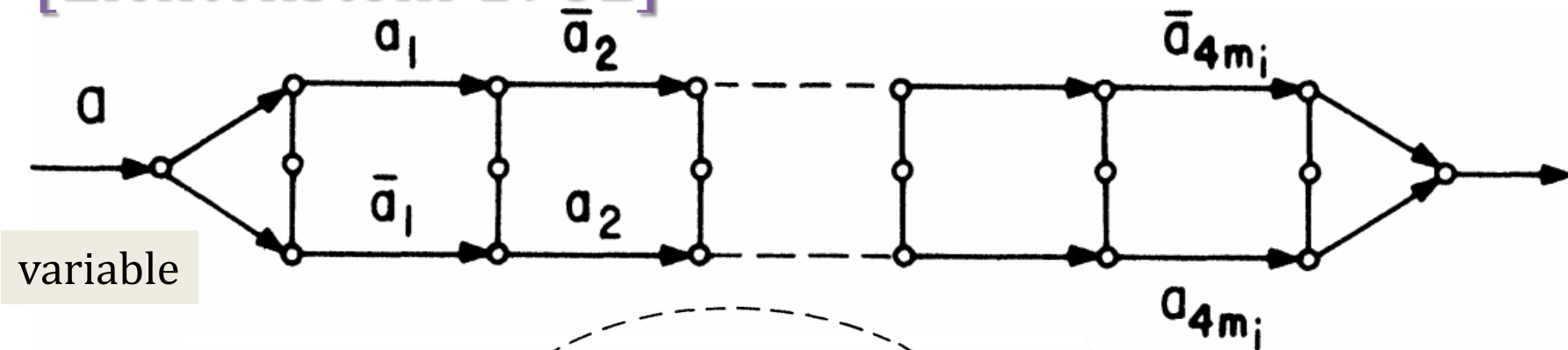


Icosian Game [Hamilton 1857]



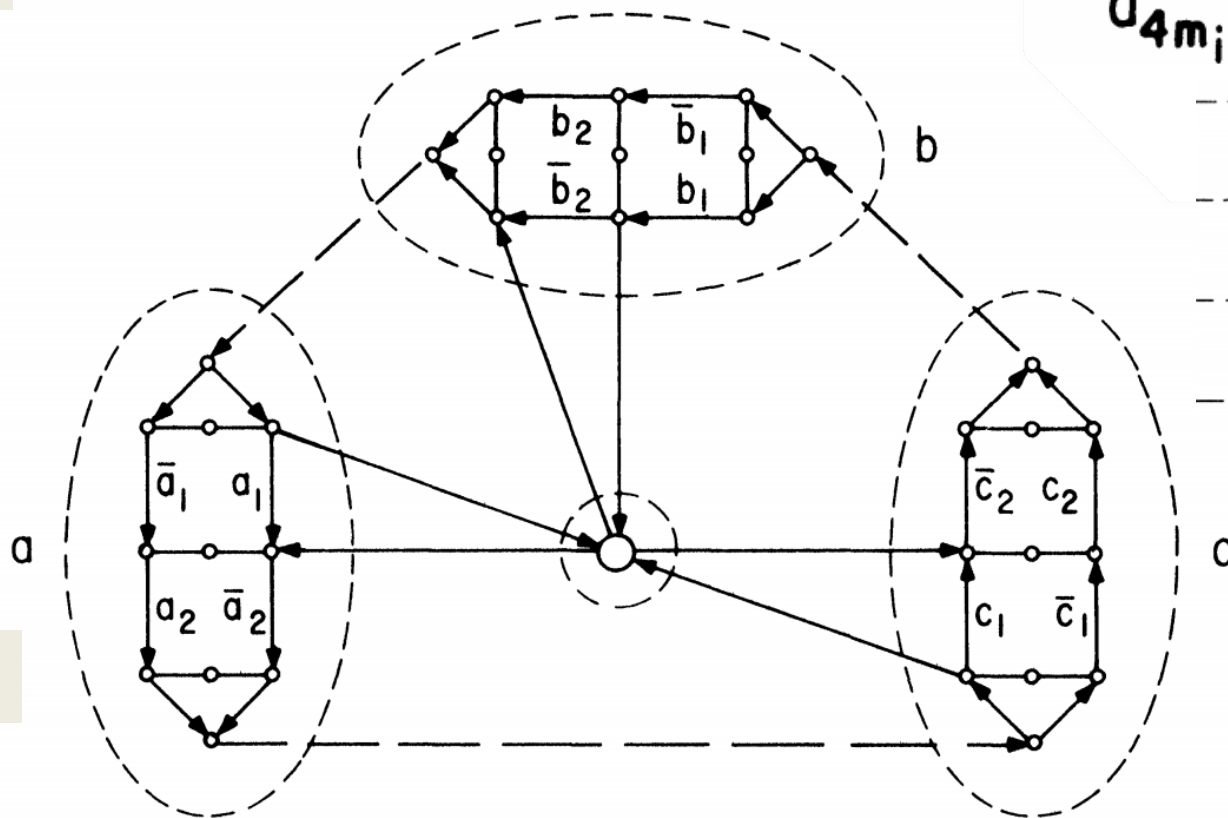
Planar (Directed) Hamiltonian Cycle

[Lichtenstein 1982]

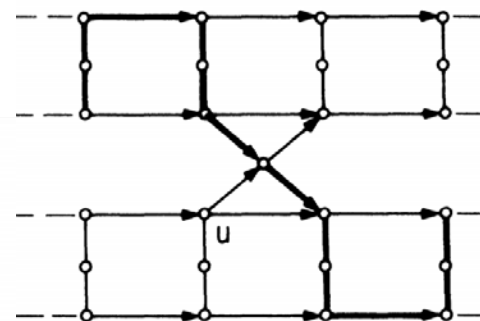


clause

$a \vee \bar{b} \vee c$

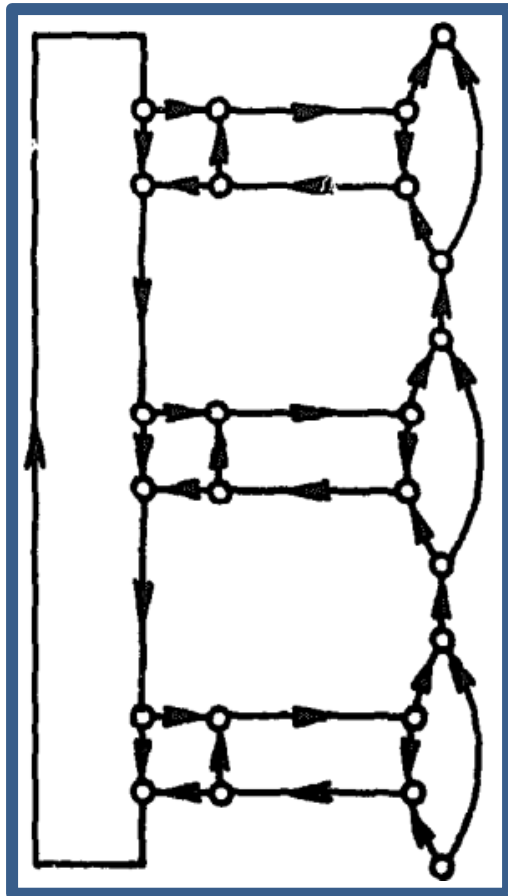


a_{4m_i}

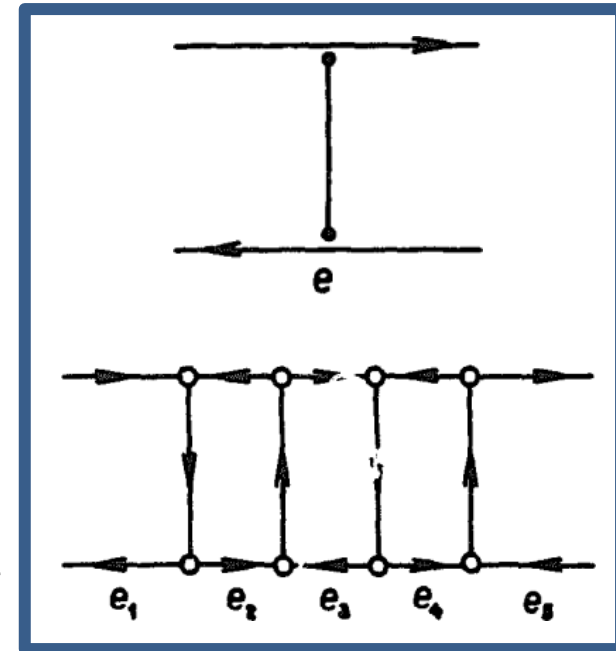
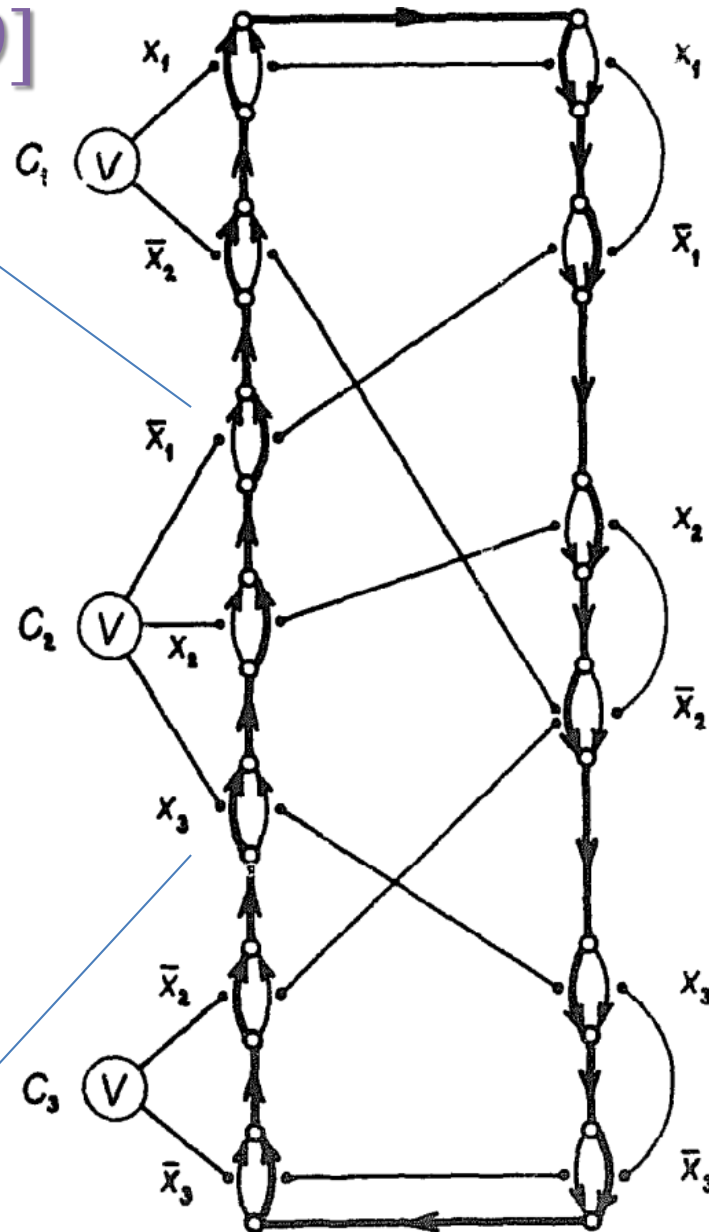


Planar Directed Max-Degree-3

[Plesník 1979]



clause gadget



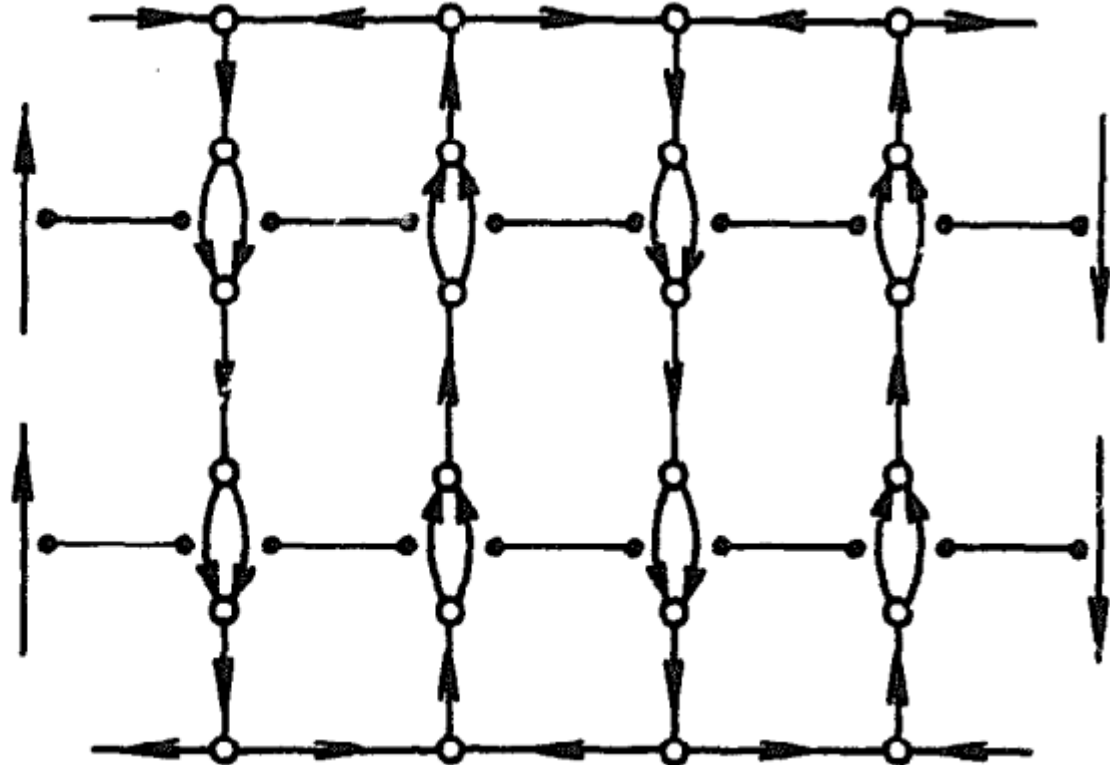
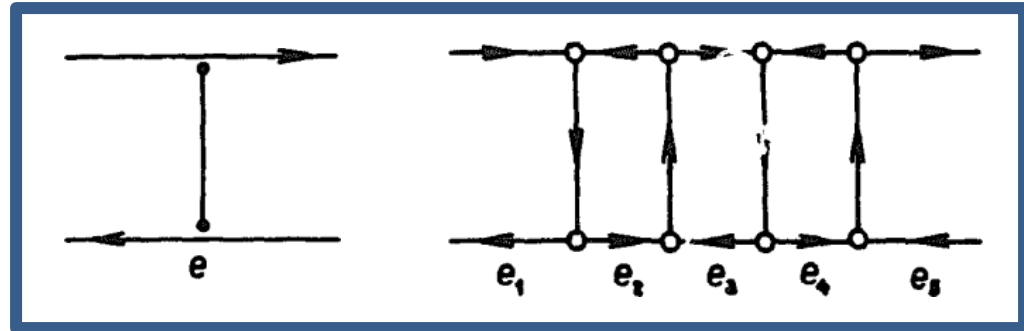
XOR gadget

$$\begin{aligned} & (x_1 \vee \overline{x_2}) \\ & \wedge (\overline{x_1} \vee x_2 \vee x_3) \\ & \wedge (\overline{x_2} \vee \overline{x_3}) \end{aligned}$$



Planar Directed Max-Degree-3

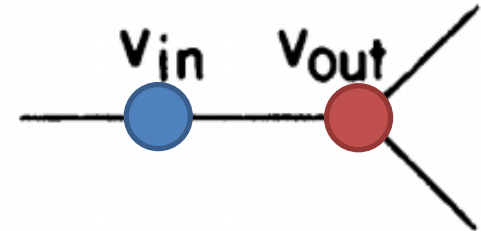
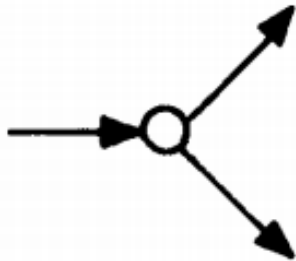
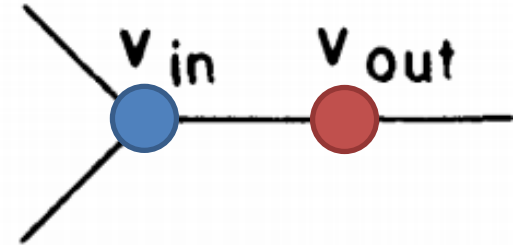
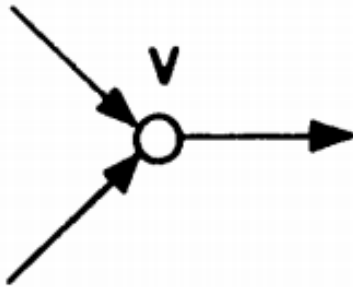
[Plesník 1979]





