Therefore they are grouped into another supernode. This facilitates in identifying disjoint tasks that can be run in parallel, without reducing the parallelism.

In order to construct the disjoint supernodes in an efficient manner, the **Disjoint-Set Forest** implementation [4] for disjoint sets, we used the “Union By Rank” and Path Compression” heuristics. The basic operations $\text{MAKESET}(v)$, $\text{UNION}(v_1,v_2)$ and $\text{FIND-SET}(v)$ have been implemented inline with the supernode concept.

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3.3 Promoting a Sync node

After creating the supernodes, the NSP problem(s) in each supernode would be generally addressed at each supernode level (except in the case of a disjoint sync node, which is addressed later). While solving the NSP issues at each supernode, we select one of the sync nodes, from the set of sync nodes in the supernode, and promote it as the sync node for the rest of the siblings in the supernode. The procedure is as follows:

- Let all the un-synced parents of all the task nodes in the considered supernode sync to the selected sync node.
- All the other siblings in the supernode would be spawned from this sync node. This would result in adding of an additional synchronization level to the SP graph. The key advantage is that no additional synchronization nodes are added, thus from task node perspective, $T_1$ is not increased.

This is shown diagrammatically in Figure: 4.

The following proves that this strategy would not violate any existing precedence relationships.

**Lemma 3** Among the candidate sync nodes $\{V\} \in$ Supernode $s$, (if any), let $v' \in \{V\}$ be a randomly chosen sync node. Then,

- All parents $\{P\}$ of $\{V\}$ can be synced to $v'$
- $\forall v \in \{V\} - v'$ can be safely spawned from the randomly chosen sync node. without violating the existing precedences.

**Proof** There does not exist a dependency between the nodes in a supernode (by definition).

$$\forall v_i, v_j \in \text{Supernode}(s), \neg \exists \text{edge}(v_i, v_j), \text{where } i \neq j$$

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Therefore one of them can be randomly chosen as a sync node, and the others can be spawned from this chosen sync node. However, the chosen sync node, acts as the new sync node for all the parents of the sibling nodes in the supernode. Since, the parents of all the sibling sync nodes, have synced to the chosen sync node, the spawning of the the sibling nodes does not violate the precedence relation. The spawning of the siblings from the chosen sync node leverages the parallelism in the execution of the nodes in the supernode.

If \( v' \) is the randomly chosen sync node, all the other siblings \( \{V\} - \{v'\} \) can be spawned from \( v' \).

Further, a special kind of NSP problem is the transitive relationship, as explained in Appendix A. Our solution to this, is to select the sync node in topological precedence as the only sync node of the spawn node in question, without violating existing precedences. The following lemma proves its validity.

**Lemma 4** If a vertex \( v' \in G \), has distinct paths \( P_i \) and \( P_j \) ( \( P_i \neq P_j \) ) to more than one sync node \( v_i \) and \( v_j \) in \( G \) and there is a path from \( v_i \) to \( v_j \), such that \( \text{Level}(v_i) \leq \text{Level}(v_j) \). This problem is resolved by sync’ing \( v' \) to the sync node \( v_j \) that precedes all the other sync nodes \( v_j \).

**Proof** From the above definition, \( \text{Level}(v') \downarrow \text{Level}(v_i) < \text{Level}(v_j) \). Since \( \exists \) distinct paths from \( v' \) to \( v_i \), \( v' \) to \( v_j \) and \( v'_i \) to \( v_j \), there is a indirect precedence between \( v' \) and \( v_j \). Therefore this resembles an serial execution sequence \( v' \succ v_i \succ v_j \). Hence without violating the existing precedence it is sufficient to model the dependencies between \( v' \), \( v_i \) and \( v_j \). This can be achieved by letting \( v' \) sync to \( v_i \) and preserving the dependency \( v_i \succ v_j \).

Now, extending form the above proof we will look at a more general case where, even when the two sync nodes does not have any transitive relationships, even though they are in different levels.

**Claim 1** If a vertex \( v' \) in SP DAG \( G' \) has already been synced to a sync node \( v_i \) in \( G' \), \( v' \) should not have an edge to another sync node \( v_j \) in \( G' \), where \( \text{Level}(v_i) \leq \text{Level}(v_j) \).

According to lemma 2, all vertices at level \( L_i \) are resolved before vertices at level \( L_j \) where \( L_i < L_j \). Therefore when the BFT is processing level \( L_i \), the sync node at level \( L_j \) would not be considered for processing. This precedence would model an execution sequence of \( v' \succ v_i \succ v_j \). Therefore as per lemma 3, a candidate sync node would be selected at level \( L_k \) which might be \( v_i \). Also, since there is no dependency between \( v_i \) and \( v_j \) there can be another vertex \( v_k \) in the same level \( L_i \) where \( \exists \) precedence relation between \( v_k \) and \( v_j \) and \( \exists \) a path from \( v' \) to \( v_j \) through \( v_k \). Then from lemma 3, all parents of vertices at level \( L_i \) including \( v' \) would sync to the sync node, which might be \( v_i \) or another. Lets look at this in two scenarios.

**Case 1:** When the sync node at level \( L_i \) is \( v_i \).

By Lemma 3, all the parents including \( v' \) would sync to \( v_i \) and the rest of the siblings of \( v_i \) would spawn form \( v_i \). Then the precedence relation between \( v' \) and \( v_j \) would be \( v' \succ v_i \succ v_j \) through \( v_i \). Therefore there is no need to represent the precedence \( v' \succ v_j \) again. Thus the precedence holds.

