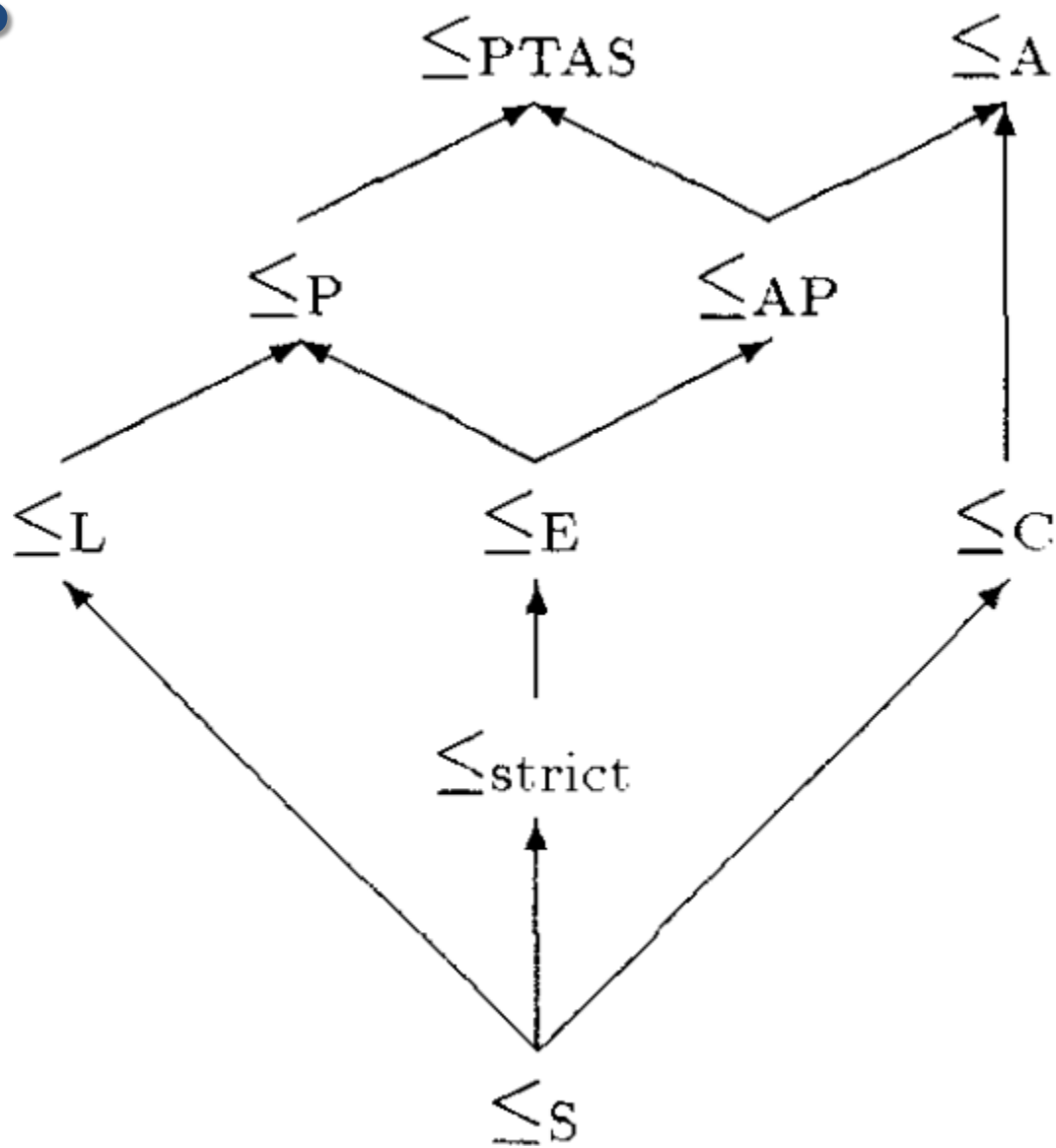




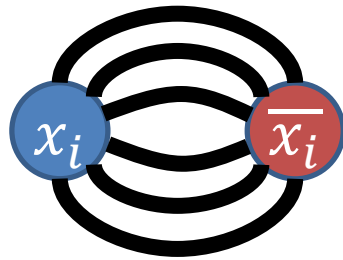
# Reductions



[Crescenzi 1997]