Planar Bipartite Max-Degree-3

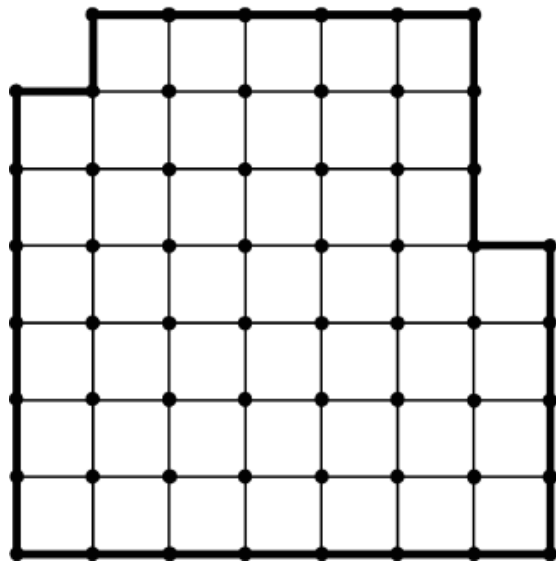
[Itai, Papadimitriou, Szwarcfter 1982]



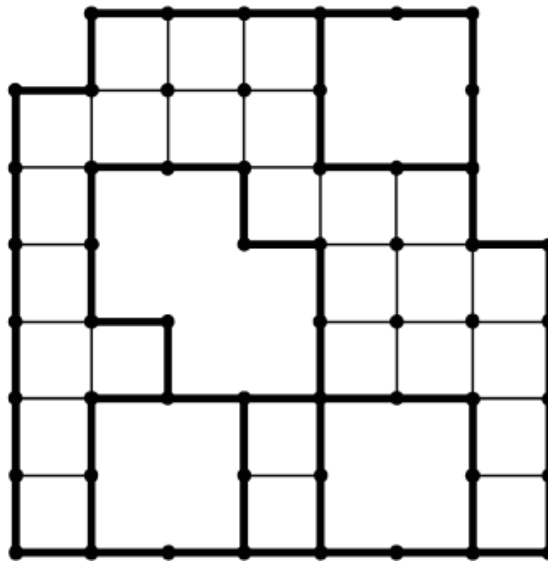


Grid Graphs

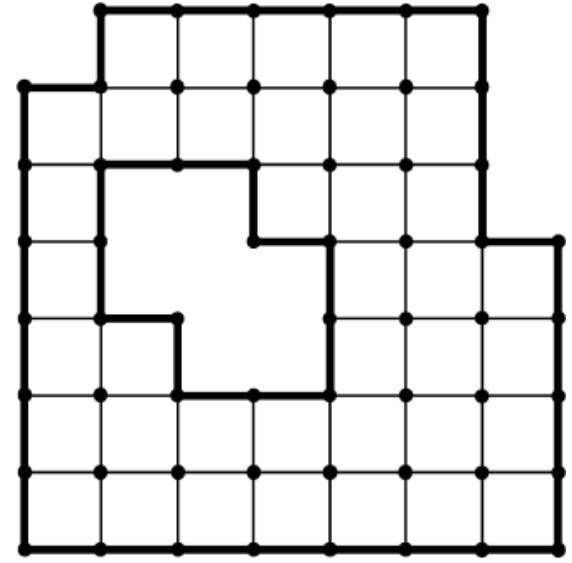
[Itai, Papadimitriou, Szwarcfiter 1982]



solid
(no holes)



holes



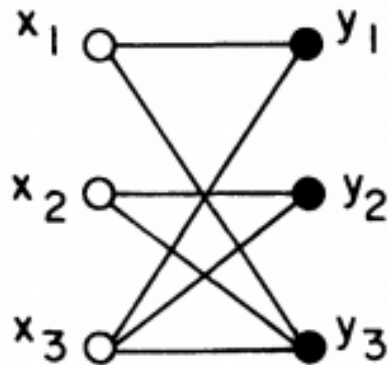
holes

figure by Arkin, Fekete, Islam, Meijer, Mitchell,
Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009

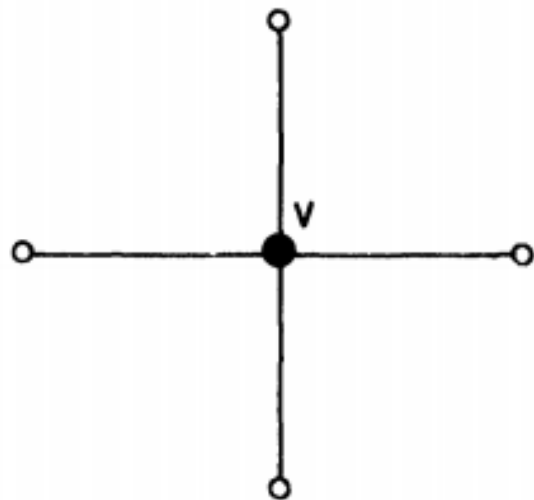
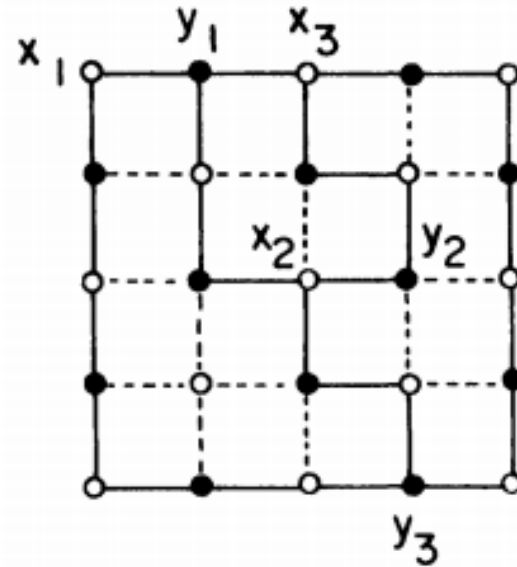


Planar Bipartite Graph Drawing

[Itai, Papadimitriou, Szwarcfiter 1982]

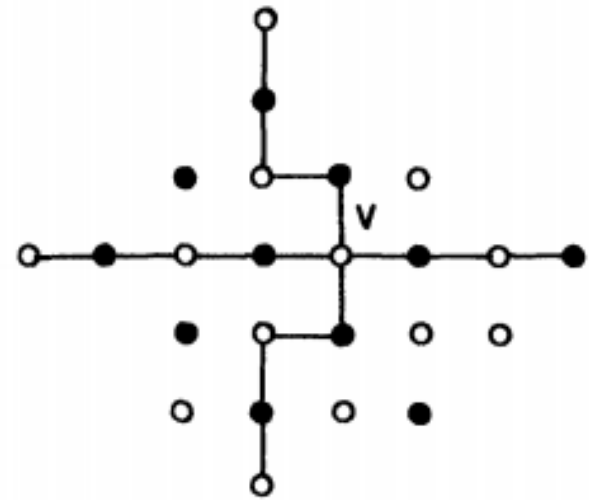


⇒
emb



⇒

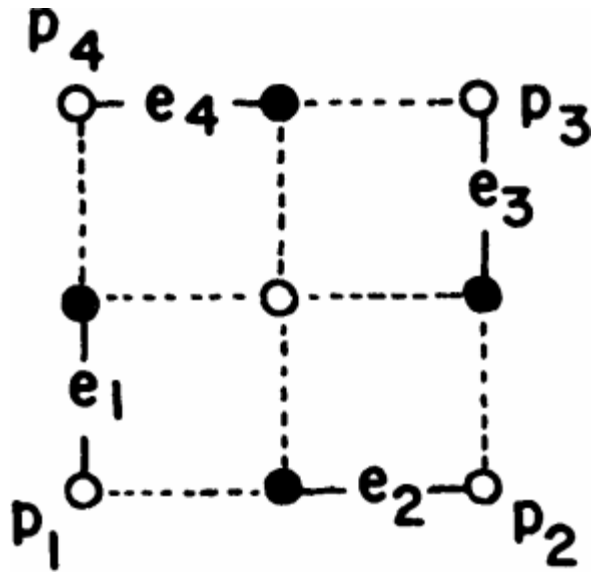
scale 3×



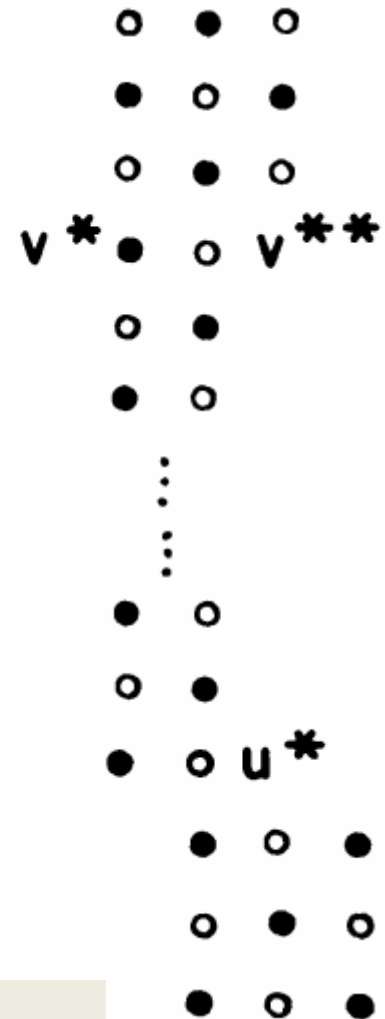


Hamiltonicity in Grid Graphs

[Itai, Papadimitriou, Szwarcfiter 1982]



vertex gadget

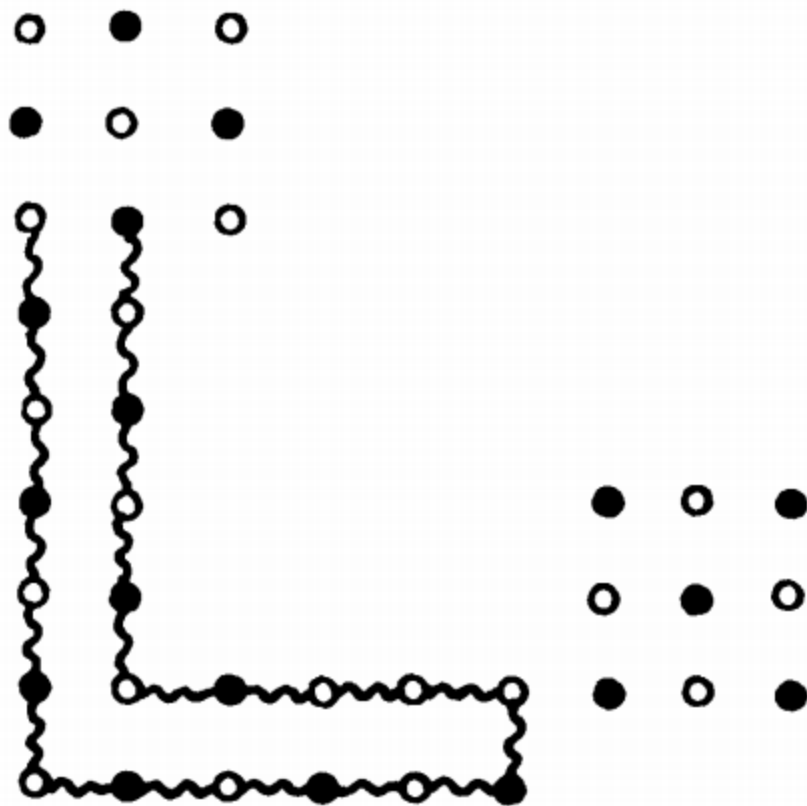


vertex-edge connections

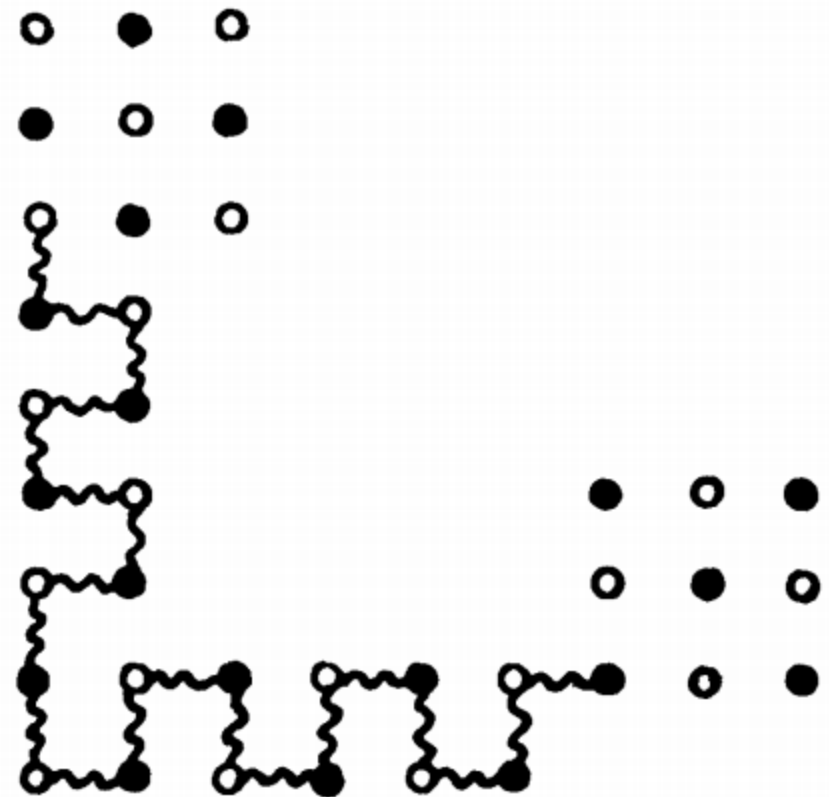


Hamiltonicity in Grid Graphs

[Itai, Papadimitriou, Szwarcfiter 1982]



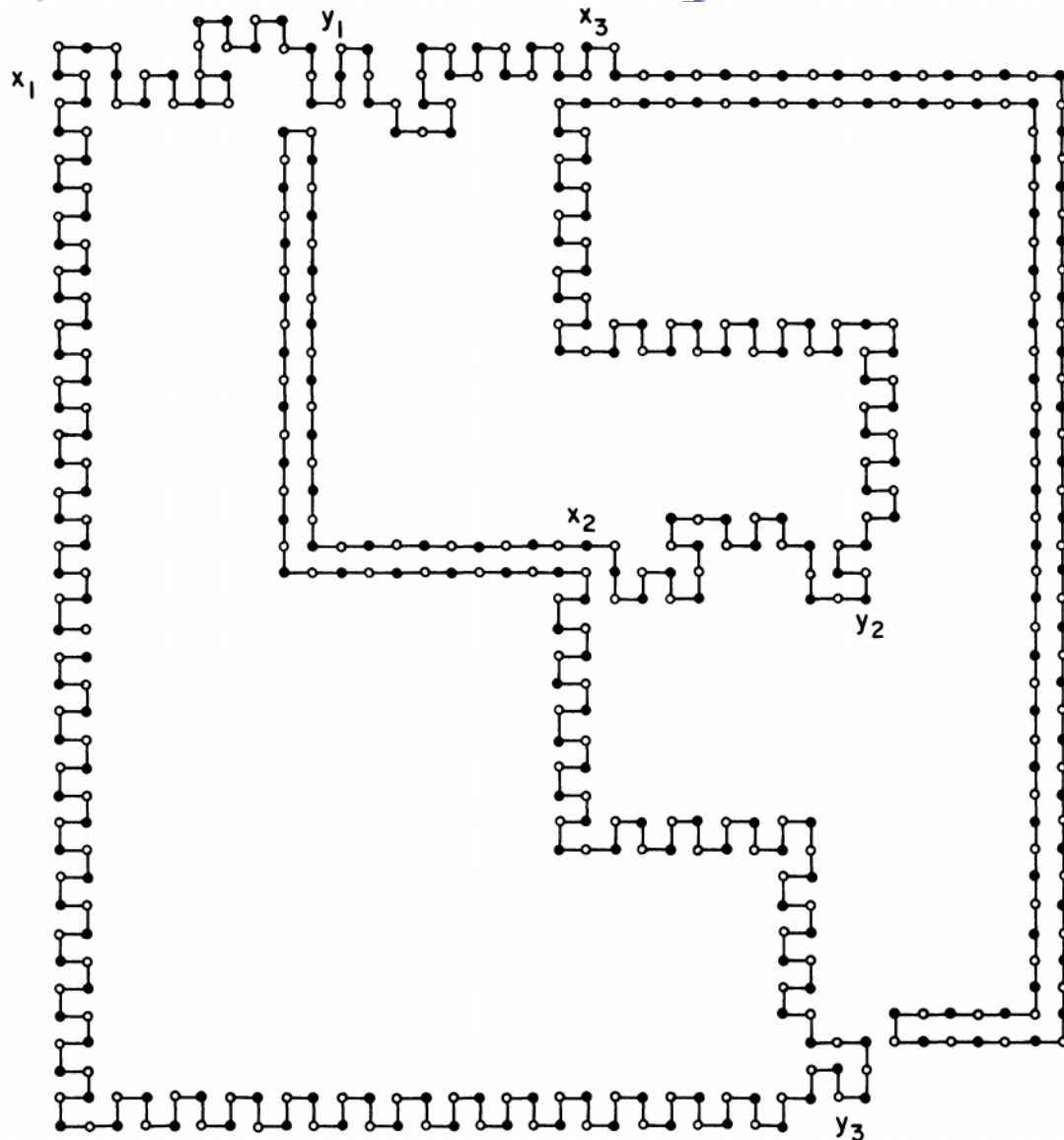
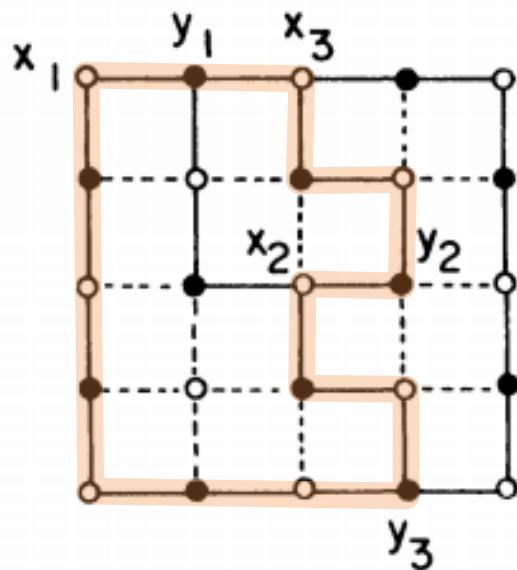
unused edge gadget