**Figure 4:** Promoting a sync node
Case 2: When the sync node at level \( L_i \) is not \( v_i \).
In this instance, another node \( v_{\text{other}} \) would be selected. Thus as per lemma 3, all parents including \( v' \) would sync to \( v_{\text{other}} \) and all siblings including \( v_i \), \( v_k \) would spawn from \( v_{\text{other}} \). Then the precedence relationship would be \( v' > v_{\text{other}} > v_i \) and \( v' > v_{\text{other}} > v_k > v_j \). Since \( v_{\text{other}} \) is the only sync node for \( v' \) as per lemma 3 and the precedence relation \( v' > v_{\text{other}} > v_k > v_j \) there is no requirement for additional precedence relation representation for the relation between \( v' \), \( v_i \) and even \( v' \), \( v_j \).
Therefore it proves the fact that, if a vertex is already synced to a vertex at level \( L_i \) it is not required to sync it again to a node at level \( L_j \) where \( L_i < L_j \).

3.4 Synchronizing to a Disjoint sync node

**Definition 20** *Disjoint sync node (DSN)*

A node is a disjoint sync node if any of its parents are disjoint vertices.

This is an instance where there are sync nodes at the current level, descending from parents that have different spawn nodes, which had distinct execution paths before the current level. This is shown in the LHS of Figure 5. Before delving to our transformation strategy, let’s try to prove that there were multiple parallel execution paths before the current level that ultimately lead to the disjoint sync node.

**Lemma 5** Let \( \{S_i\}, \{S_j\} \) be the spawn sets of vertices \( v_i, v_j \) respectively. If \( \{S_i\}, \{S_j\} \) are disjoint sets i.e. \( \{S_i\} \bigcap \{S_j\} = \phi \), then \( v_i, v_j \) can have different execution paths until they sync up to sync node \( s \) such that, \( \text{Level}(s) > \text{Level}(v_i) \) and \( \text{Level}(s) > \text{Level}(v_j) \).

**Claim 2** The vertices \( v_i, v_j \) have parallel execution paths.

**Proof** Let \( G_i \) and \( G_j \) be the sub-graphs rooted at the disjoint vertices \( v_i \) and \( v_j \). Since the spawn sets maintain the spawn vertices from which the vertex \( v_i \) was spawned, if they are disjoint, then there does not exist a dependency between the vertices \( v_i \) and \( v_j \). Therefore subgraphs rooted at \( v_i \) and \( v_j \) can be executed in parallel until a sync node is encountered, or if there exists a dependency between the vertices in the sub-graph. The execution paths are disjoint as long as no dependency exists between the sub-graphs or a sync node is encountered. In either of the cases, the disjoint spawn-set vertices \( v_i \) and \( v_j \) can be synced to the common sync point, until which parallel execution can proceed.

Then we propose the following synchronization mechanism to resolve the NSP problem when there exists at least one Disjoint sync node.

**Proposition 1** If there exists a disjoint sync node \( d \), in one of the supernodes \( s_i \in \{S\} \) for particular level \( l \) and let \( \{V\} \) be the set of all the vertices at level \( l \), then we propose that we would select \( d \) as the sync node for level \( l \), and all the vertices \( P \in G' \) that are not synced yet, to be synced to disjoint sync node \( d \).

Further we would propose that all the vertices \( v \in (V - d) \) could be spawned from node \( d \). Both of these operations would not violate the existing precedences in the NSP DAG \( G \).

**Claim 3** We claim that this proposition would not violate any existing precedence relationships in the original NSP DAG.

**Proof** First this is a simplistic work around for preserving the precedence when multiple disjoint paths are synced. When disjoint paths are synced, there can be multiple sync structures aggregated together, thus making the graph irreducible to a SP structure. Therefore a less complex way of preserving the precedence is, letting all the vertices \( P \in G' \) that are not synced yet (to preserve one sync node constraint for each task node - lemma 4), to be synced to \( d \). Further from lemma 2, there are no inter-dependencies between the vertices in \( \{S\} \). Therefore spawning vertices \( v \in \{S - d\} \) from \( d \) would not violate any dependencies. Also we know that levels of all the vertices \( v \in \{S\} \) are less than all the vertices that are already resolved [lemma
Further, since the disjoint sync node is the only sync node for the considered level, if there are transitivity relationships between vertices that lie above and below the disjoint level needs to be resolved. The following lemma addresses this issue.

**Lemma 6** For all vertices v, and disjoint sync node d, let Level(v) ≟ Level (d). For all the ancestors of v, who have an edge to v, which topologically precede d, the edges from the ancestor to v can be expunged in G'.

**Proof** We know that, for a disjoint node, all the nodes that topologically precede the disjoint node are definitely synced to the disjoint-sync node [proposition 1]. Thus in every topological ordering of the nodes, the ancestor which is topologically ordered before the disjoint-sync node would always be synced to the disjoint-sync-node. Hence, even by omitting the parent-child node link, we maintain the graph properties of precedence by the Contra positive rule of transitivity.

