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Problem Set 6

Due: Tuesday, March 19, 2019 at noon

Problem 6.1 [Reduction Properties]. Suppose you have a polynomial-time one-call reduction r from NP search problem A to NP search problem B, which converts each problem-A instance x having a solutions into a problem-B instance $r(x)$ having $f(x, a)$ solutions.

(a) What must be true about f for r to preserve NP-hardness (i.e., A being NP-hard implies B is NP-hard)?

(b) What must be true about f for r to preserve #P-hardness (i.e., $\#A$ being #P-hard implies $\#B$ is #P-hard)?

(c) What must be true about f for ASP A being NP-hard to imply ASP B is NP-hard?

(d) Given an NP search problem A, define *AASP A* to be the following decision problem: given an instance of A and two distinct solutions of that instance, is there a third distinct solution? What must be true about f for AASP A being NP-hard to imply AASP B is NP-hard?

(e) Besides the function r converting instances of A into instances of B, we require another function to conclude that ASP B is NP-hard. What must the other function do? What about for AASP?