used edge gadget

Hamiltonicity in Grid Graphs

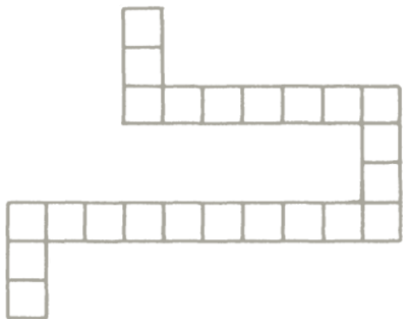
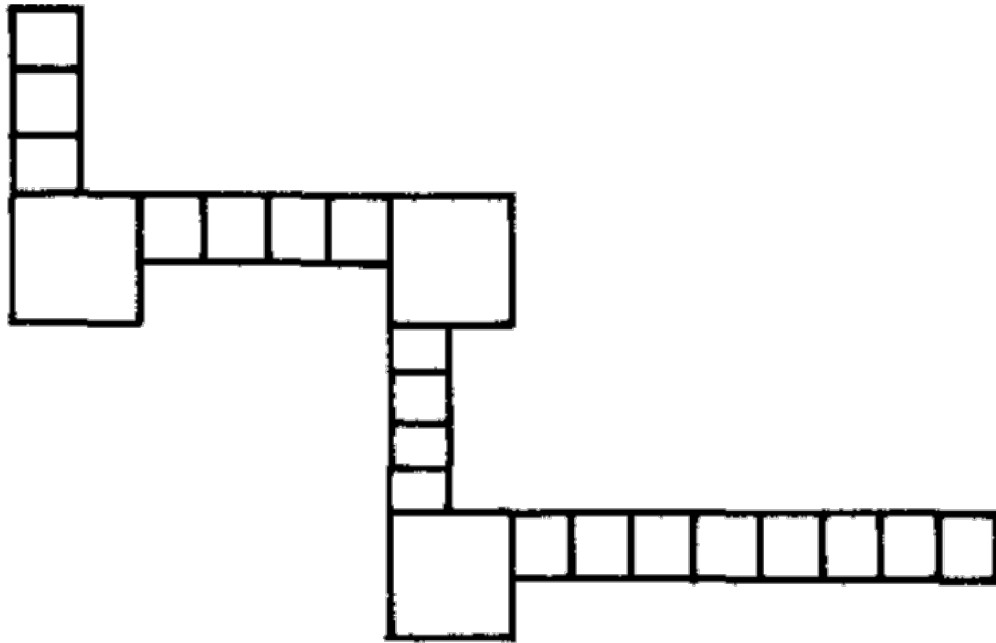
[Itai, Papadimitriou, Szwarcfter 1982]



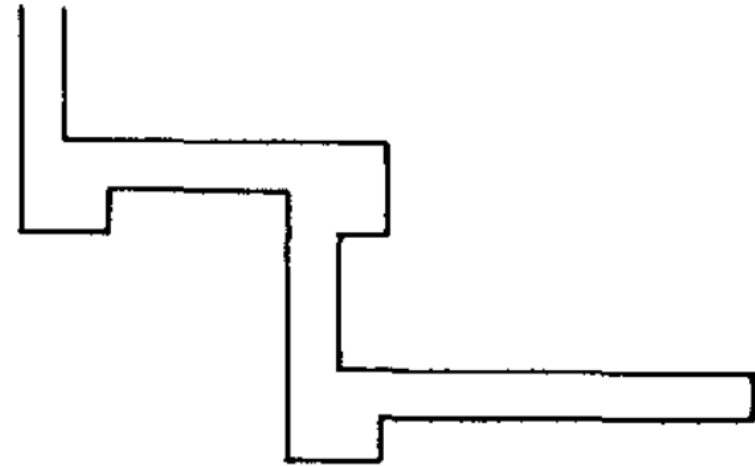


Max-Degree-3 Grid Graphs

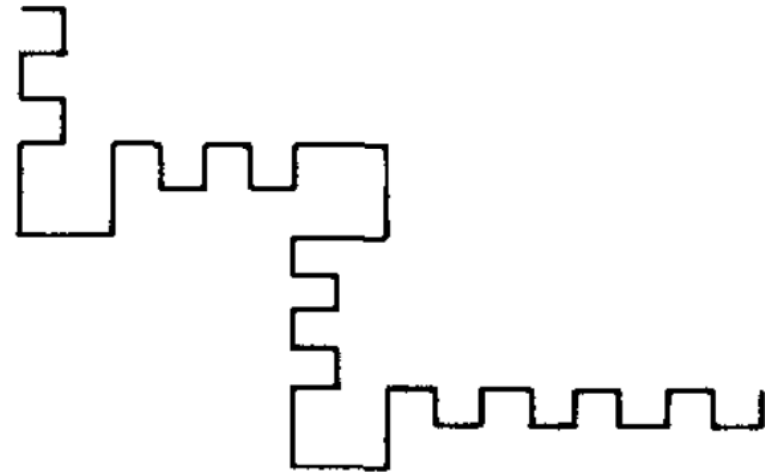
[Papadimitriou & Vazirani 1984]



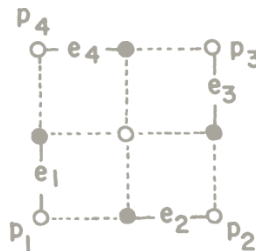
edge gadget



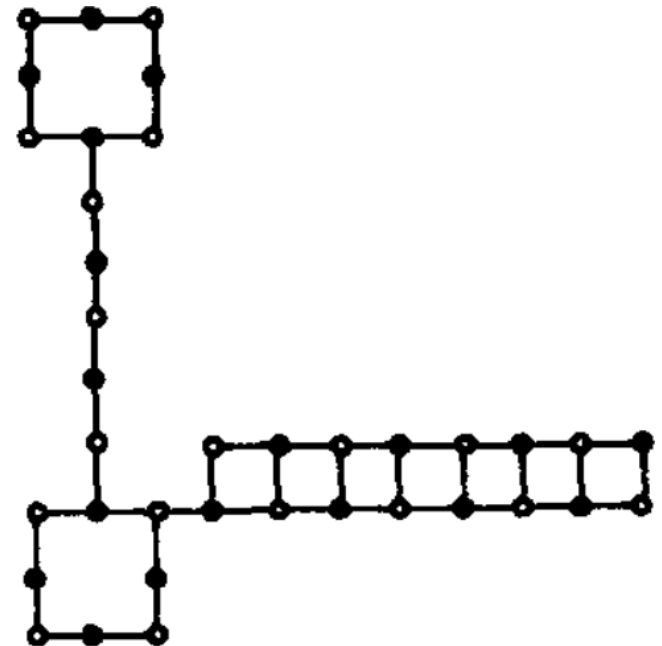
Return Path



Cross Path



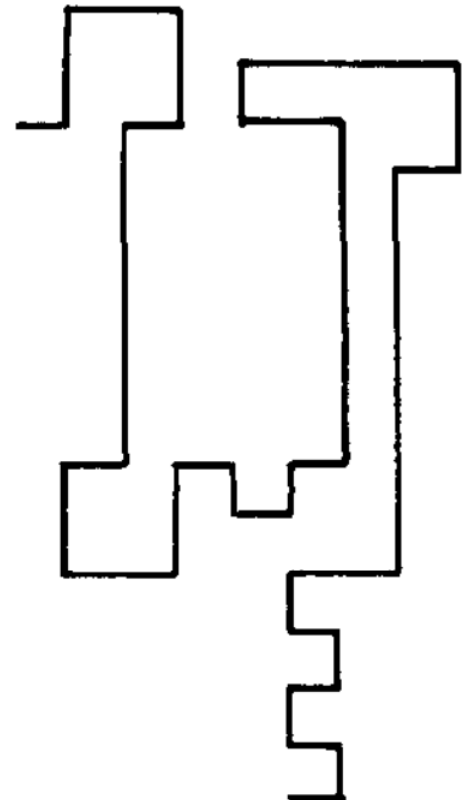
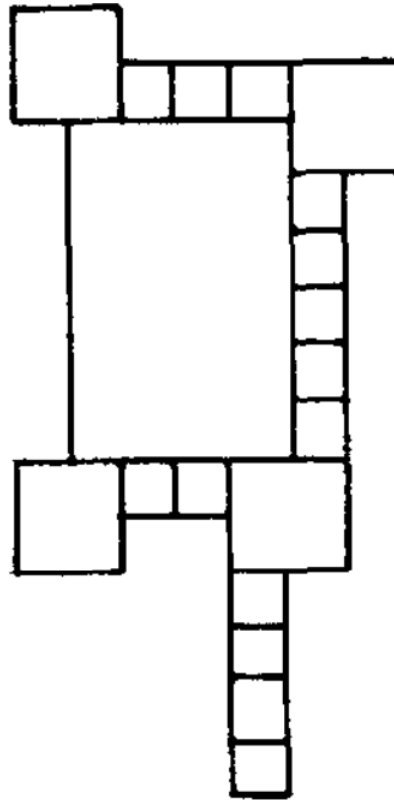
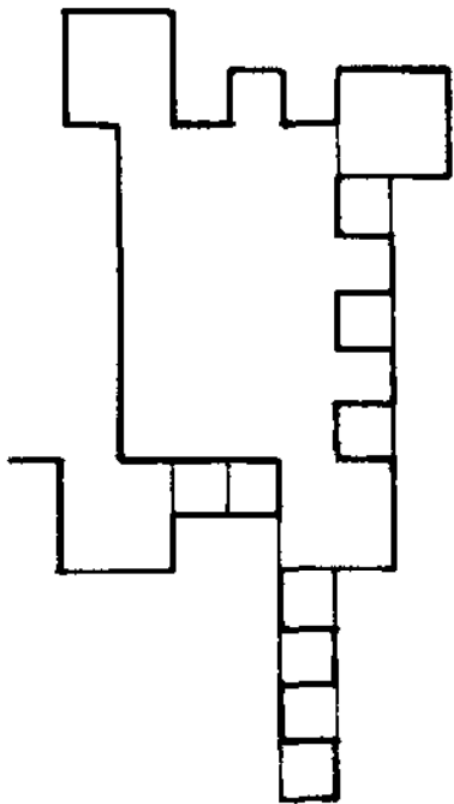
vertex-edge co



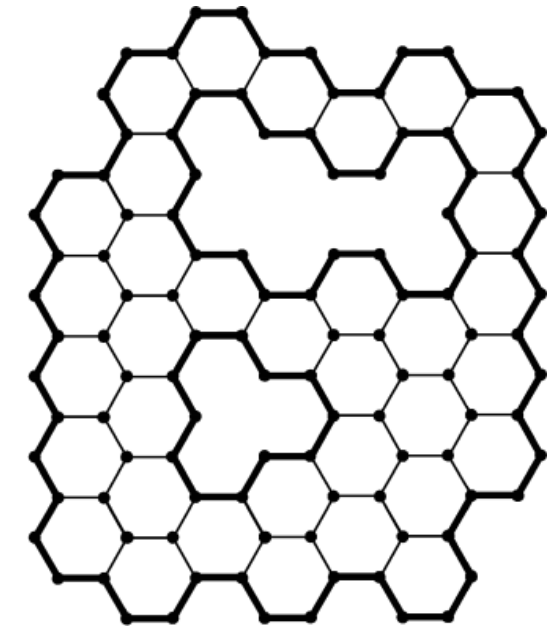
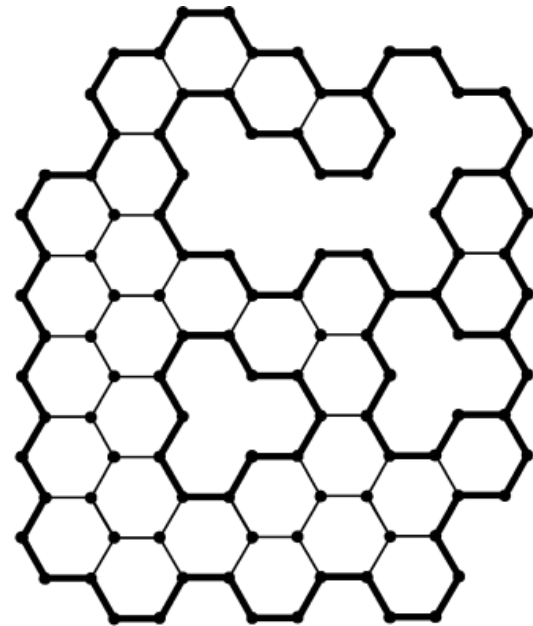
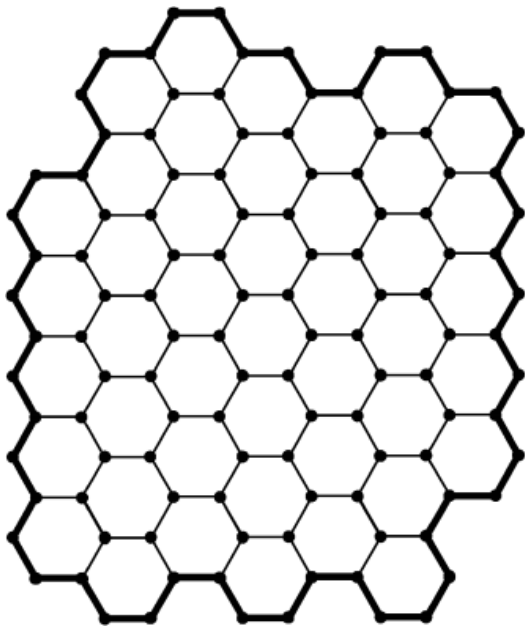
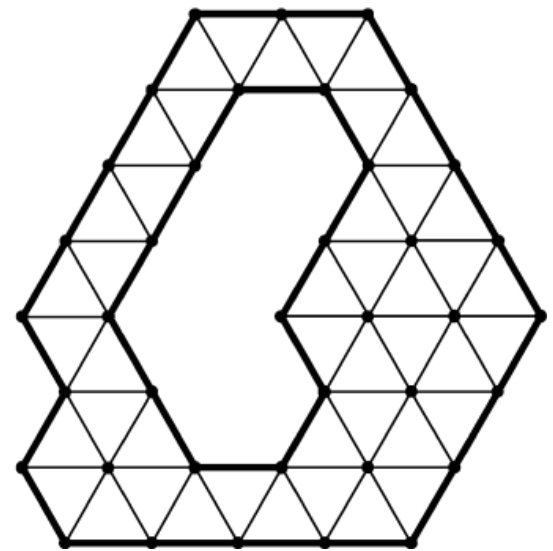
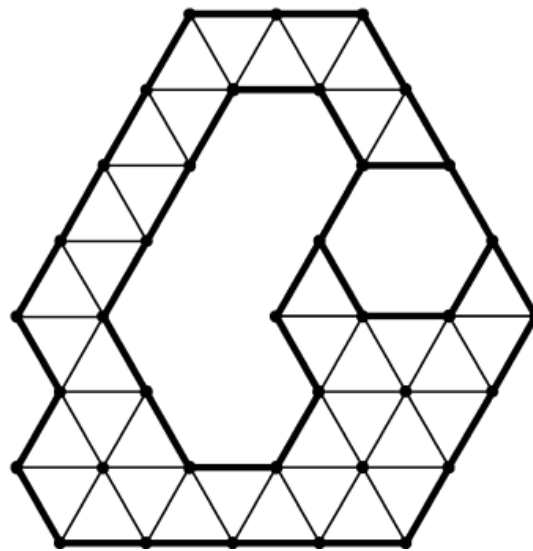
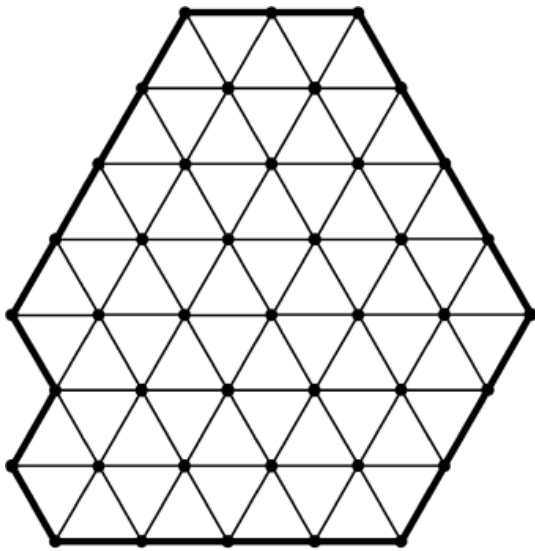