**Contra positive of the rule of transitivity:**

\[
\text{Level}(\text{Ancestor}) < \text{Level}(\text{Disjoint-sync-node}) = \begin{cases} 
\text{If Level(Ancestor) < Level(disjoint - sync - node) and Level(Disjoint - sync - node) < Level(v)} 
\end{cases}
\]

Let’s look at it in the 2 possible scenarios:

**Case 1:**
A vertex v such that Level(v) > level(D) and Level(D) > Level(Parent(v)). As proven earlier, all nodes v' where Level(v') ñ Level(D) are directly or indirectly synced to D. Then in the original NSP the edge(Parent(v), v) would be in transitive relationship with \{(Parent(v), D) ñ (D, v)} . Therefore this results in an redundant dependency edge(Parent(v), v). Therefore the edge dependency \{(Parent(v), v)} can be expunged.

**Case 2:**
The vertex v with parents v_i and v_j, such that Level(v_i) > Level(D) and Level(D) > Level(v_j). There is a dependent edge relationship like: (D → v_k → v_{k+1} → v_{k+2}...v_{k+n} → v_i) .

As proven earlier, all nodes n such that Level(n) < Level(D) are directly or indirectly synced to D. The when the two dependency edges for v, v_i → v and v_j → v are considered, there is a transitive relationship between v_j → D → v_k → v_{k+1} → v_{k+2}...v_{k+n} → v_i, and v_j → v. Therefore the two transitive relationships can be reduced to a single edgev_i → v due to the fact that the other edge is already synched to a previous level. Therefore since the precedence relationship between v → v_i(Parents(v)) and D are preserved as it is in the NSP DAG, the disjoint set creation and link removal preserves the correctness of the graph.

Now lets examine the following example. As per the figure below, The sync node 9 has parents 5,6 that have different spawn nodes. Thus node 9 would be considered as a *disjoint sync node*. We can see that even though nodes 8, 10 have different parents, that result in different supernodes, the parents of 8, 10 are also synched to node 9. Further, the nodes 8, 10 are spawned from node 9. Even though this results in an reduction in parallelism, it ensures a proper SP synchronization structure for the level.

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3.5 Flow of the Algorithm at a high level

After delving on the basic concepts behind the algorithm, following flow chart shows the overall algorithm at a high level.

**Figure 5:** sync to a disjoint sync node

**Figure 6:** Flow of the Algorithm
4 Algorithm

This section looks at the intricate details of the algorithm and its implementation mechanism.

**Description:**
This is the main entry point to the algorithm. This performs the breadth first traversal.

\[\text{BFT}(\text{NSP} - \text{DAG} G)\]

1. for ∀ level \( L_i \) of the NSP DAG G
2. do
3. let \( V \leftarrow \) set of all vertices at level \( L_i \)
4. \text{CreateDisjointSets}(\{V\})
5. let \( \{S\} \leftarrow \) supernodes created from \text{CreateDisjointSets}
6. \text{CONVERT-SP}(\{S\})
7. End

**Description:**
Creates the set of disjoint supernodes that correspond to the disjoint sets of vertices for the level \( L_i \) during the BFT. The algorithm is based on the disjoint-set forest algorithm with the following heuristics.

1. **Union by rank** - During \text{UNIONSUPERNODES} \((S_1, S_2)\) of two super nodes, the root with fewer nodes is pointed to the root of the tree with more nodes. Rank of a super node is denoted by \( \text{rank}[S] \).
2. **Path compression** - During \text{FINDSUPERNODE}(v), make each node on the find path in the disjoint forest point directly to the root without changing any ranks. Let \( P[v] \) denote the parent of \( v \) in the disjoint-set forest.

\[\text{CreateDisjointSets}(\text{Vertices } \{V\})\]

1. \( \{V_{all}\} \leftarrow \{V\} \cup \text{Parents}(\{V\}) \)
2. for ∀ \( v \in \text{Parents}(\{V\}) \)
3. do \text{MAKESUPERNODE}(v, \text{false})
4. for \( \forall v \in \{V\} \)
5. do \text{MAKESUPERNODE}(v, \text{true})
6. for \( \forall \text{ edge } p \rightarrow v \) where \( p \in \text{Parents}(\{V\}) \) and \( v \in \{V\} \)
7. do
8. \( S_v \leftarrow \) supernode of \( v \)
9. \( S_p \leftarrow \) supernode of \( p \)
10. \( \{S\} \leftarrow \text{UNIONSUPERNODES}(S_v, S_p) \)
11. for \( \forall p \in \text{Parents}(\{V\}) \)
12. do
13. remove \( p \)
14. for \( \forall v \in \{S\} \)
15. do \text{ADDPARENTSPAWNVECTOR}(v)
16. End

**Description:**
Creates a supernode \( s \), with the representative Vertex \( v \) with a rank 0.
MAKESUPERNODE (Vertex v, bool isChild)
1   s ← Ø
2   D bit of s ← 0
3   if (v isChild)
4      then
5         if ∀ parent of v’s C bit == 1
6             then
7              S ← S ∪ v
8              D ← GETDISJOINTPARENTSTATUS(v)
9              if (D)
10                 then
11                    S bit of S ← 1
12
13        else
14         if (C-bit of v =1)
15          then
16              S ← S ∪ v
17
18         rank[s] ← 0.
19   return s.

Description:
Unites two supernodes p and v, where the supernodes v,p are disjoint prior to the union generation.

UNIONSUPERNODES (supernode p, supernode v)
1   S_p ← FINDSUPERNODE(p)
2   S_v ← FINDSUPERNODE(v)
3   Let D_p be the D bit of supernode p
4   Let D_v be the D bit of supernode v
5   Let D be the D bit of joined supernode.
6   if (rank[S_p] > rank[S_v])
7      else
8       P[S_p] ← S_v
9       if (rank[S_p] == rank[S_v])
10          then rank[S_v] ← rank[S_v] + 1
11       D ← Bitwise-OR(D_p,D_v)
12 End

Description:
Finds the supernode where the vertex v is represented currently

FINDSUPERNODE(Vertex v)
1   if (v ≠ P[v])
2      then P[v] ← FINDSET(P[v])
3   return P[v].