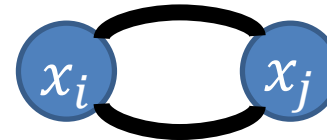
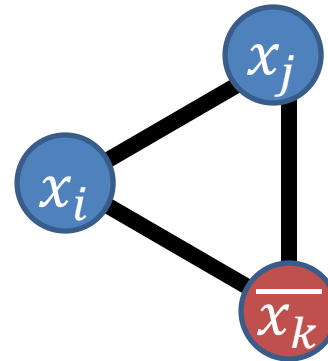
# Max Cut

[Papadimitriou & Yannakakis 1991]



$2k$  parallel edges  
for  $k$  occurrences

variable



NAE clause

# Approximability of CSP

[Khanna, Sudan, Trevisan, Williamson 2001]

- Max CSP
  - $\in$ PO if setting all vars. false or all vars. true satisfies all clause types
  - $\in$ PO if all clauses in DNF have 2 terms, one all positive & one all negative
  - APX-complete otherwise

# Approximability of CSP

[Khanna, Sudan, Trevisan, Williamson 2001]

- Max Ones:
  - EPO if setting all vars. true satisfies all
  - EPO if CNF of Dual-Horn subclauses ( $\leq 1$  negated)
  - EPO if  $\leq 2$ -X(N)OR-SAT: linear eqns., 2 terms, over  $\mathbb{Z}_2$
  - APX-complete if  $\leq$  X(N)OR-SAT (not 2-)
  - Poly-APX-complete if CNF of Horn subclauses
  - Poly-APX-complete if 2CNF
  - Poly-APX-complete if setting all or all but one variable false satisfies each constraint
  - 0 vs.  $>0$  NP-hard if setting all vars. false satisfies
  - feasibility NP-hard if none of above (& not previous case)

# Approximability of CSP

[Khanna, Sudan, Trevisan, Williamson 2001]

- Min CSP:
  - $\epsilon$ PO if setting all vars. false or all vars. true satisfies all clause types
  - $\epsilon$ PO if all clauses in DNF have 2 terms, one all positive & one all negative
  - APX-complete if  $\underbrace{\text{OR}(O(1) \text{ variables})}_{O(1)\text{-hitting set}}, \underbrace{\neg x_1 \vee x_2}_{\text{implication}}$
  - Min UnCut-complete if  $\epsilon$  2-X(N)OR-SAT  
Min CSP(XOR) - APX-hard &  $O(\log n)$ -approx.
  - Min 2CNF-Deletion-complete if 2CNF  
Min CSP(OR, NAND) - APX-hard &  $O(\log n \log \log n)$ -apx.
  - Nearest Codeword-complete if  $\epsilon$  X(N)OR-SAT (not 2-)  
Min CSP( $x_1 \oplus x_2 \oplus x_3, \bar{x}_1 \oplus x_2 \oplus x_3$ ) -  $\Omega(2^{\log^{1-\epsilon} n})$ -inapprox.
  - Min Horn Deletion-complete if Horn or Dual-Horn  
Min CSP( $\bar{x}_1 \vee x_2 \vee x_3$ ) -  $\Omega(2^{\log^{1-\epsilon} n})$ -inapprox.  $\in$  Poly-APX
  - $\Delta$  vs.  $>0$  is NP-complete otherwise

# Approximability of CSP

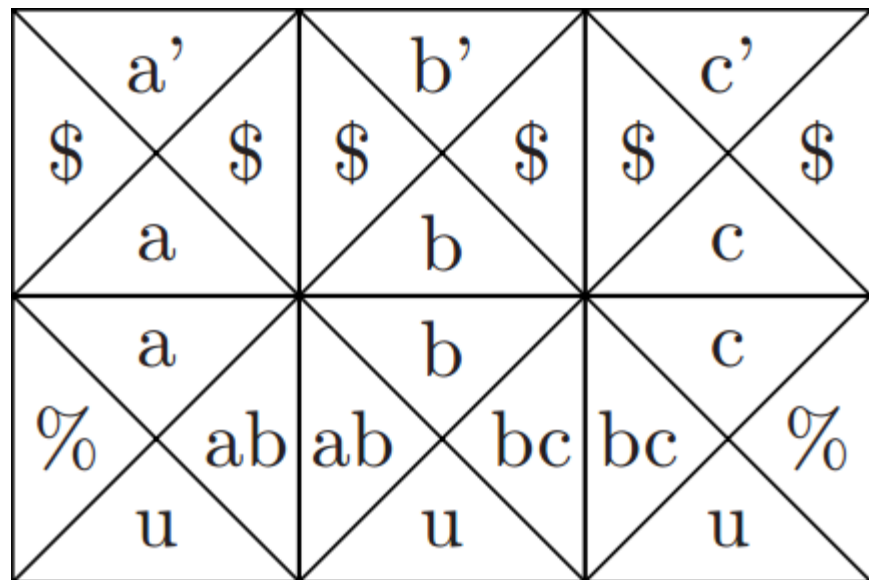
[Khanna, Sudan, Trevisan, Williamson 2001]