Max-Degree-3 Grid Graphs

[Papadimitriou & Vazirani 1984]



forced-edge vertex-edge connections



solid

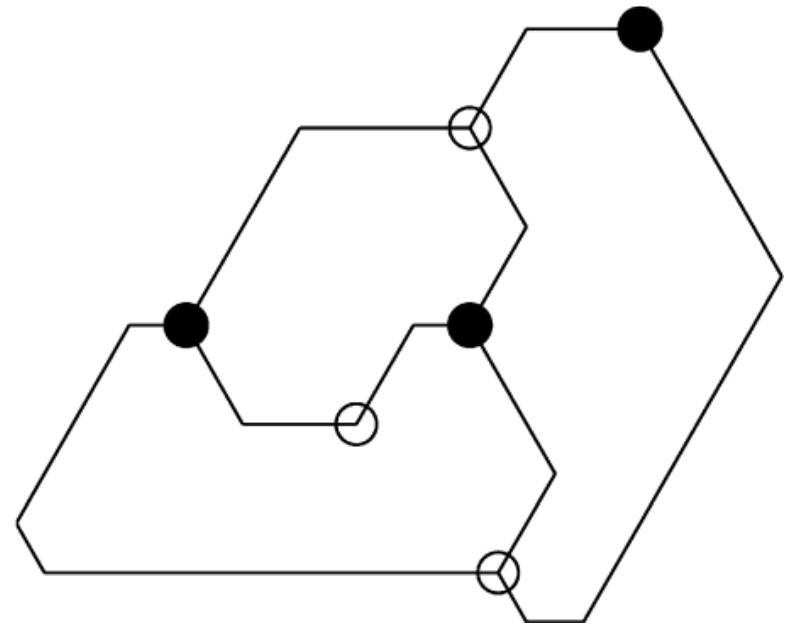
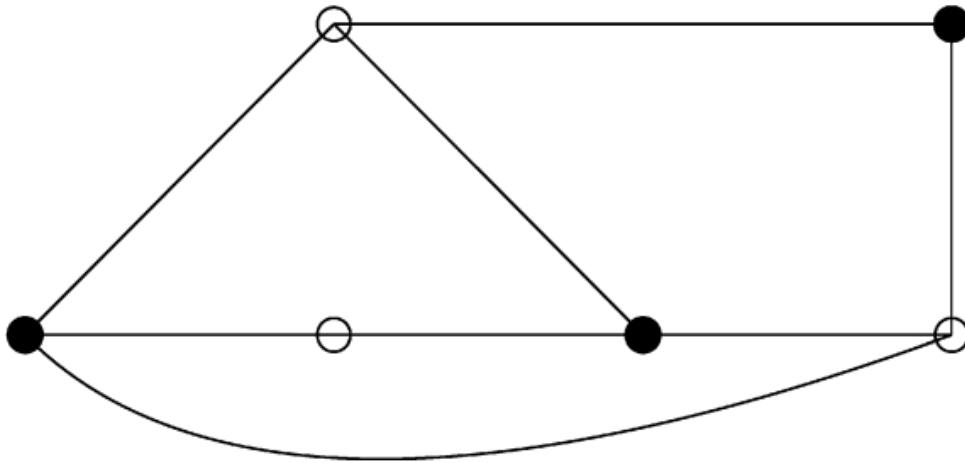
Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009

polygonal



Hamiltonicity in Triangular Grid Graphs

Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009

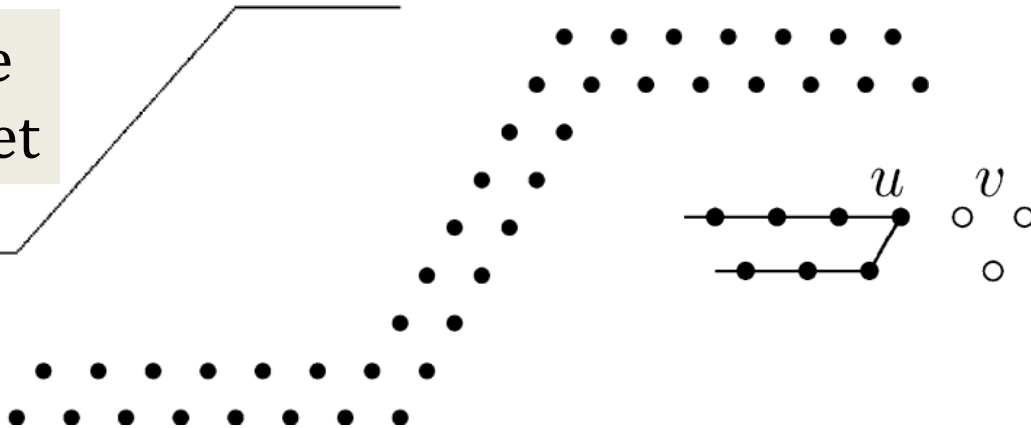




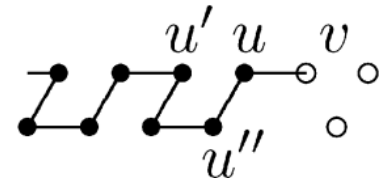
vertex gadget



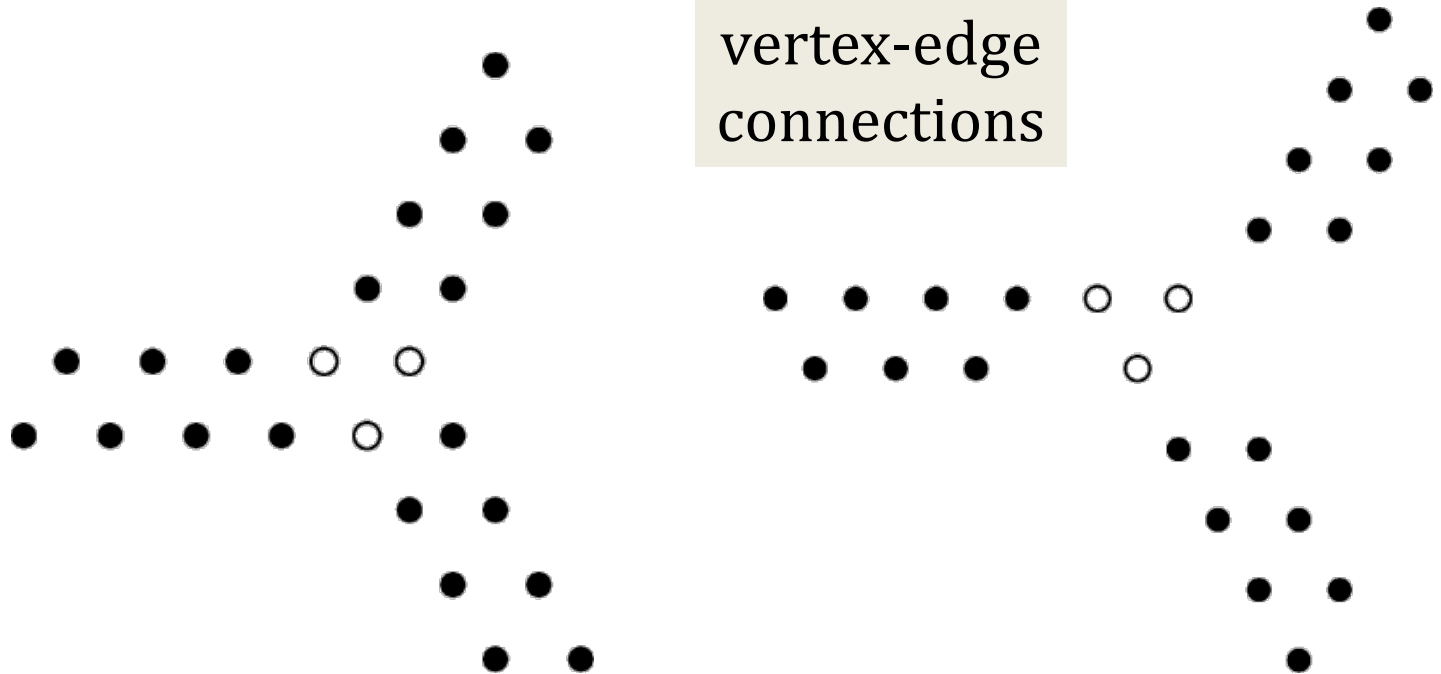
edge
gadget



Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009

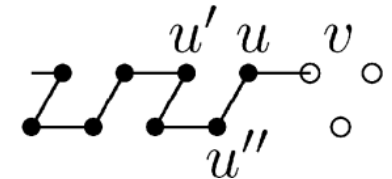
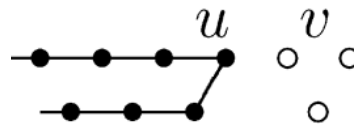
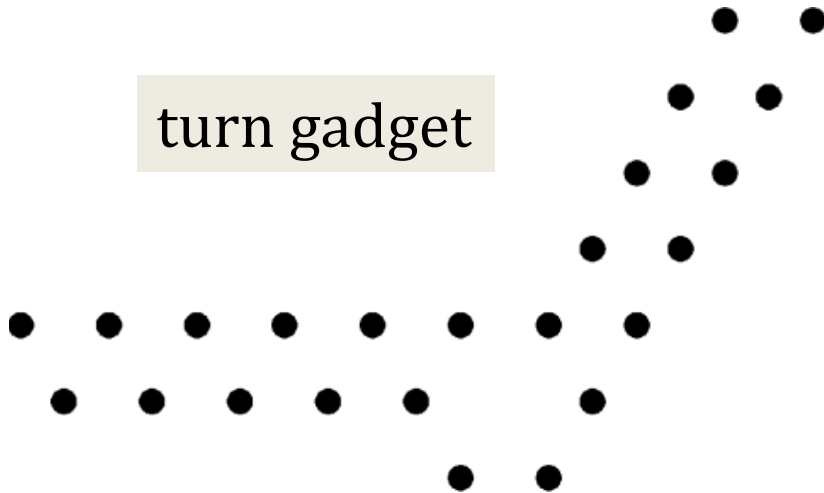


vertex-edge
connections



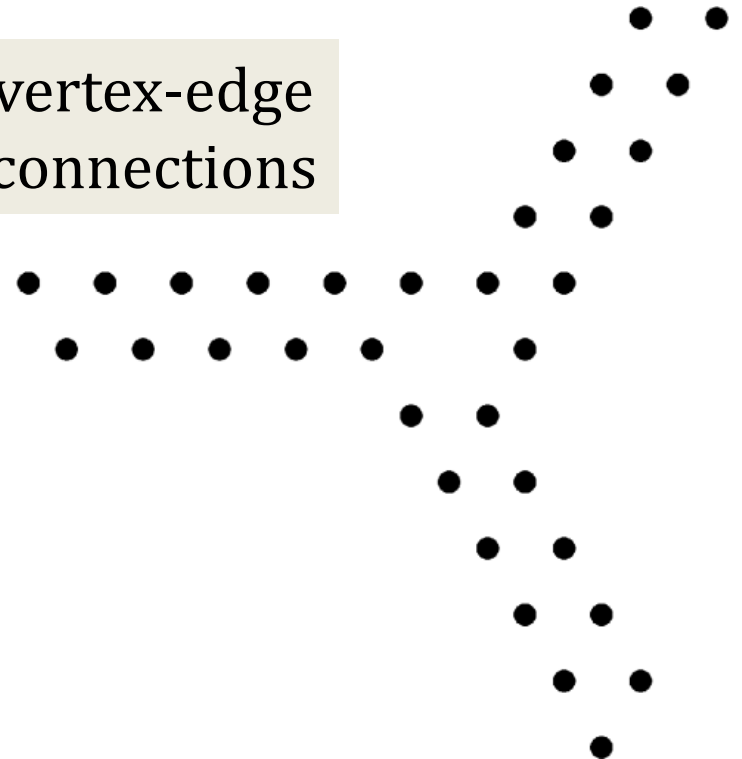


turn gadget



Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009

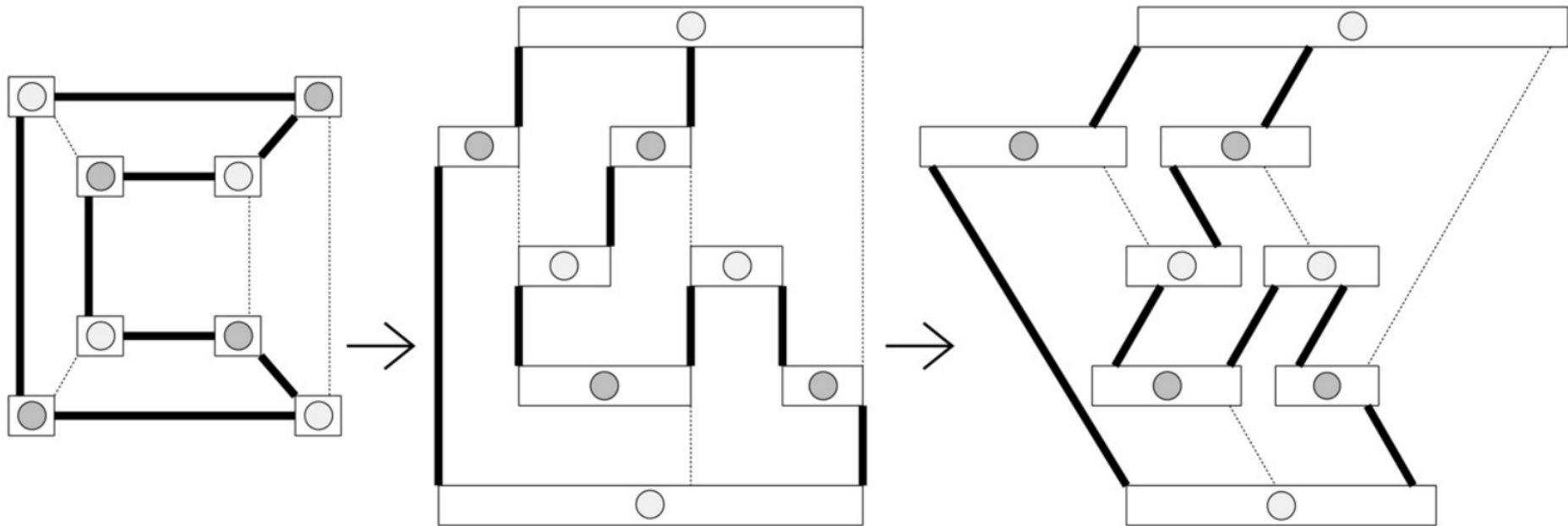
vertex-edge
connections





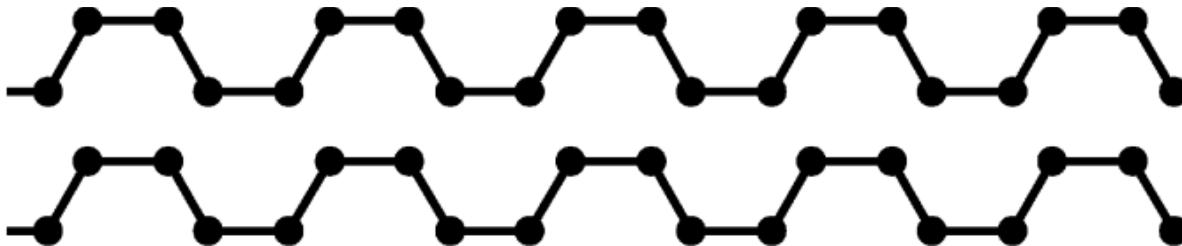
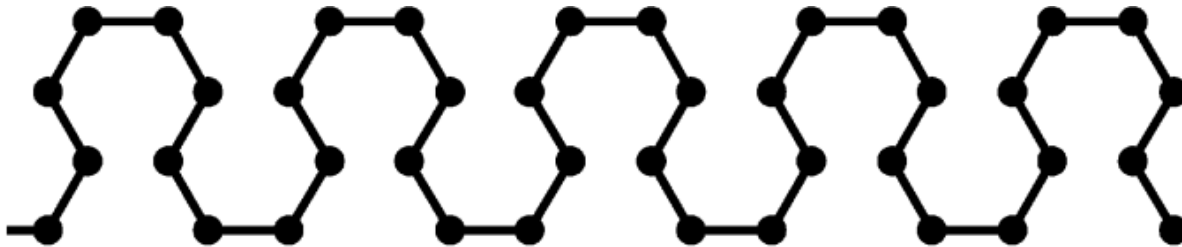
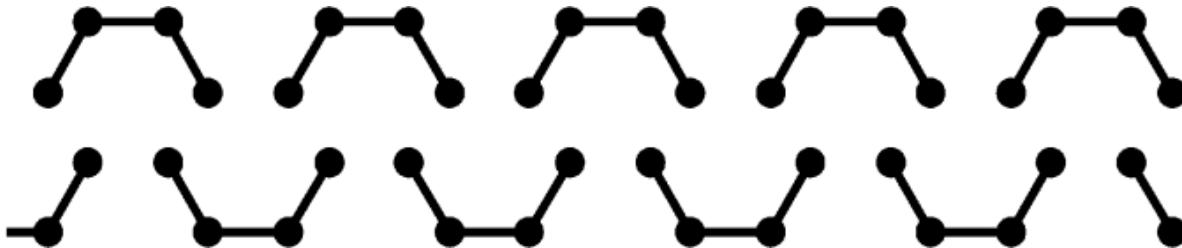
Hamiltonicity in Hexagonal Grid Graphs

Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009





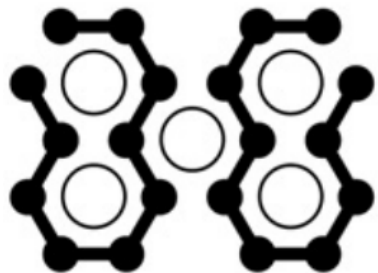
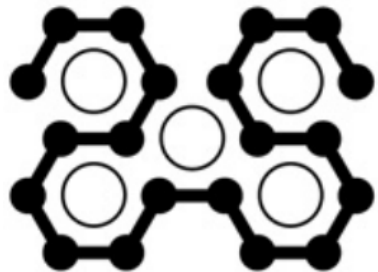
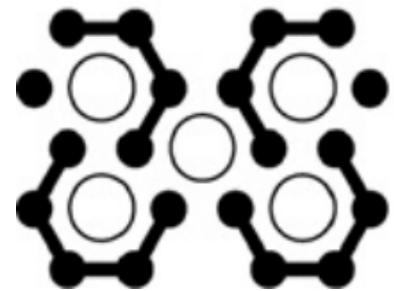
Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009



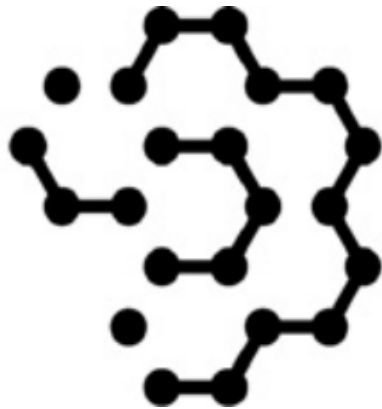
edge gadget



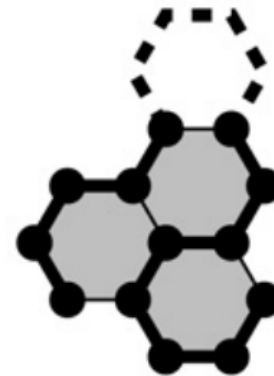
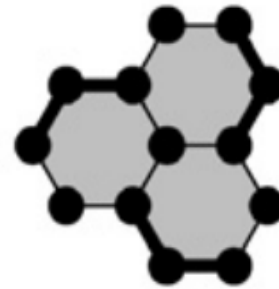
U-turn



turn gadget



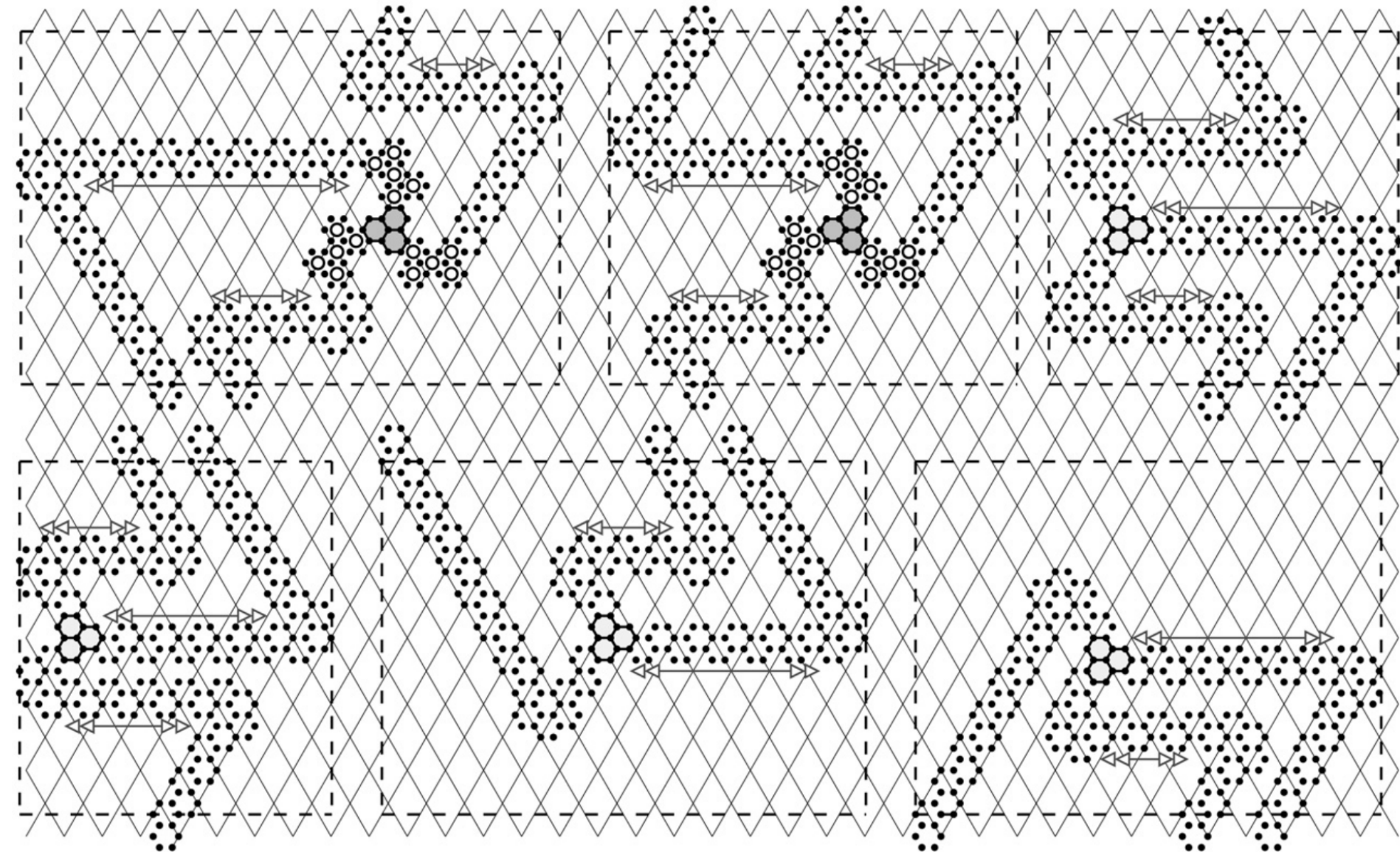
vertex core



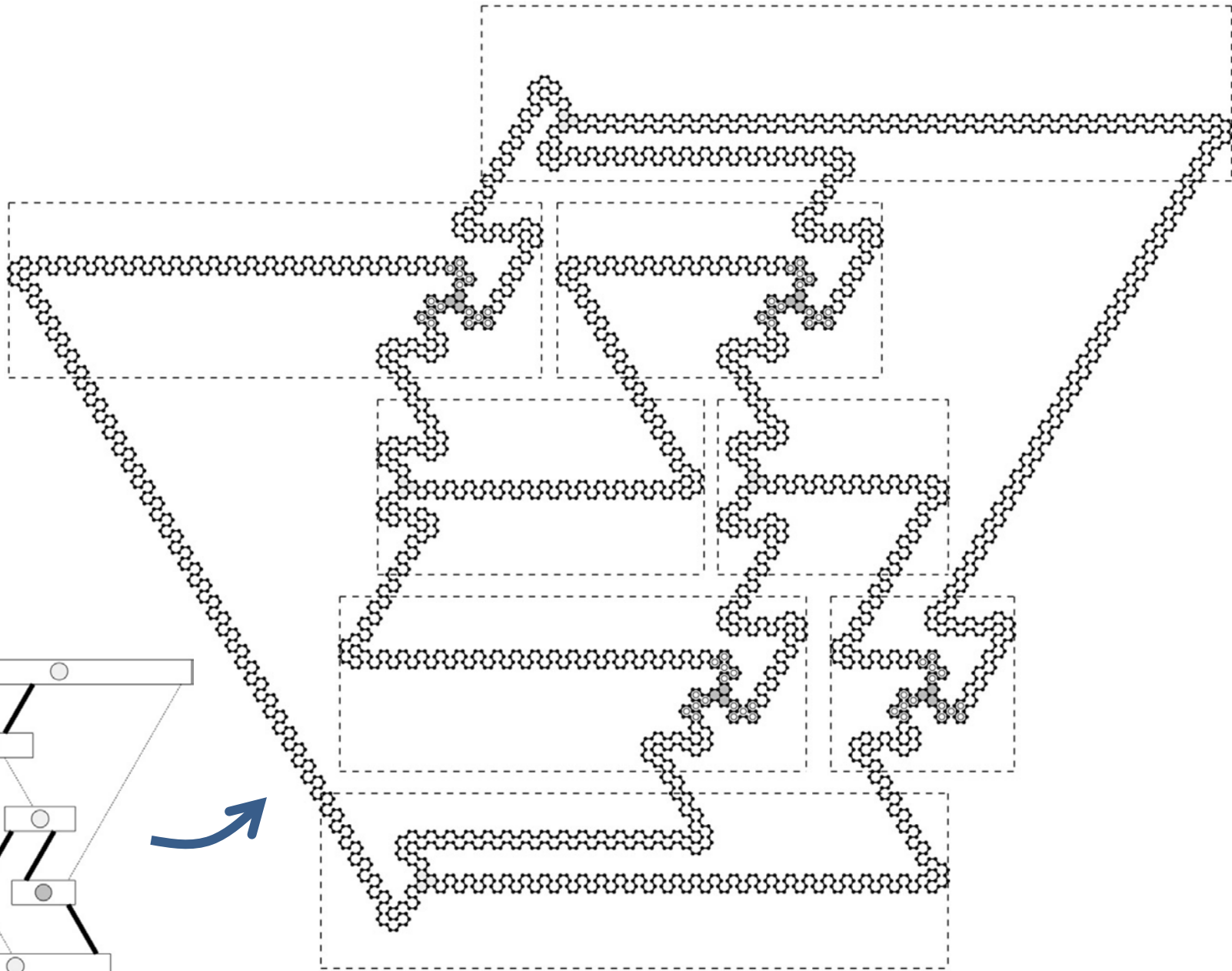
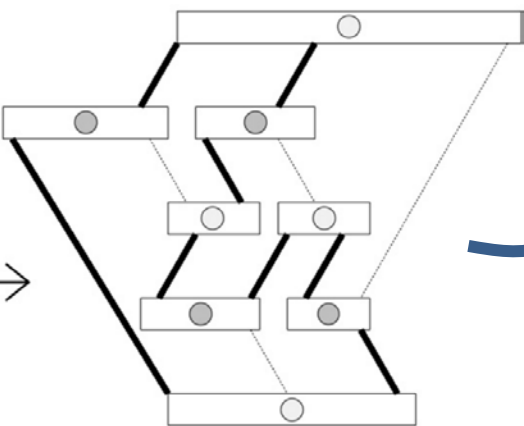
Arkin, Fekete,
Islam, Meijer,
Mitchell, Núñez-
Rodríguez,
Polishchuk,
Rappaport, Xiao
2009



vertex gadgets

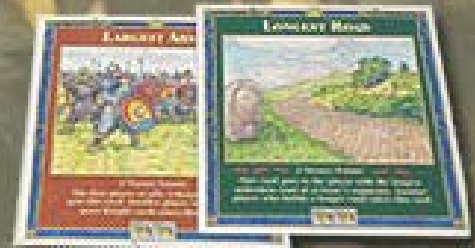


Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009



Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009

THE SETTLERS OF CATAN



> 15 million sold

[Klaus Teuber 1995]

Settlers of Catan Mate-in-1 is NP-Complete

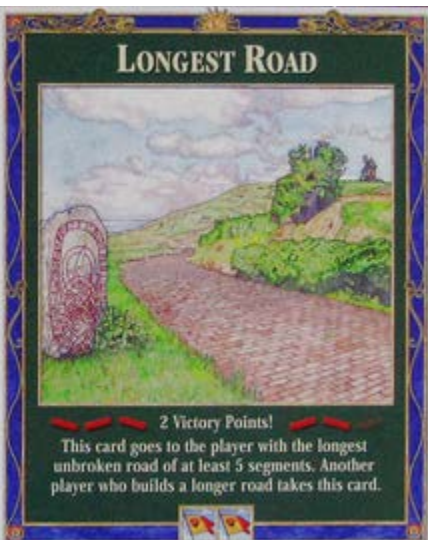
[Demaine, van Eyck, McKay 2011]

- Reduction from Hamiltonicity in hex grids

[Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2008]

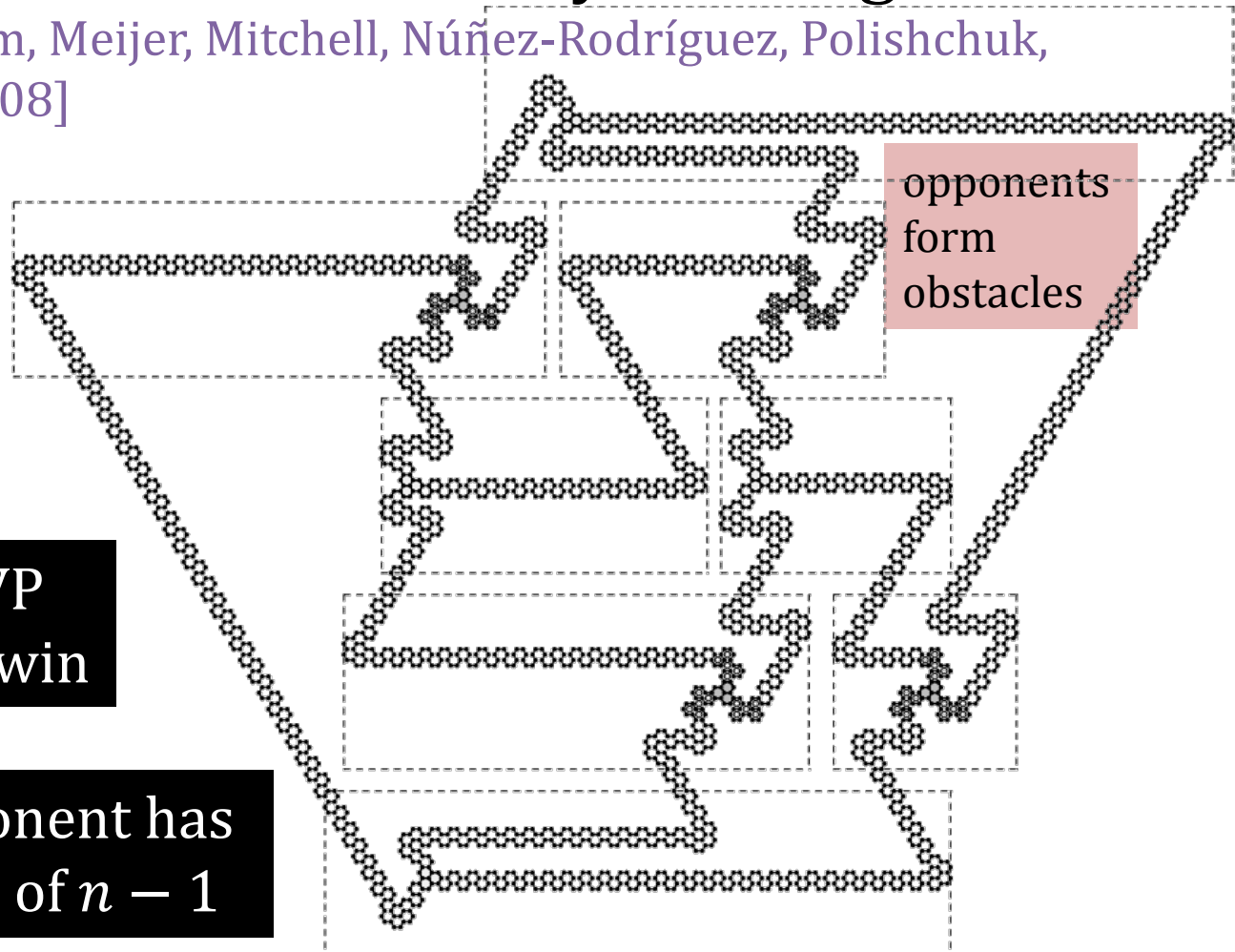


$\times \infty$
(buys
roads)