Description: Resolves the vertices V ∈ {S} and adds them to the resultant SP graph G’.
**CONVERTSP(SuperNodes \{S\})**

1. Let \( v_i \) be the disjoint sync node returned by FINDSYNC
2. for \( \forall \) vertex \( v \in \{S\} \)
   
   do
   
   If ( \( \text{Parents}(v) == \emptyset \) )
   
   then \( G' \leftarrow G' \cup v \).
   
   Bit Vector of \( v \leftarrow <1,1> \)
   
   else
   
   do
   
   If(\exists \) disjoint supernode \( d \in \{S\} \)
   
   then
   
   Let \( s' \leftarrow \) first supernode \( \in \{S\} \).
   
   \( v' \leftarrow \text{findsync}(s',\text{isDisjointNode}) \)
   
   else
   
   do
   
   for \( \forall s' \in S \)
   
   if ( \( s_s = \text{findsync}(s',\text{isDisjointNode}) \neq \emptyset \) )
   
   then
   
   \( \text{ATTACHSYNCNODE}(s_s,s_s) \)
   
   else
   
   for ( \( \forall \) vertex \( v \in s' \) )
   
   \( \text{ATTACHNODETOSP}(v,\text{PARENTS}(v)) \)
   
   End

**Description:** Finds and returns a random sync node from a supernode \( s \).

**FINDSYNC(Supernode s, bool IsDisjointNode)**

1. \( S_d \leftarrow \text{GETDISJOINTSETSYNCNODE}(S) \)
2. if \( S_d \neq \emptyset \)
   
   then \( \text{SETGLOBALLEVEL} \)
   
   \( \text{IsDisjointNode} \leftarrow \text{TRUE} \)
   
   return \( S_d \)
3. elseif \( \exists v \in S \)
   
   if \( \text{GETAUGMENTEDINDEGREE}(v) > 1 \) and \( \text{Level(\text{Parent}(v))} > \text{GLOBALLEVEL} \)
   
   then Select the first candidate \( S_y \).
   
   Return \( S_y \).
4. else
   
   Return \( \text{null} \)

**Description:** Attaches the selected sync node to the parent vertices of all the vertices in the particular supernode.
**Description:** Attaches a disjoint sync node to all the vertices with out-degree 0 of the SP DAG G', including the parents of all the vertices in the current level.

**SetGlobalLevel(Vertex v)**

1. `GlobalLevel ← Level(v)`
2. End

**Description:** Sets the global disjoint level of the DAG G', to the level of Vertex v.

**GetDisjointSetSyncNode(Supernode s)**

1. for ∀ candidate sync nodes v ∈ S
2. do
3. if GetDisjointParentStatus(v) is TRUE and Level of at least one Parent(v) > GlobalLevel
4. then return v
5. End

**Description:**

Finds the first candidate disjoint sync node (if any) d out the vertices in supernode s.

**AttachSyncNode(Vertex v, Supernode S,)**

1. `v' ← { Parents(v), such that v ∈ S and R − bit(Parents(v)) = 1}`
2. if Logical-OR(R-bit(Parents(v))) = 0
3. then
4. `v'' ← v' ⋃ Parents(v) where R − bit(Parents(v)) = 0`
5. ATTACHNODETOSP(v, v'')
6. elseif Logical-OR(R-bit(Parents(v))) = 1
7. then
8. `v'' ← v' ⋃ Parents(v) where R − bit(Parents(v)) = 1`
9. ATTACHNODETOSP(v, v'')
10. if `{S − v} ≠ φ`
11. then V_p ← `{S − v} ∈ S`
12. ∀v_i ∈ V_p ATTACHSYNCNODE(v_i, v)
13. End

**Description:**

Finds the first candidate disjoint sync node (if any) d out the vertices in supernode s.

**AttachDisjointSyncNode(Vertex v, Supernodes {S},)**

1. Let `{V'}` be all the predecessors P_i of all the vertices v in all the supernodes `{S}` such that R − bit(p_i) = 1.
2. if Logical-OR(R-bit(Parents(v))) = 0.
3. then v'' ← V' ⋃ Parents(v) where R − bit(Parents(v)) = 0
4. elseif Logical-OR(R-bit(Parents(v))) = 1
5. then
6. `v'' ← v' ⋃ Parents(v) where R − bit(Parents(v)) = 1`
7. ATTACHNODETOSP(v, v'').
8. Let `{V''}` be all the vertices in all supernodes `{S}` excluding the disjoint node v.
9. if `{V''} ≠ φ`
10. then ATTACHNODETOSP(v, V'') ▷ attach the rest of the vertices ∈ `{V''}` to v
11. End

**Description:**

Sets the global disjoint level of the DAG G', to the level of Vertex v.

**Description:**

Finds the first candidate disjoint sync node (if any) d out the vertices in supernode s.
getDisJointParentStatus(Vertex v)
1   Let spawn vector of v ← ⊘
2   for ∀ parents p of v
3       do
4           if (spawnVector of p ≠ non-empty spawnVector of v)
5              then
6                  return TRUE
7              else
8                  spawnVector of v ← spawnVector of p ∪ spawnVector of v
9                 Return FALSE.