- Min Ones:

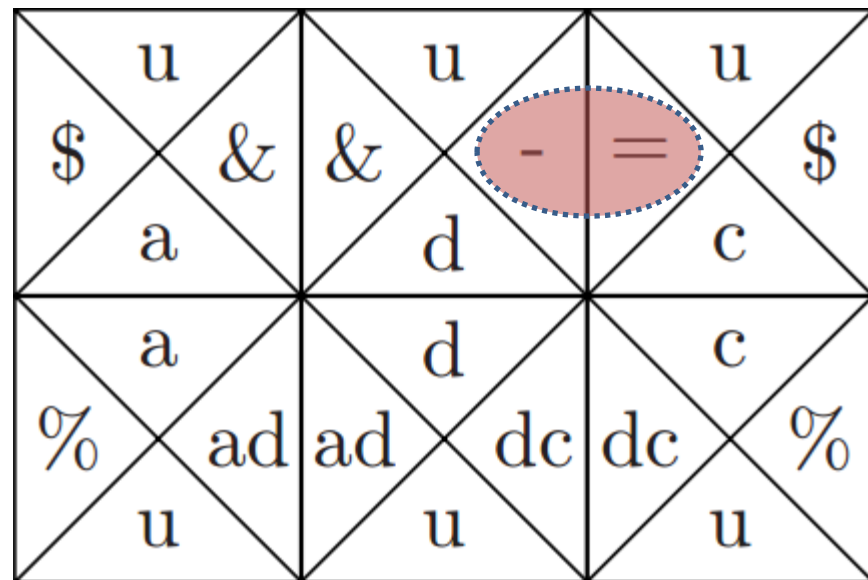
- EPO if setting all vars. false satisfies all
- EPO if CNF of Horn subclauses ( $\leq 1$  positive)
- EPO if  $\leq 2$ -X(N)OR-SAT
- APX-complete if 2CNF
- APX-complete if  $O(1)$  hitting set + implication
- Nearest Codeword-complete if  $\leq X(N)$ OR-SAT (not 2-)
- Min Horn Deletion-complete if CNF of Dual-Horn
- Poly-APX-complete if all vars. true satisfies - if weighted:  
hard to approximate by any factor
- feasibility NP-hard otherwise

# Edge Matching Puzzles

[Antoniadis & Lingas 2010]