2 VP
to win

opponent has
road of $n - 1$



Settlers of Catan Mate-in-0 is NP-Complete

[Demaine, van Eyck, McKay 2011]

- Reduction from Hamiltonicity in hex grids

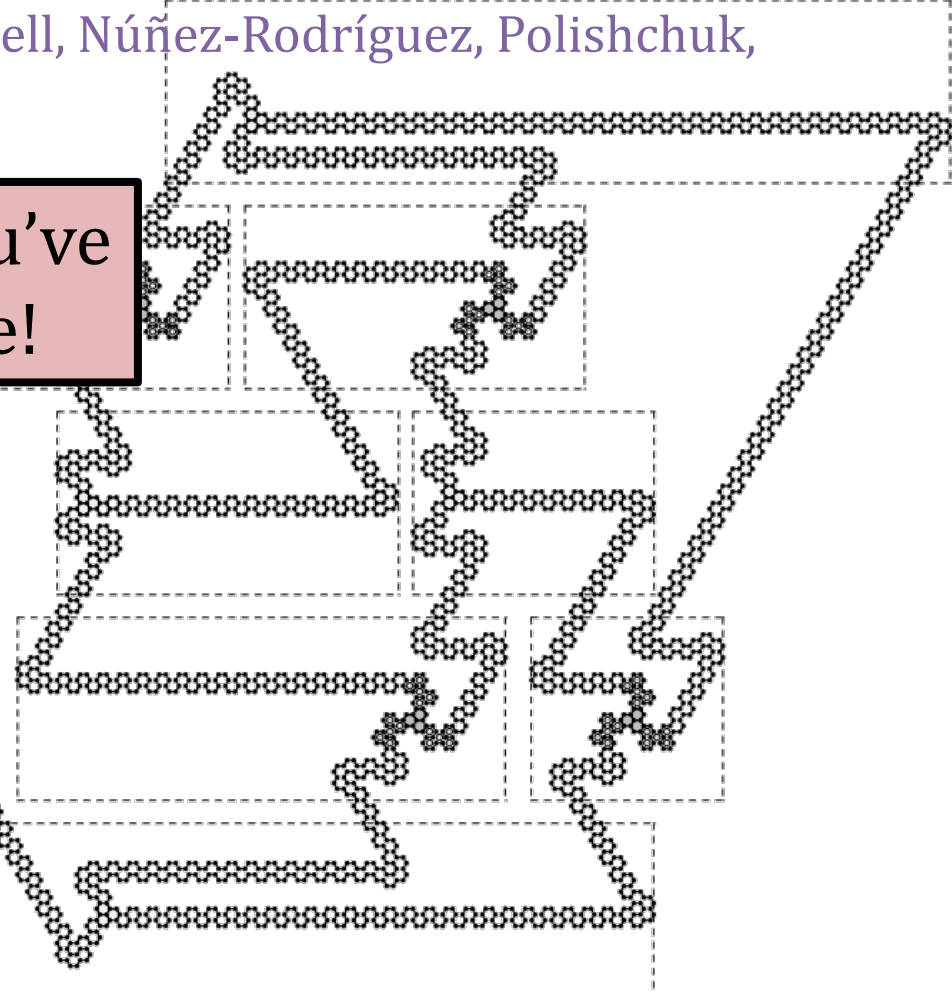
[Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009]

In fact, deciding whether you've **already won** is NP-complete!

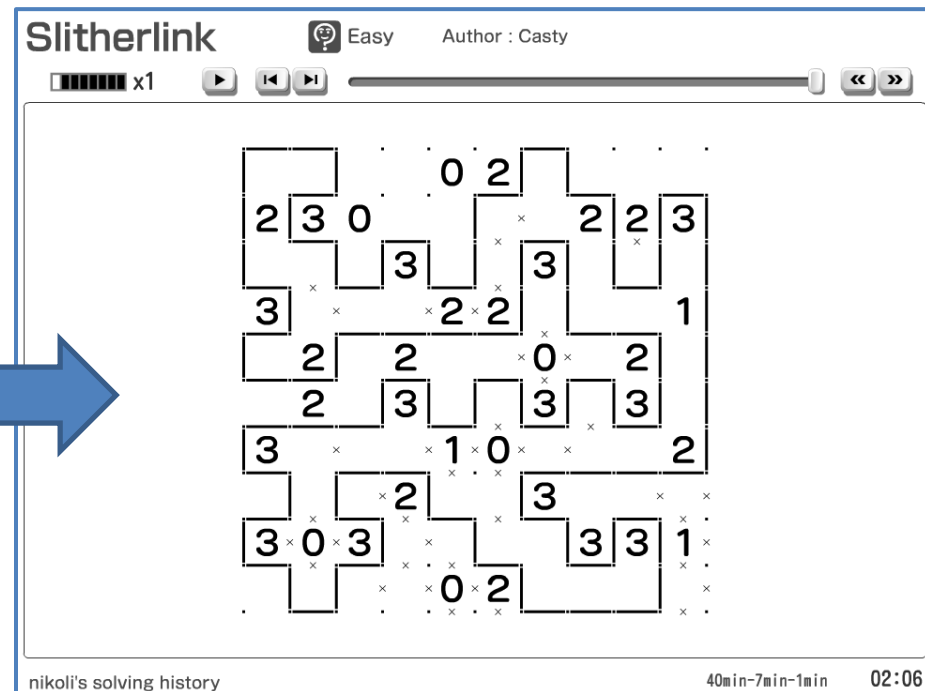
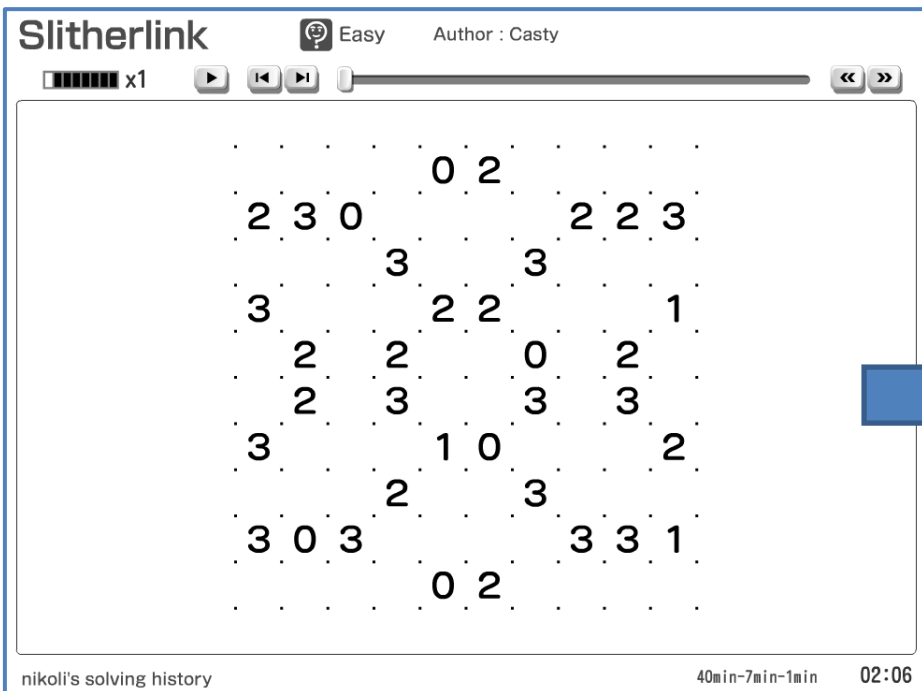


2 VP
to win

opponent has
road of $n - 1$



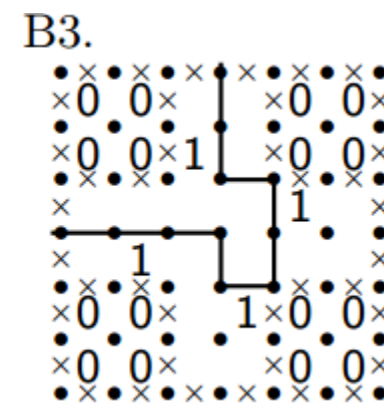
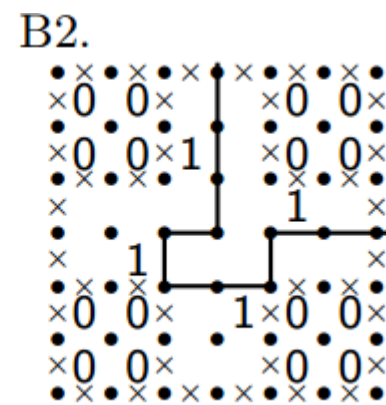
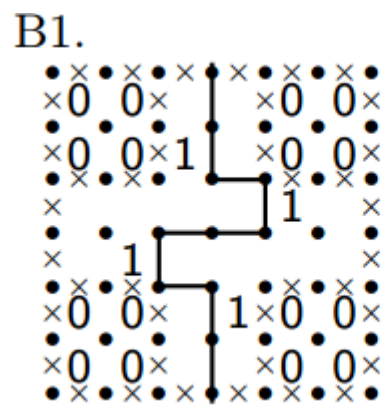
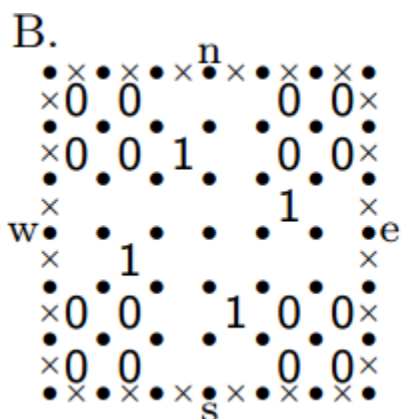
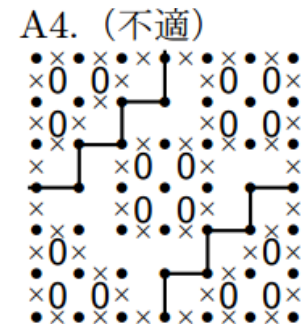
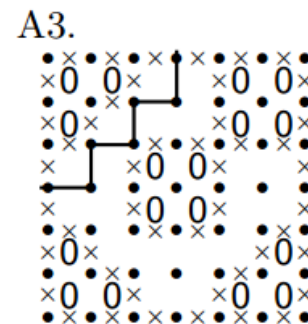
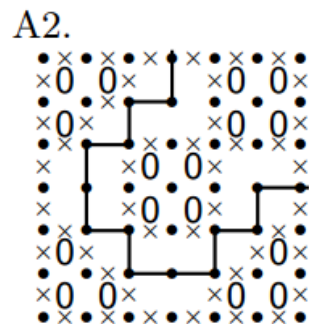
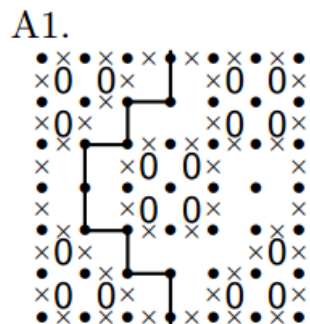
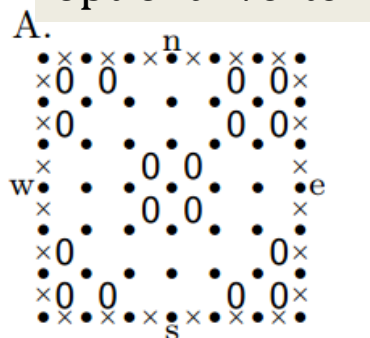
Slitherlink [Nikoli 1989]



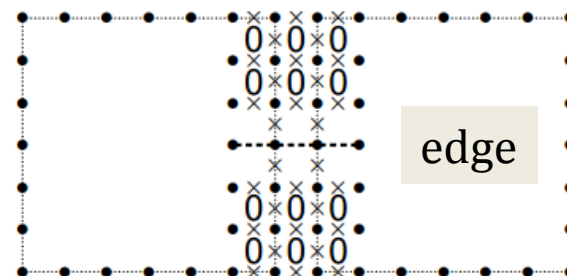
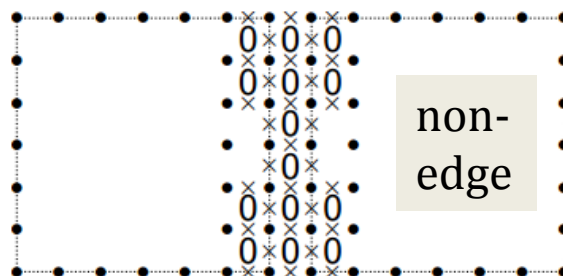


Slitherlink is NP-complete

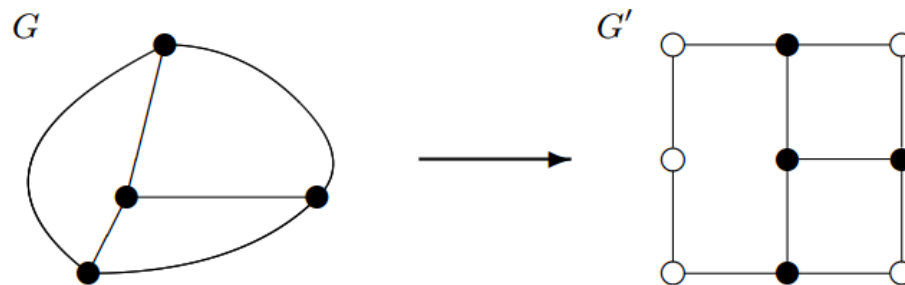
optional vertex



required vertex

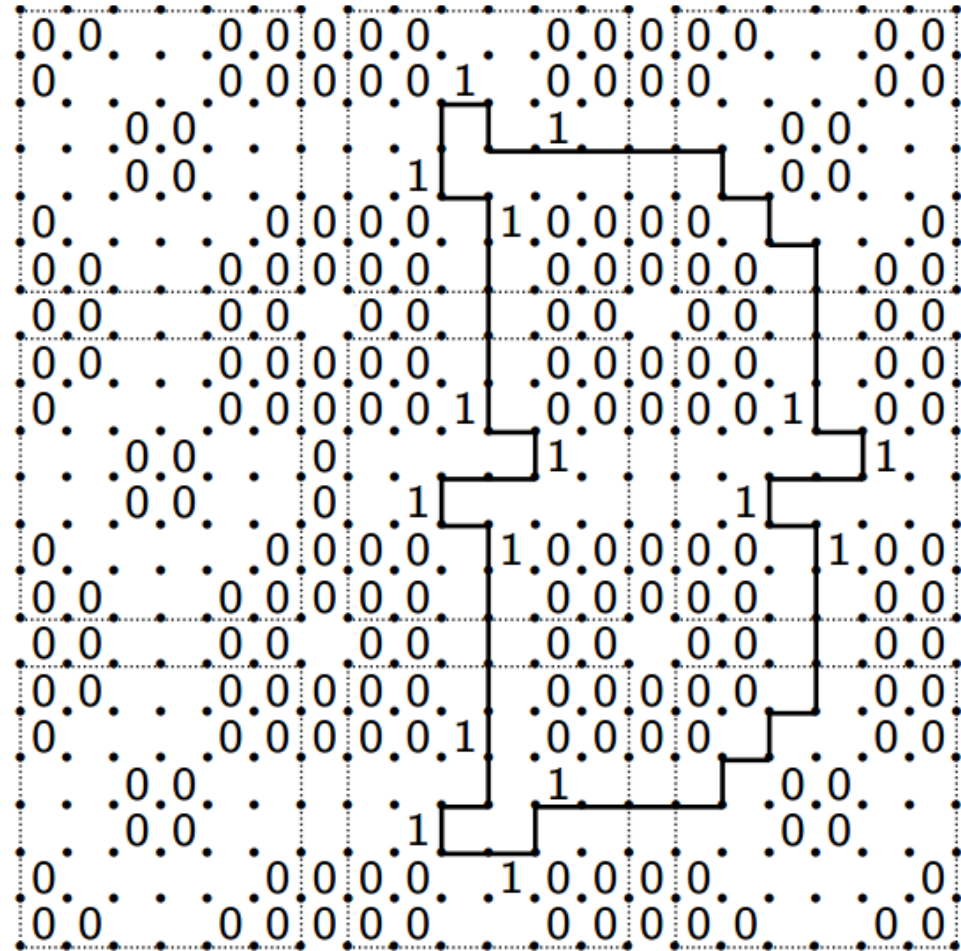
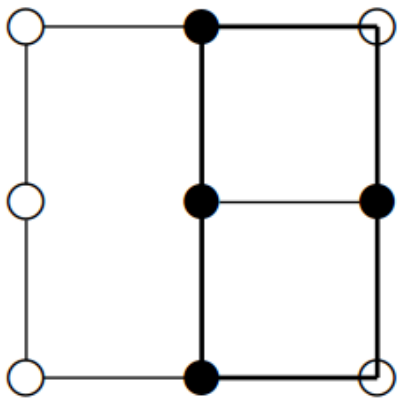
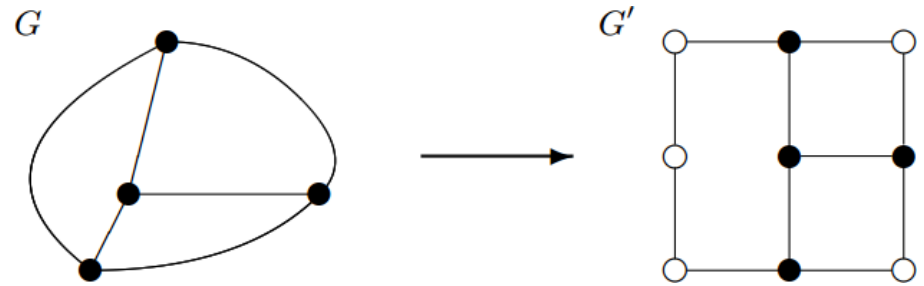


