6.892: Algorithmic Lower Bounds, Spring 2019

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Problem Set 2

Due: Tuesday, February 19, 2019 at noon

Problem 2.1 [Problem Set Scheduling]. During the semester, you will have to plan when to work on problem sets from this and other courses, subject to constraints: you cannot start working on a problem set until it is released, you must finish a problem set before it is due, and you can work on only one problem set at a time. We can formalize these constraints in the following problems:

Sequencing with Release Times and Deadlines: Given a set T of tasks, where each task $t \in T$ has a positive integer length ℓ_t, cannot be started until a nonnegative integer r_t (its *release time*), and must be completed before a positive integer d_t (its *deadline*), is there a feasible one-processor schedule for T? In this problem, once the processor starts a task, it must finish that task before starting another task.

Preemptive Sequencing with Release Times and Deadlines is the same problem, except the processor is allowed to suspend work on its current task and start on another task at any time. A task t is completed when the processor has spent ℓ_t total time on the task.

(a) Prove that Sequencing with Release Times and Deadlines is weakly NP-hard by reduction from Partition.

(b) Prove that Sequencing with Release Times and Deadlines is strongly NP-hard by reduction from 3-Partition.

(c) Prove that Preemptive Sequencing with Release Times and Deadlines is in P by giving a polynomial-time algorithm that solves it. Explain why your proofs for the non-premptive case do not hold under preemption.