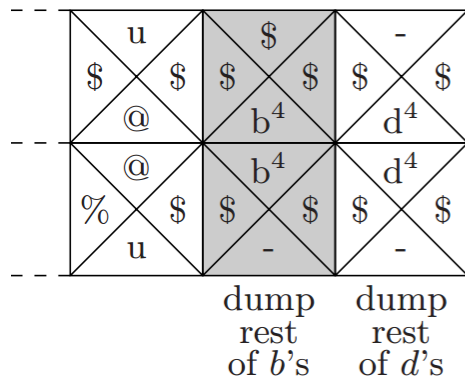
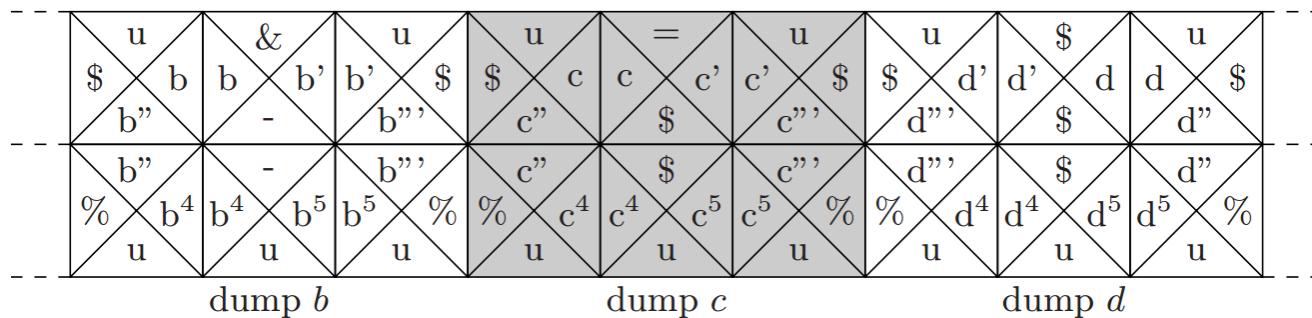
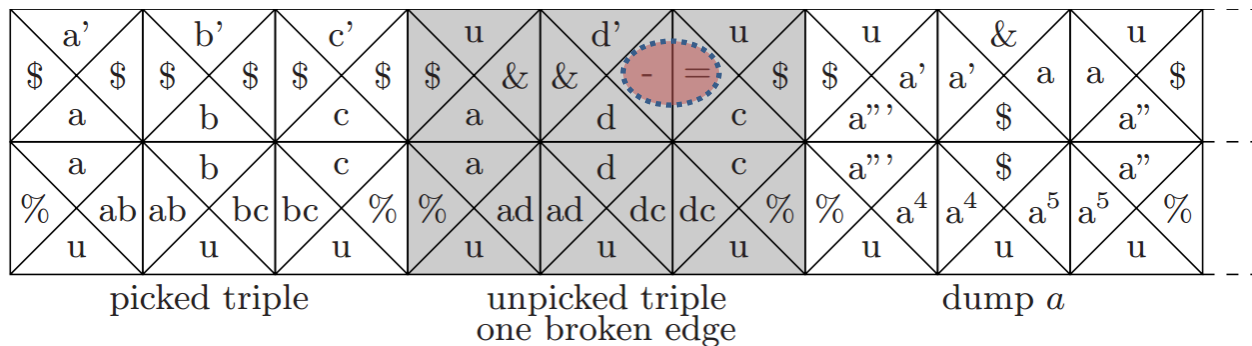
picked triple  
 $(a, b, c)$



unpicked triple  
 $(a, d, c)$

# Edge Matching Puzzles

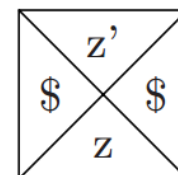
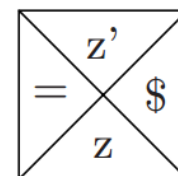
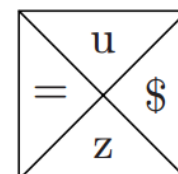
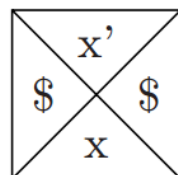
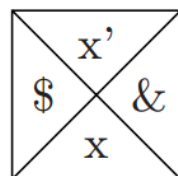
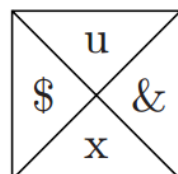
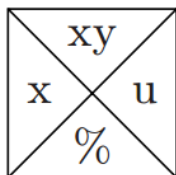
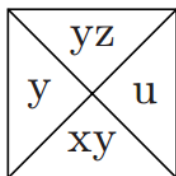
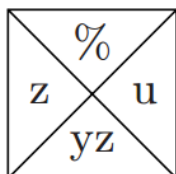
[Antoniadis & Lingas 2010]





# Edge Matching Puzzles

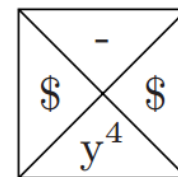
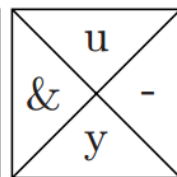
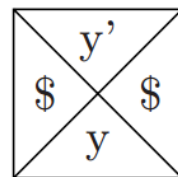
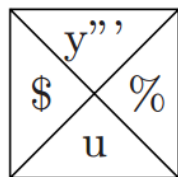
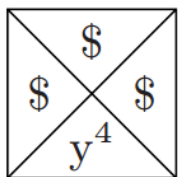
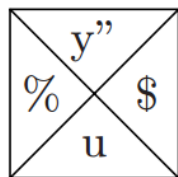
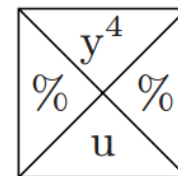
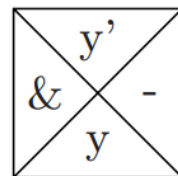
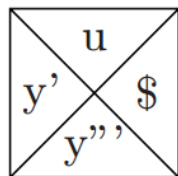
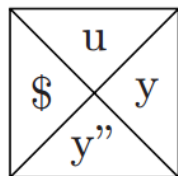
[Antoniadis & Lingas 2010]