[Yato 2000]





Slitherlink is NP-complete

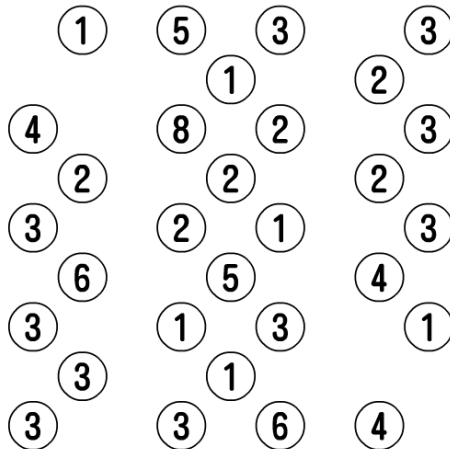


[Yato 2000]

Hashiwokakero [Nikoli 1990]

Hashiwokakero Easy Author : SAKAMOTO, Nobuyuki

||||| x1

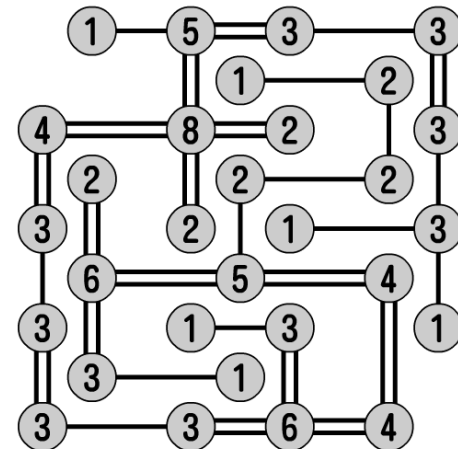


nikoli's solving history

00:31

Hashiwokakero Easy Author : SAKAMOTO, Nobuyuki

||||| x1



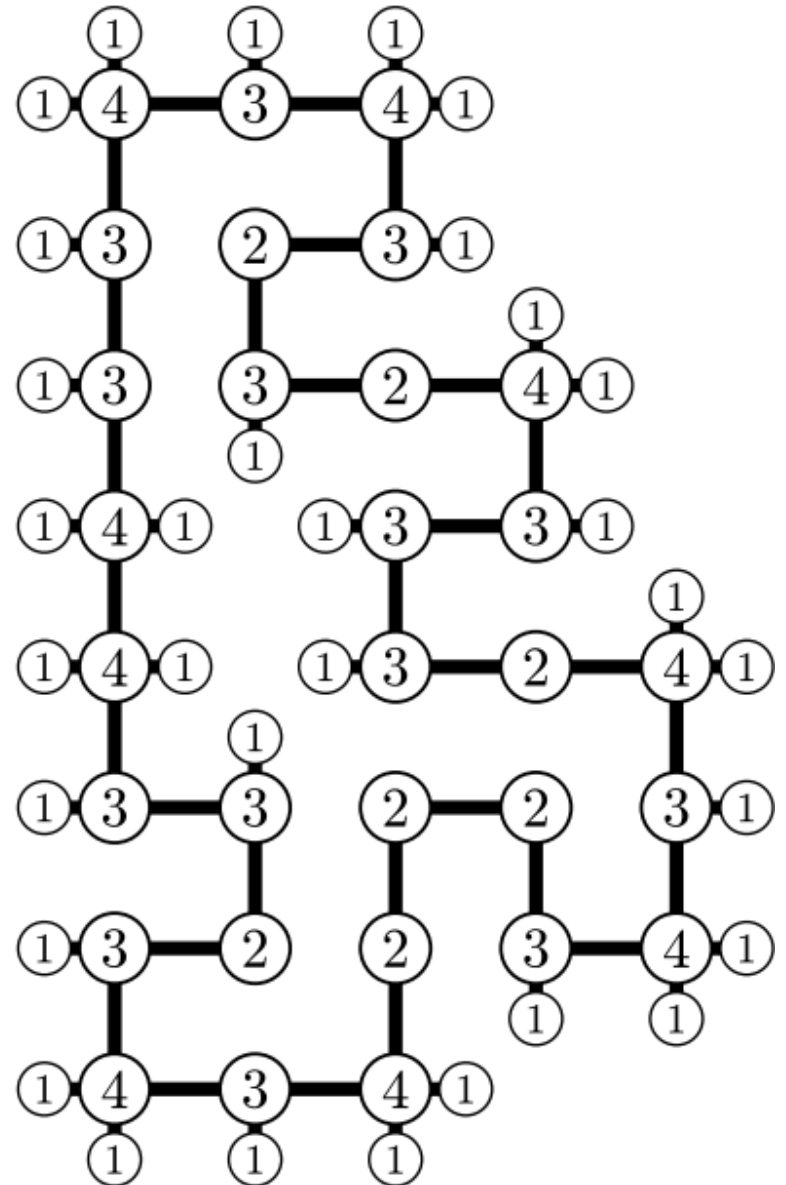
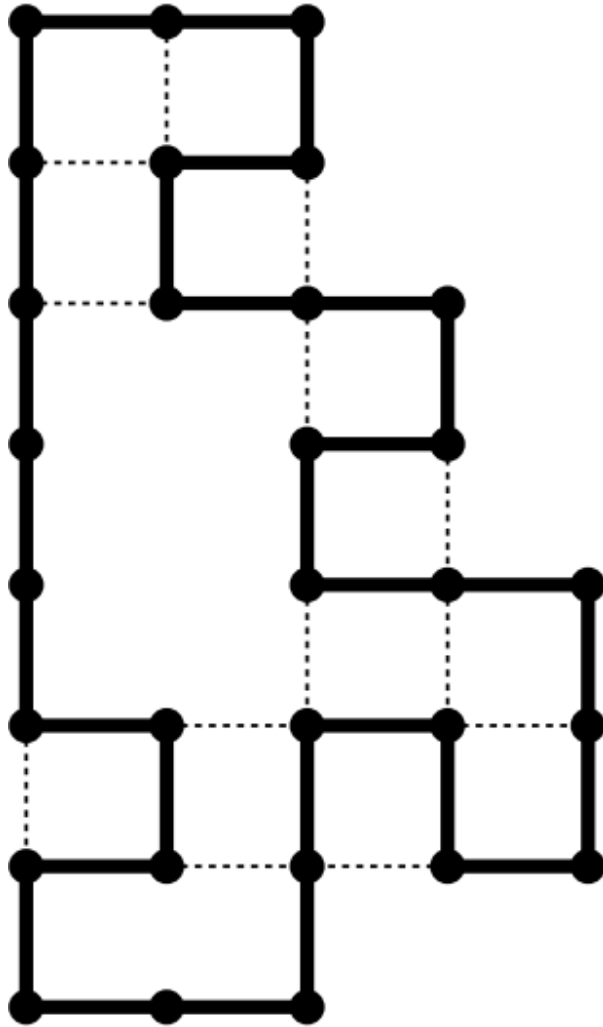
nikoli's solving history

00:31

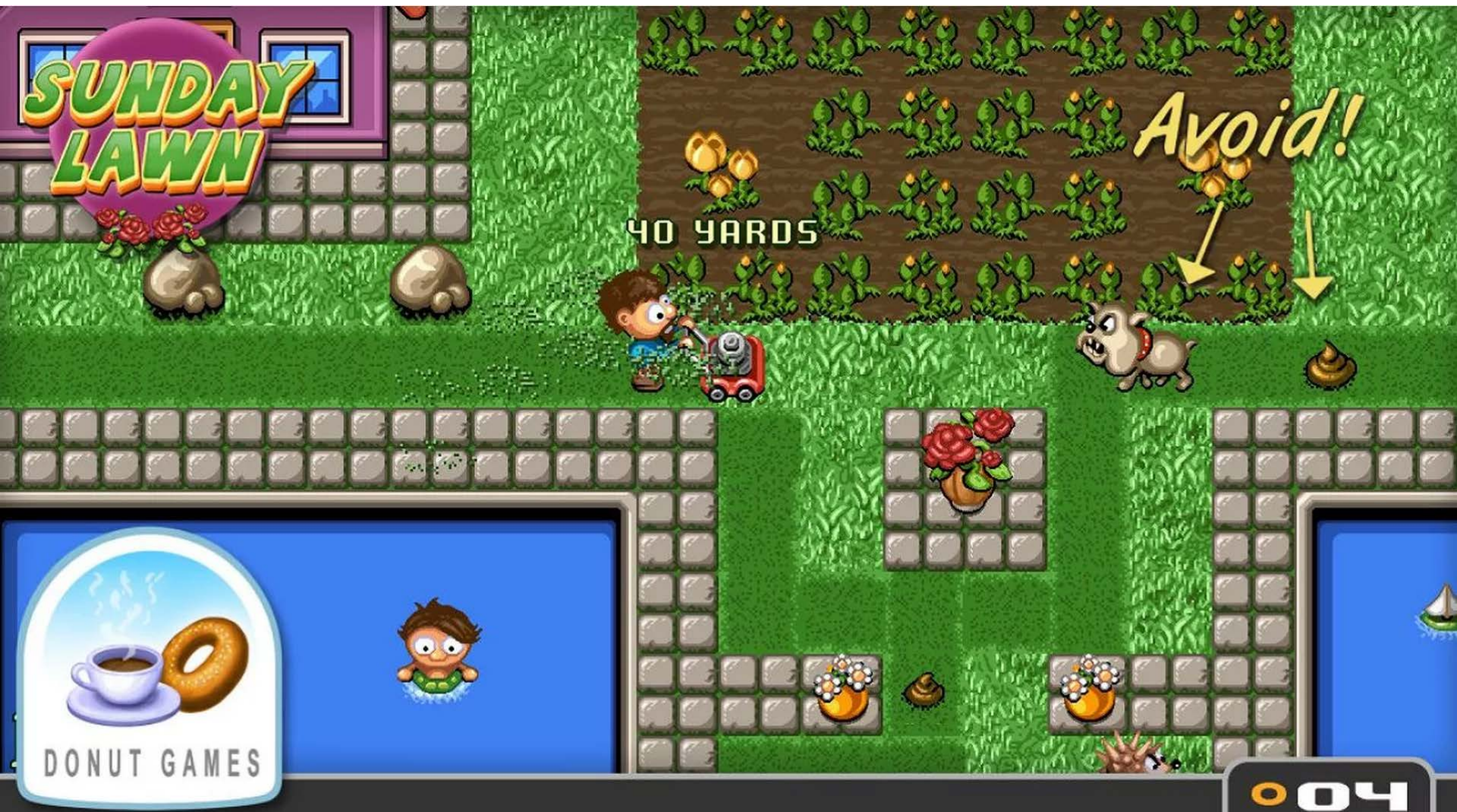


Hashiwokakero is NP-Complete

[Andersson 2009]



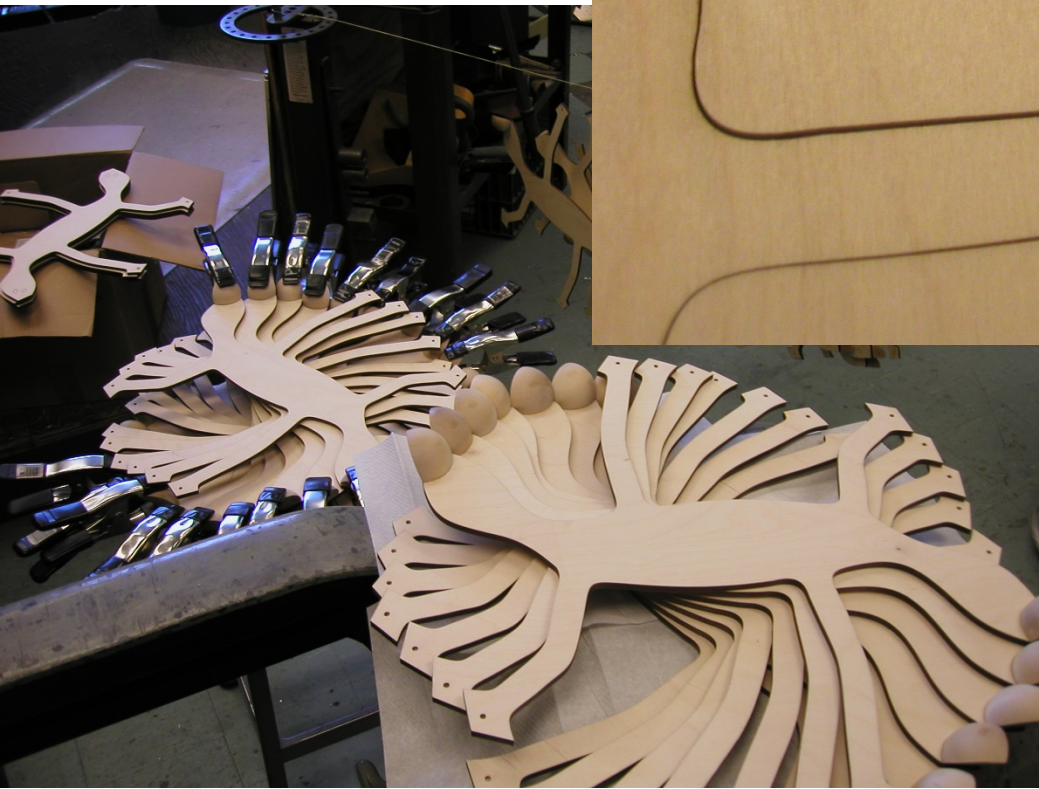
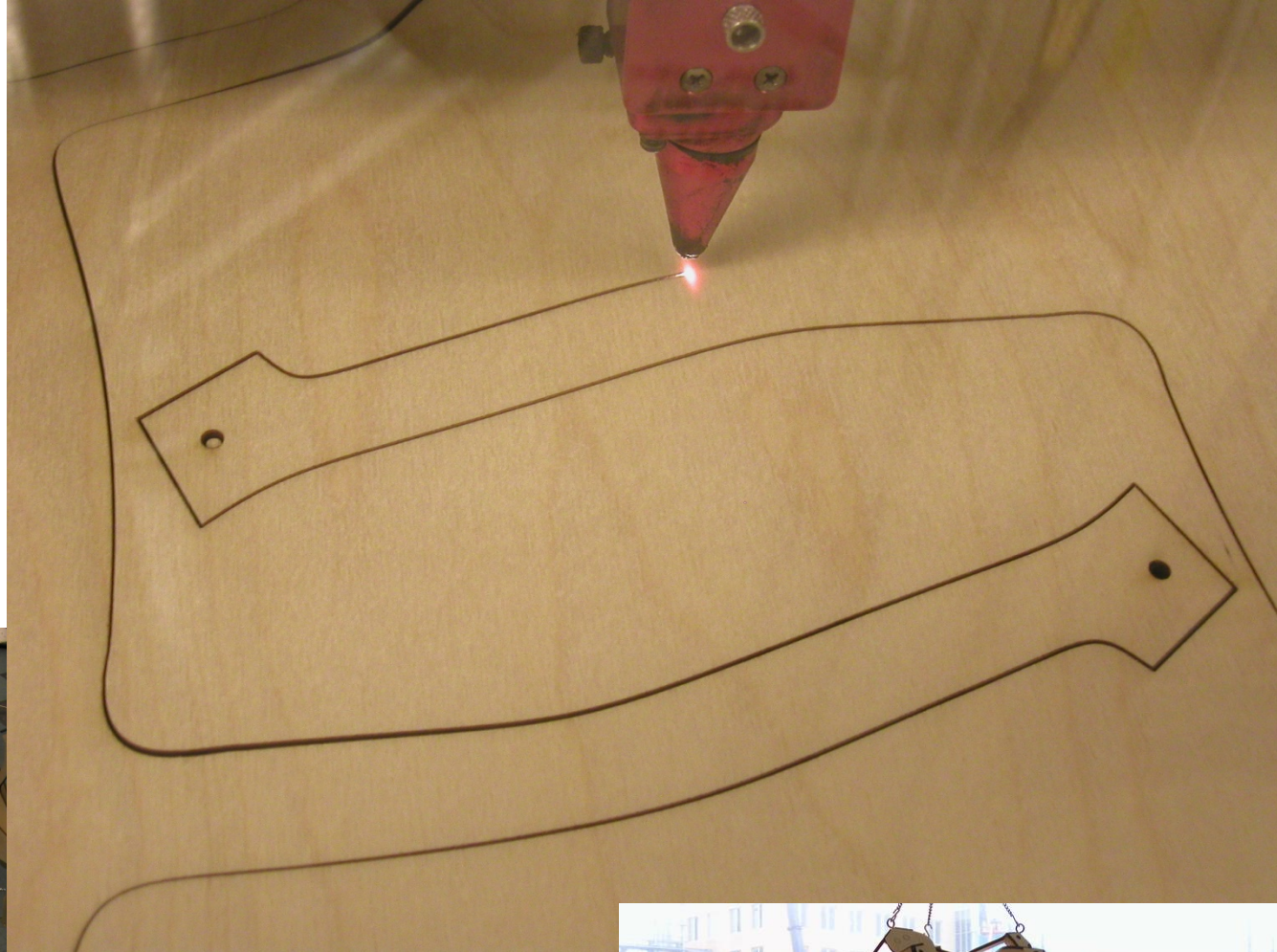
Lawn Mowing