Description:
Returns the number of parents of a vertex v, that have already been resolved.

getAugmentedInDegree(Vertex v)
1   For Parents(v)
2       do
3           if C = 1
4              then degree ← degree + 1
5           return degree
6   End

Description:
Adds a vertex v to the SP DAG G'.

attachNodeToSP(Vertex v, Parents {P})
1   Let {Pold} be the original parents of v in G.
2   if GlobalLevel = 0
3       then Set the edge pi → v for parents pi ∈ {P}.
4       ∀ pi ∈ {P} Set R = 0.
5   elseif |P| = 1 and Level(P) = GlobalLevel.
6       then Set the edge P1 → v.
7       Reset R ← 0 for pi.
8   else
9       for pi ∈ {P}
10          if Level(P) > GlobalLevel
11             then
12                Set the edge pi → v.
13                R ← 0 for pi.
14       Set R ← 1 for v.
15       Set C ← 1 for v.
16   End

Description:
creates the spawn vector for vertex v.

addParentSpawnVector(Vertex v, Parents {P})
1   for pi ∈ {P}
2     if Outdegree(pi) > 1
3        then spawnVector(v) ← pi ∪ spawnVector(v).
4     else
5        spawnVector(v) ← spawnVector(pi) ∪ spawnVector(v).
6   End
5 Proof of Correctness of the Algorithm

The proof of correctness addresses the following two issues.

- Preservation of precedence during the conversion from NSP DAG G to SP DAG G’
- SP properties of the SP DAG G’.

In order to prove that the generated graph G’ is correct from the aspects of preserving precedence and SP property, the following inductive approach is taken. Two functions are defined in the inductive approach.

Let G be the original NSP DAG and G’ be the SP DAG created at level \( L_i \) where \( 0 < L_i < \text{Depth}(G) \). Let a precedence relationship between two vertices \( v_1, v_2 \) be denoted as \( \text{Pred}(v_1, v_2) \) where \( v_2 \in \text{Predecessor}(v_1) \).

- Let precedence\((G, G’, L_i)\) be the property:
  \[ \forall \text{Pred}(v_1, v_2) \text{ where } v_1, v_2 \in G \text{ and } v_1 \in \text{supernode } s \in \{S\} \text{ at } L_i \text{ and Level}(v_1) = L_i \Rightarrow \forall \text{Pred}(v_1, v_2), v_1, v_2 \in G’ \text{ and level}(v_1) = L_i. \]
- Let SPness\((L_i)\) denote that from level 0 to \( L_i \) \((0 < L_i < \text{Depth}(G’))\) in G’, the SP property has been guaranteed.

Now let’s look at the induction based proof.

- **Base Case**
  Let consider the level \( L_i = 0 \). Then as per lemma 1, 2 vertices \( \{V\} \) at a level L would be resolved before traversing to the next level \( L_2 \) \((L < L_2)\). Therefore when \( L_i = 0 \), the root node \( v \) at level 0 would be resolved to G’. Also since node count = 1 and \( \text{Pred}(\text{root}_1, \text{root}) = \text{root} \), precedence\((G, G’, 0)\) holds. Further from the first definition of a SP DAG, G’ is a SP DAG. Thus SPness\((0)\) holds.

- **Induction Hypothesis**
  Let assume that at level \( L_i \) \((0 < L_i < \text{Depth}(G))\) the two functions hold. Let them be SPness\((L_i)\) and precedence\((G, G’, L_i)\).

  Now let’s consider G where level \( = L_{i+1} \) \((0 < L_i < \text{Depth}(G))\).

  In order to ascertain the correctness at Level \( L_{i+1} \), all different scenarios of graph transformations that could be applied in a particular level is considered, case by case.

- **Case 1**
  **There are no sync nodes in the supernode**

  **Proof**
  Let consider precedence\((G, G’, L_{i+1})\) first. We know precedence\((G, G’, L_i)\) holds. Now as per Lemma 2, vertex at a level \( L_i + 1 \) will only be added when all the resolved vertices \( v \) at \( L_i \) are resolved and added to G’. Therefore \( \forall \text{Pred}(v_1, v_2) \) for nodes \( v_2 \) in level \( L_i \) and nodes in \( L_{i+1} \) in G’ hold the same as for G after resolving. Therefore precedence\((G, G’, L_{i+1})\) holds.

  Also since there are no sync nodes, all the vertices at level \( L_i \) would be parallel to each other. From the third definition of SP DAG, task nodes that are in parallel constitute a SP structure. Thus SPness\((L_i)\) also holds.

- **Case 2**
  **There exists at least one sync node \( v’ \) out of all the nodes \( V \) at level \( L_{i+1} \) in a particular supernode \( s \).**

  **Proof**
  At each supernode \( s \) where there are sync node(s) \( v’ \), as per lemma 3, every parent of all the vertices of \( s \) are synced to a selected node \( v’ \). Also the vertices in \( \{s - v’\} \) are spawned from vertex \( v’ \). Since there are no dependencies among \( v’ \) and vertices in \( (s - v’) \) (lemma 2) there are no precedence relationships among \( v’ \) and vertices in \( (s - v’) \). Therefore as per lemmas 3, moving the vertices \( (S - v’) \) below \( v’ \) as spawn edges, does not break the existing precedence even

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though it creates a new synchronization structure. Further every parent of vertices in s, is synced to one and only vertex v'. Thus selecting a sync node and promoting it as the only sync node for every vertex in S does not violate the existing precedence for Level L_{i+1}. Also we know from the induction hypothesis that \textbf{precedence}(G,G',L_{i}) holds. Therefore \textbf{precedence}(G,G',L_{i+1}) holds.

