Problem 5.1 [Consecutive Sets]. Prove that the following problem is NP-complete.

CONSECUTIVE SETS: Given a collection of (unordered) subsets $S_1, S_2, \ldots, S_n$ of a finite alphabet $\Sigma$, and a positive integer $k$, is there a string $w$ over the alphabet $\Sigma$ with length at most $k$ such that, for each $S_i$, the elements of $S_i$ occur (in any order) as some consecutive characters $w_i, w_{i+1}, \ldots, w_{i+|S_i|-1}$ of $w$?

Hint: Reduce from some version of Hamiltonicity.