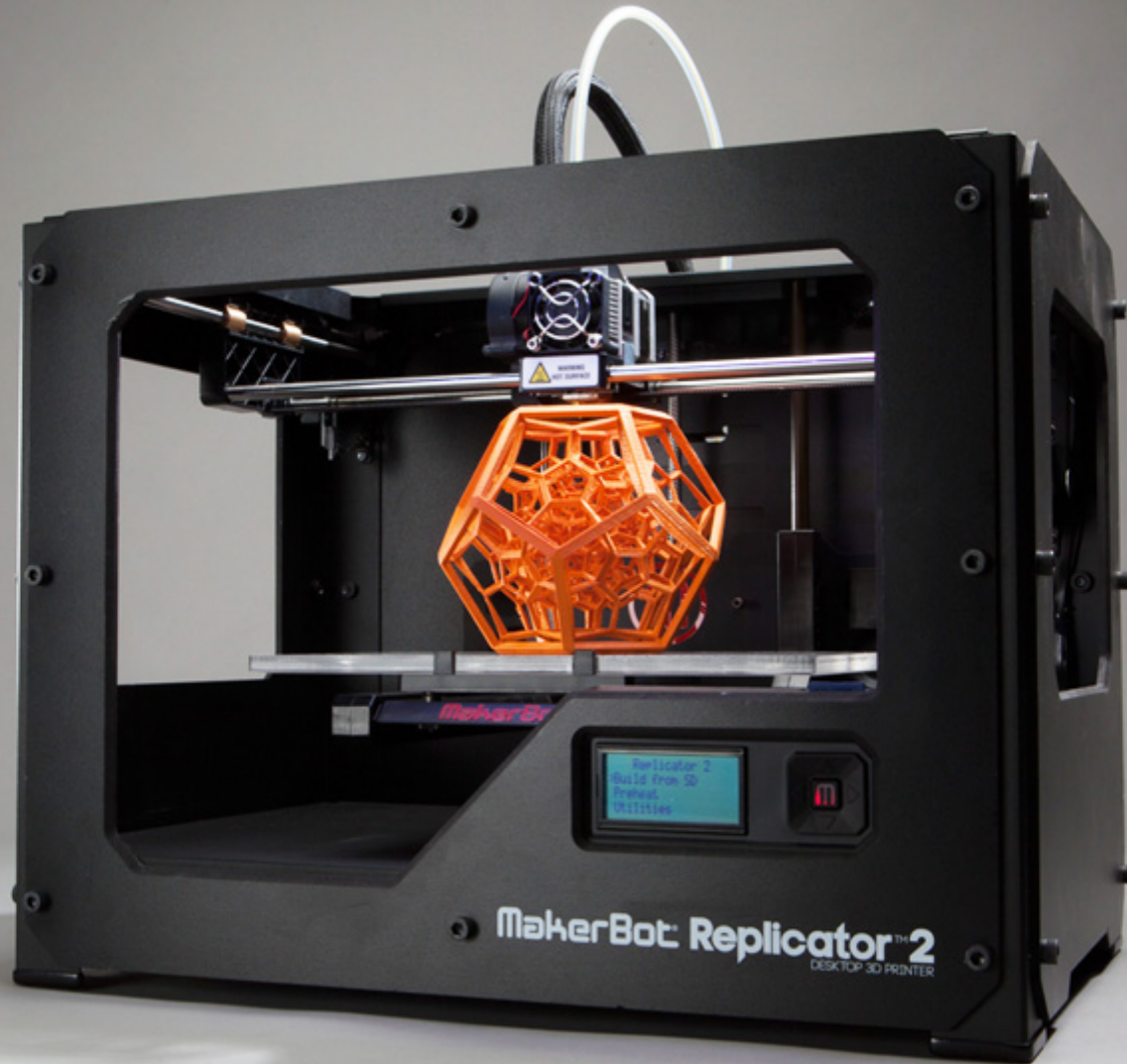


Laser Cutting

George Hart at MIT,
2003



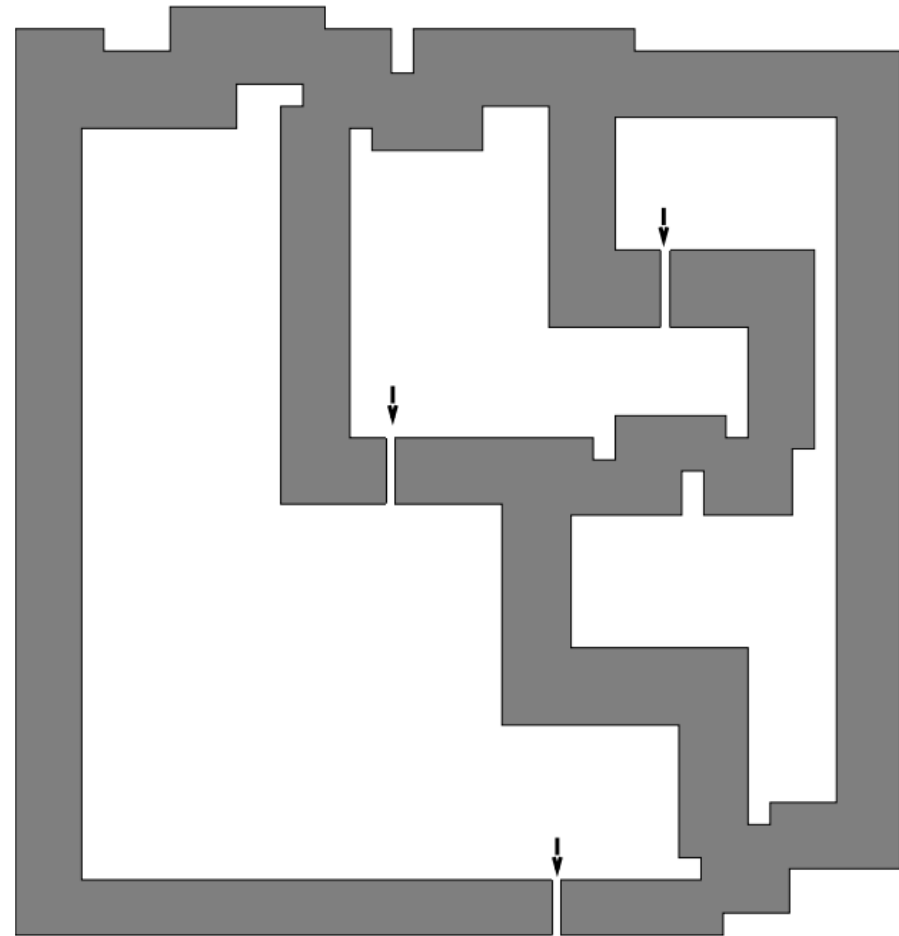
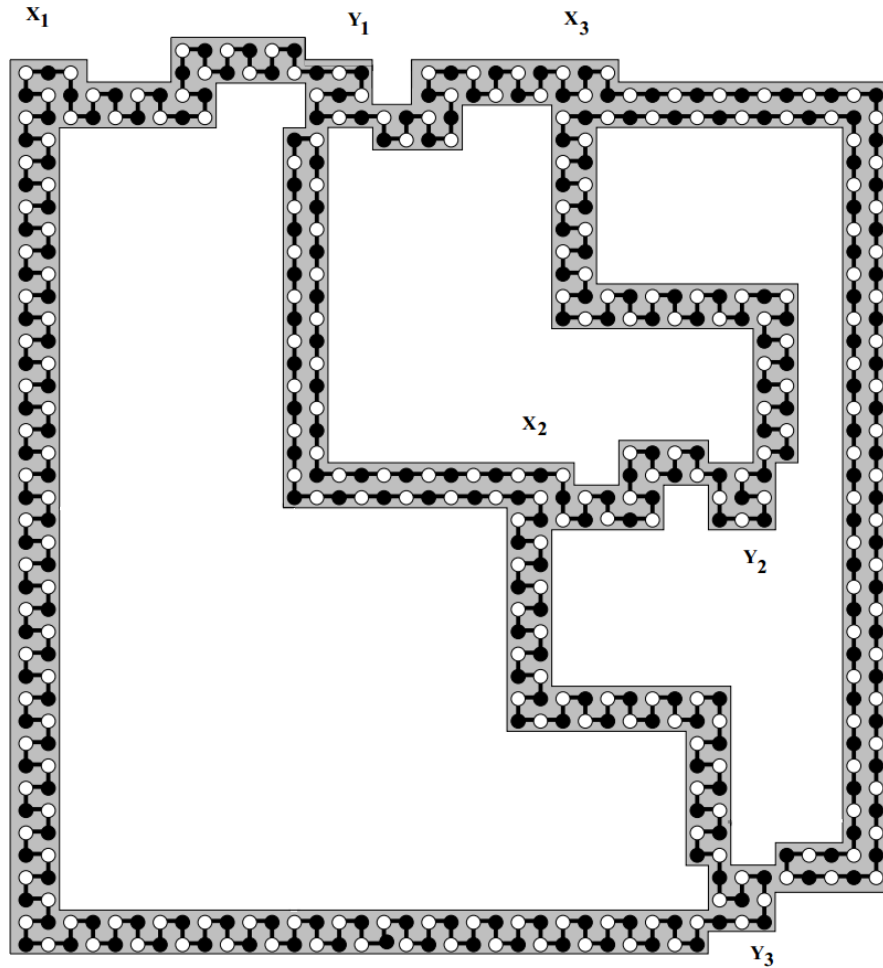
3D Printing





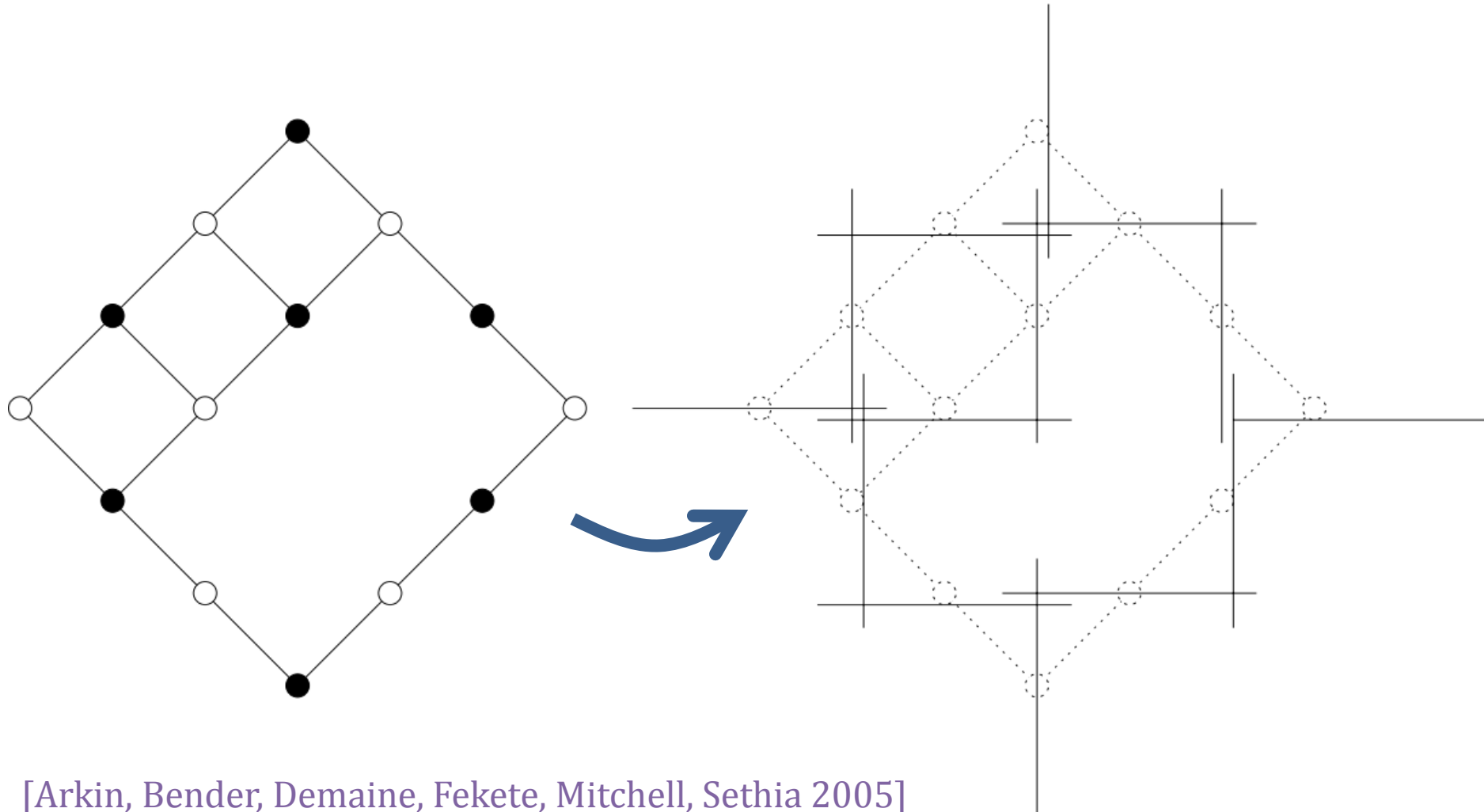
Milling & Lawn Mowing

[Arkin, Fekete, Mitchell 2000]





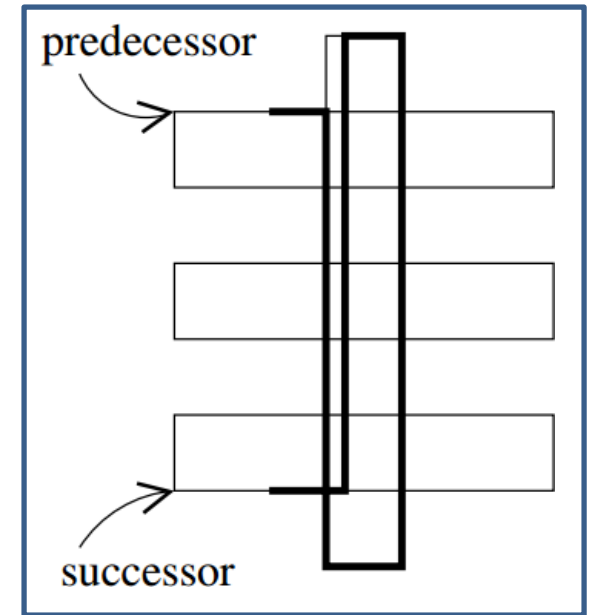
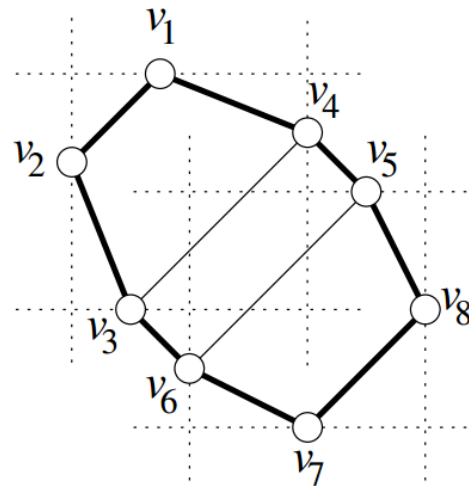
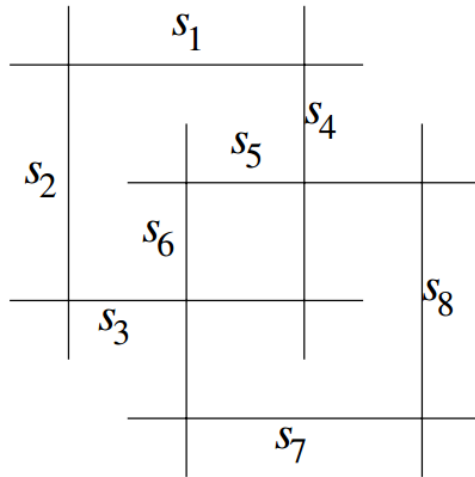
Hamiltonicity in Unit Orthogonal Segment Intersection Graphs



[Arkin, Bender, Demaine, Fekete, Mitchell, Sethia 2005]



Minimum-Turn Milling



[Arkin, Bender, Demaine, Fekete, Mitchell, Sethia 2005]