Now let’s look at the proof of SP property , which would be analyzed in 3 aspects.

- **Case 2.1** There are more than one sync node v at level L_{i+1} in supernode s, and there are vertices \{P\} where \{P\} \subset \text{Predecessor}(v) who have already been resolved but not synced to any node s' where Level(s') \textless L_{i+1}.

\textbf{Proof}

As per lemma 3, a single node v_s, from all vertices \{V\} \in s is selected where all un-synced parents \{P_s\} \subset \{V\} would sync to v_s . Then from lemma 4 there won’t be any p_s \in P_s that syncs to more than one Sync node. Therefore this transformation satisfies the SP property for level L_{i+1}. But From the inductive hypothesis, sp property is held up to level L_i. Then using inductive hypothesis we can claim that up to level L_{i+1}\textbf{SPness}(L_{i+1}) holds.

- **Case 2.2** There are more than one sync node v at the considered supernode s and there are spawn nodes \{P\} where \{P\} \subset \text{Predecessor}(v) who have already been resolved. Then there exists some nodes \{P_s\} \subset \{P\} that sync to a node v \in s as well nodes \{V\} where Level(\{V\}) < Level (v) and level(\{P_s\}) < Level(v') and there are paths from \{V\} to v.

\textbf{Proof}

As per the lemma 4 and proof of claim 1,a transitivity issue like this, for a spawn node p_s \in P_s could be resolved by selecting one sync node s , where its level is the minimum out of all the sync nodes of p_s .Then the node p_s would have already been synced to s. Then the same spawn node would not sync to v. Therefore when node v is selected as the candidate node, the predecessor p_s would not need to sync to v, thus preserving SP property for level L_{i+1}. But From the inductive step, SP property is held up to level L_i. Then using inductive hypothesis we can claim that up to level L_{i+1}\textbf{SPness}(L_{i+1}) holds.

- **Case 2.3** There are more than one sync node v at a considered super node s at L_{i+1}, and there are spawn nodes \{P\} \subset \text{Predecessor}(\{V\}) who have already been resolved. Then some nodes \{P_s\} \subset \{P\} , sync to a node v \in s as well as node v' where Level (v') \textgreater Level (v).

\textbf{Proof}

This is the inverse of case 2.2.Therefore vertices \{P_s\} would sync to the candidate node v rather than v', where Level (v) = L_{i+1} < Level(v') - (lemma 4 and proof of claim 1). Therefore vertices \{P_s\} would only have one sync node, which removes any transitivity issues. Further when the resolving level equals level (v') \textgreater L_{i+1}, then \{P_s\} is already synced to v. Therefore from case 2, it would not sync again to v'. Thus at level L_{i+1} the SP property is preserved. But From the inductive hypothesis, SP property is held up to level L_i. Then we can claim that up to level L_{i+1}\textbf{SPness}(L_{i+1}) holds.

- **Case 3**

There are more than one sync node V at level L_{i+1} and at least one is a \textit{Disjoint sync node}.

\textbf{Proof}

Let v' be a vertex in super node s \in \{S\} at level L_{i+1}. When a sync node v' at level L_{i+1} is disjoint , then as per the proof of claim 3 and lemma 5,

- All the parents of all the vertices in all the supernodes at L_{i+1} and
- All predecessors of nodes at L_{i+1} which haven’t being synced in G’

would be synced to the disjoint node v. Thus as per previous cases, since there is only one sync node at level L_{i+1}, SP property is preserved and \textbf{SPness}(L_{i+1}) is held.
Further, from the proof of Claim 3, at level $L_{i+1}$ precedence is preserved. Therefore as above the precedence relationships up to level $L_{i+1}$ is held.

6 Complexity Analysis of the Algorithm

The complexity analysis is done in a breadth wise manner.
For the worst case, let’s assume the following.

- The DAG $G = (V,E)$ with $|V| = n$ and $|E| = m$, is densely connected. i.e Each level is a fully connected bipartite sub graph. Then from graph theory[5] we could assume that $|V| \approx |E|^2$.
- Average number of nodes at each level = $b$ (Branching Factor). Therefore number of edges at each level = $b^2$.

At each level of the NSP DAG, the algorithm performs the following main functions:

- **Supernode creation.**
  The supernode creation is based largely on the disjoint set algorithm utilizing **disjoint set forest** implementation. At each level, both parents and current vertices would be considered in creating the disjoint sets. From [1] the worst case time complexity for $m$ disjoint-set operations on $n$ elements is $O(m.\alpha(n))$. In the worst case, at each level, $2b$ elements would be considered (nodes at the current level + their parents). Therefore the number of operations $m$ would be
  - $2b$ MAKESUPERNODE operations.
  - $(2b-1)$ UNIONSUPERNODES operations.
  - $2(2b-1)$ FINDSUPERNODE operations.

This sums up to $m = 8b - 3$ operations.

Time complexity for disjoint set operations:

$$O((8b - 3).\alpha(2b))$$  \hspace{1cm} (1)

But each node would be checking for disjointness and coloring status of its parents in MAKESUPERNODE method. Therefore this would involve $O(b^2)$. Hence the time complexity for supernode creation for each level is:

$$O(b^2).$$ \hspace{1cm} (2)

Extending this to all the levels and $n$ vertices, the loose upper bound time complexity for supernode creation for the task graph would be:

$$O(n^2).$$ \hspace{1cm} (3)

- **NSP to SP conversion.**
  As per the algorithm, at worst case the following complexities can be observed for each level.