(a)  $\forall$   
triple

(b)  $\forall x \in X$

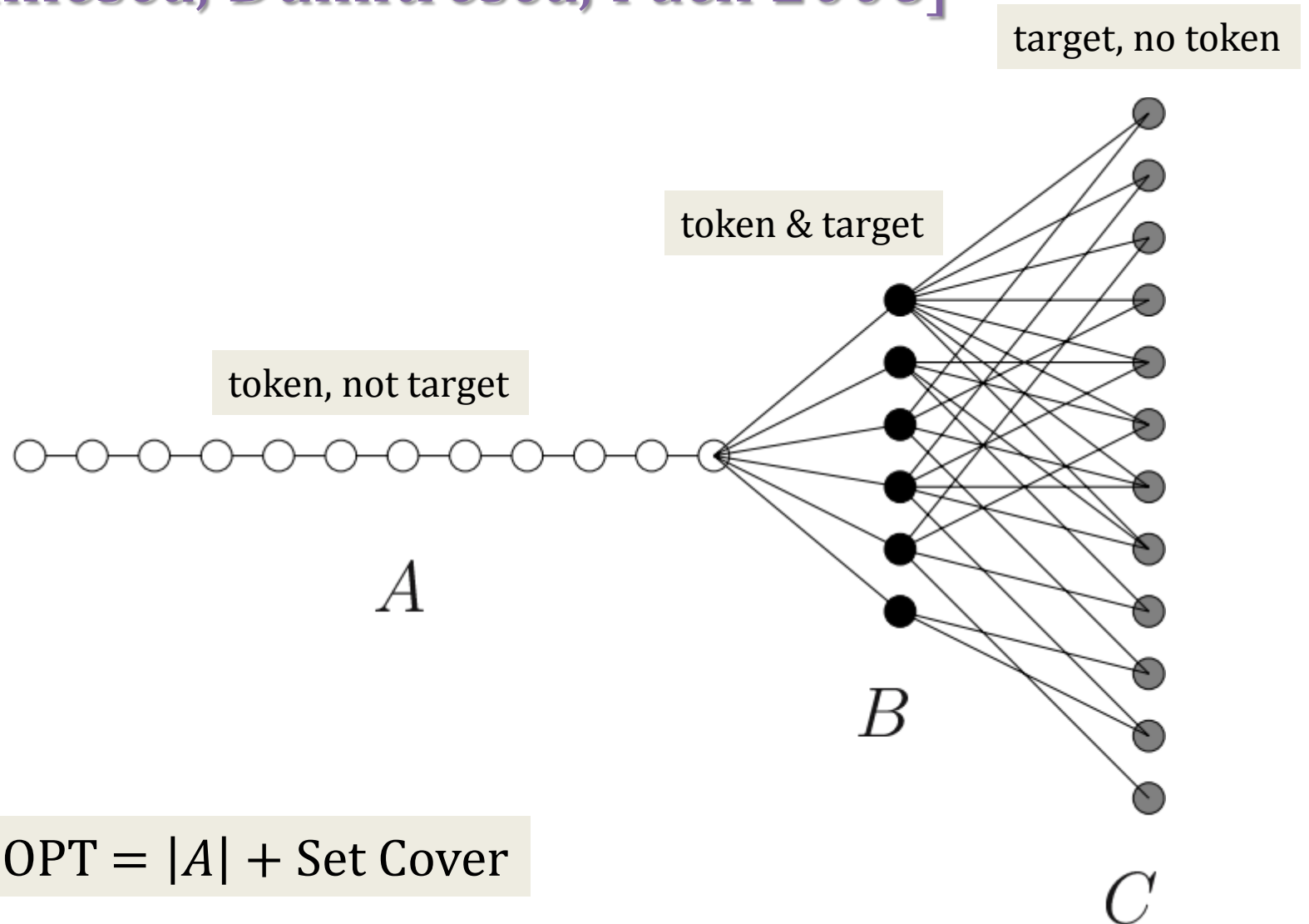
(c)  $\forall z \in Z$



(d)  $\forall y \in Y$