  - **Disjoint sync node exists.** Then as per the algorithm, both FINDSYNC and ATTACHDISJOINTSYNCNODE have upperbounds of $O(b^2)$.
  - **Sync node exists.** Then both FINDSYNC and ATTACHSYNCNODE have upperbounds of $O(b^2)$.

Therefore loose upper bound in conversion for each level would be:

$$O(b^2).$$ \hspace{1cm} (4)

Then extending to all the levels in the NSP DAG, as before the overall loose upper bound for NSP to SP conversion would be:

$$O(n^2).$$ \hspace{1cm} (5)

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Considering (1) and (2) the overall Time complexity for the overall algorithm in converting a NSP DAG with \( n \) vertices would be:

\[ O(n^2). \]  

(6)

7 Qualitative assessment of the transformation

7.1 Critical path increment

7.1.1 Upper bounds

The algorithm processes the NSP DAG in a breadth first manner. When the traversal depth is at level \( L_i \), all the nodes \( \{V\} \) where \( \text{Level}(V) < L_i \) are already resolved to a SP DAG. As per the definition of supernode, at each level \( L_i \), all disjoint sub graphs of \( G \), that belongs to different supernodes, would be resolved in parallel. Therefore irrespective of the number of disjoint sub graphs at level \( L_i \), a sync node would be selected for synchronizing the NSP problems at level \( L_i \) for each sub graph. Therefore if there are \( n \) such supernodes, there would be one sync node promoted for each supernode, which would be in parallel to each other. Therefore irrespective of \( n \), only one level of additional sync nodes would be added for each level of a NSP problem.

Further when there are NSP problems across disjoint parents of the nodes at level \( L_i \) ( i.e disjoint sync nodes), irrespective of other NSP problems in the individual supernodes, the algorithm selects a disjoint sync node to synchronize all the nodes at level \( L_i \) ( proposition 1 ), thus incrementing the critical path length by 1 for the level \( L_i \). Therefore at worst case, for a graph with a depth of \( L \), for every level \( L_i \) ( \( 0 < L_i < L \) ) there can be at least one NSP problem. Therefore at each level, a specific synchronization is required. This results in a critical path increment of \( 1 \times L \).

- Original depth of the NSP graph = \( L \)
- Additional path length added in worst case = \( L \)

Therefore we claim that the upper bound in the increment of Critical path length is \( 2L \) for any type of NSP Graph with computational depth of \( L \).

7.1.2 Lower Bound

For completeness, lower bound invariably would result in when an SP DAG is processed by the algorithm. Since there are no NSP problems at any level \( L_i \) ( \( 0 < L_i < L \) ) that needs transformation, the graph would not be transformed. Therefore the lower bound would be \( L \).

7.2 Increase in Critical path vs reduction in parallelism

When a task node is selected as an sync node for a level \( L \) to solve an NSP problem our algorithm creates a new level of synchronization in the task graph. Even though it does not increase the overall work \( (T_1) \) it increases the critical path \( T_\infty \).

As an example if we assume that there are \( n \) nodes at level \( L_i \), selecting one candidate node as the sync node \( v \) would result in other \( (n-1) \) siblings spawning out from \( v \). The resultant would be:

- Increase in critical path by 1.
- Reduction in the parallelism at level \( L_i \) from \( n \) to \( (n-1) \).

In the case where this selected vertex \( v \) has a significant larger task load, then the rest of the \( (n-1) \) tasks get delayed until \( v \) is finished. Therefore as a variant to our existing algorithm we would like to explore the following heuristical approach.

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7.3 Variant of the Algorithm using heuristics

If the loads of every task node in a supernode can be measured, we suggest the following heuristics in selecting the sync node rather than selecting a first sync node as the synchronization node.

First let’s define cost $SOH_c$ -As the overhead incurred in adding a new task node to $G'$.

**Assumption 1** Let’s assume that the synchronization cost of the parent nodes on a sync node would be similar , when either an existing task node or a new synchronization node is used. Thus is not used in the comparative cost analysis below.

The suggestions are as follows:

- Select the sync task node $v_{lo}$ with the lowest work load , say $V_{c_{lo}}$ where $\text{cost}(v_{lo}) \leq \forall \text{ vertex } v \in \text{current supernode}$. This would result in a lesser increment to $T_\infty$ than selecting an arbitrary node, from the point of actual execution time of the Critical path inline with the definition in [2].
- Include a new synchronization node $v_s$ where all parents of all vertices in supernode would sync to, when $V_{c_{lo}} \gg SOH_c$. The following remarks needs to be considered.
  - This would result in an lesser increment in the actual execution time since $T_\infty + V_{c_{lo}} \geq T_\infty + SOH_c$.
  - But would result in an increase in the Total work since the addition of $SOH_c$. The new Total work would be $T_1 + SOH_c$.

8 Experiments and analysis

In order to verify the correctness and the quality of conversion, from the aspects of increase in critical path, precedence preservation and SP properties, several experiments were carried out. They were categorized under two aspects.

- Modelling of task graphs that represent actual applications
- Asses the conversion quality by using random NSP Graphs with different topologies and work loads.

8.1 Actual Task Graph modelling

In this empirical analysis, several graph structures were utilized. Further, to assess the quality we compared our transformations with a previous transformation algorithm suggested by Gemund et.al [6].

8.1.1 Fully connected Bipartite Graph

We achieved commendable results for this transformations. At each level of the graph with a NSP issue, we were able to achieve SP transformation by addition of one level. Therefore in a Bipartite graph with $n$ levels, the maximum increase in the critical path was $2n$. 

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The following graph shows the comparative results.

![Figure 8: comparison of Bipartite graph conversion](image)

**8.1.2 Paired Synchronization Graphs**

This can be modelled in parallelization of loops where a part of the loop body contains loop-carried dependencies. Our comparative results were very attractive. When we experiment out with NSP problem at only one level with multiple levels of SP subgraphs following it, our algorithm managed to narrow down the increment of $T_\infty$ to 1. But critical path increment of [6] increased linearly proportionate to the number of SP levels below the NSP level as given in the figure 9.
The following graph shows the comparative results.

![Paired Synchronization example](image)

**Figure 9:** Paired Synchronization graph transformation

The following graph shows the comparative results.

![Paired synchronization example](image)

**Figure 10:** comparison of Paired Synchronization conversion

### 8.1.3 Macro Pipeline

The Macro Pipeline graphs can be seen in wave-front computation applications. Let's denote the width (and the breadth) of the task graph as $M$, then our algorithm managed to restrict the Critical path increase to $2M-4$, where as in the comparison [6] showed more increase in the critical path. Further there was no increase in the Total Work $T_1$. Following graph shows a comparison with [6][7]
8.1.4 Blumofe’s lower bound example [8]

This represents a NSP DAG with computationally deep and shallow tasks in the same portion of computation. The model in a multi-threaded computation would be as in Figure 13. The root thread spawns $m$ threads of subcomputations. Then this could be represented in a NSP DAG as shown in Figure 14.

When we convert the NSP DAG to a SP DAG, we observed that the critical path increase was $(m-1)$ where $m$ denotes the subcomputations spawned from the root thread. Figure 15 shows the transformed SP DAG.
Figure 13: multi-threaded computation model

Figure 14: NSP DAG representation of the computation
8.2 Random Graph Analysis

In order to verify the correctness and transformation quality of our algorithm, we utilized random Graphs generated by Waseda University, Japan [9]. Two sets of graphs were utilized each having around 100 graphs. One set comprised of 50 node task graphs while the other included task graphs of 100 nodes each. The resultant graphs were verified for:
- Preservation of precedence
- SP property of the transformation.

Following shows the comparative results. The results generated from the random graphs were encouraging to the fact that, the increase in Critical path was always less than twice the critical path length of the NSP DAG. The summarized results for 100 node random task graphs were as follows:

![SP DAG representation of the computation](image-url)
8.3 Verification of the results

Keeping inline with the theoretical verification of the algorithm we took effort in verifying the quality of the transformation empirically. The following steps were taken.

- Verification of preservation of precedence in the transformation through a precedence checking program.
- Verification of SP properties through a SP property checker program. The Series-parallel digraph recognition algorithm proposed by Tarjan et. al [10] was implemented in order to verify the SP properties in the transformed task graphs.

9 Conclusion

We were able to successfully implement a new algorithm that gives a more qualitative NSP to SP conversion with a lesser time complexity compared to the previous findings. Further after implementing the algorithm, we managed to verify the correctness and the quality through converting actual NSP computation models and random task graphs. Following summaries our achievements.

- Upper Bound for increase in $T_\infty$ is $2T_\infty$.
- Upper Bound for increase in $T_1$ is 0.
- Time Complexity of the Conversion Algorithm is $O(n^2)$
9.1 Future Work and Enhancements

Following describes the enhancements that could have been applied in the project and projects the future scope of any possible extensions.

- The current scope address the NSP DAG to SP DAG conversion from a theoretical standpoint. But the conceptual transformation can be verified by modelling the transformation in an actual parallel multi-threaded environment like Cilk. This would facilitate in analyzing the actual increase in $T_\infty$ and $T_1$ according to the cost model of the underlying environment.

- The current algorithm results in unbounded in-degree when synchronizing levels with disjoint sync node. This results in a bottleneck for that particular level through the disjoint synchronization node. Another possible exploration area would be refine the algorithm to avoid such bottleneck.

- Even though we were able to perform the transformation with significant effects on the time bounds, we haven’t performed a thorough analysis of the effects on space bounds after the transformation. This is essential since its would be fruitless to achieve significant time bounds with relatively inefficient space bounds.

- As there exists parallel algorithms and implementations to verify the ”SP-ness” of a arbitrary DAG, a possible extension would be to delve into a parallel implementation of the algorithm. This would involve in analyzing possible vertical/horizontal partitioning of the task graphs to disjoint sets (similar to the Supernodes) and applying the current algorithm on those disjoint task graph sets.

- Further, we would like to verify the correctness of the algorithm with more models of actual task graphs as well as random task graphs with more variety and task nodes. Further it would be relevant to verify the correctness using task graphs with different task loads rather than unit work loads. This would enable us to compare this with the variant we have discussed and analyse the pros and cons of each method.

10 Appendix

10.1 Appendix A - NSP problems

This section outlines the following NSP problems,( Where any NSP problem could be decomposed into [6]) that are addressed in our Algorithm.

- A spawn node syncs to sync nodes that are in a transitive relation. The sync nodes are in different levels in the NSP DAG. This can be called a transitive relation problem.

- Spawn node(s) syncing to more than one sync node that are in the same level (e.g. simple bi-partite graph).

References


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