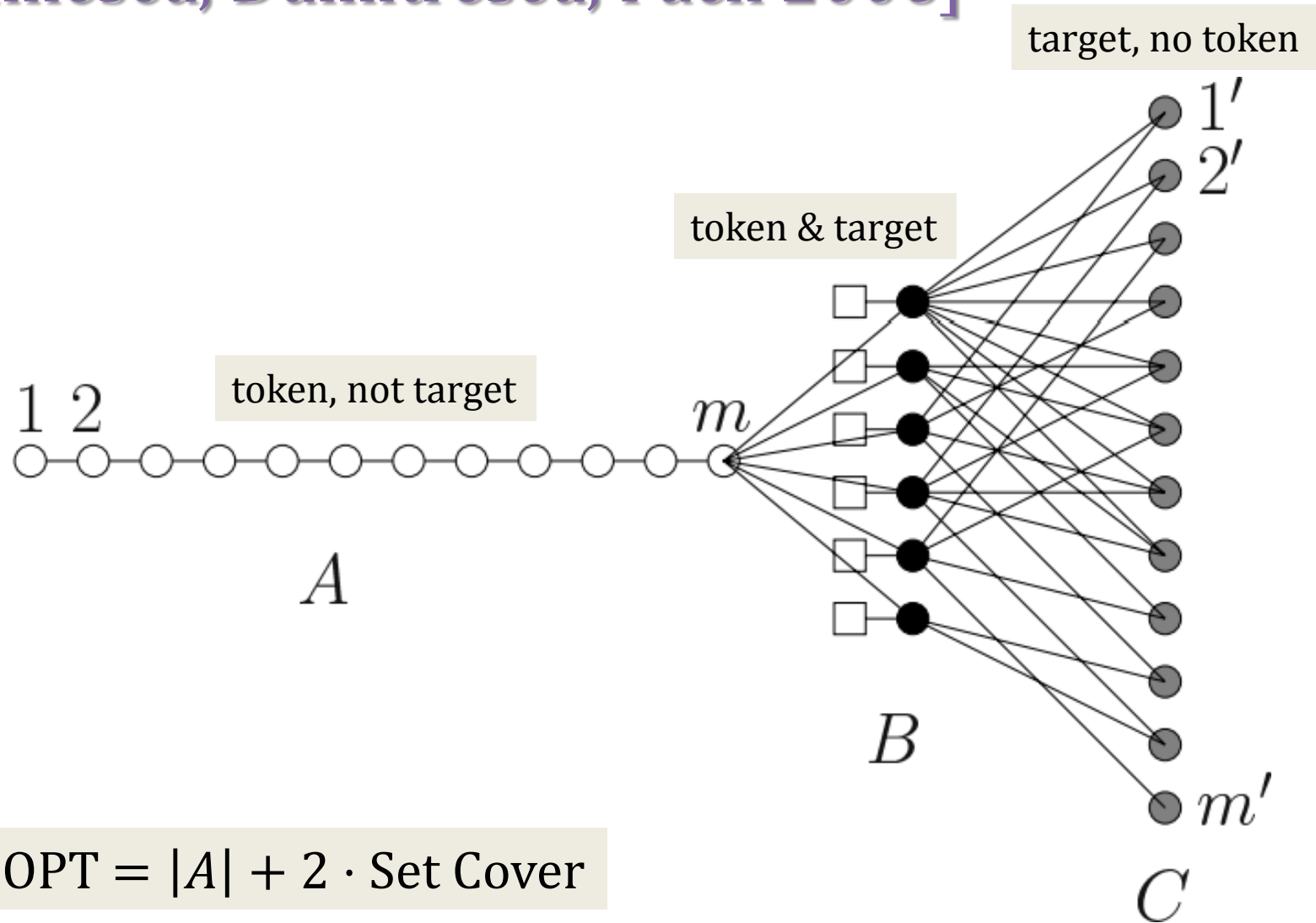
# Token Reconfiguration

[Calinescu, Dumitrescu, Pach 2006]



# Token Reconfiguration

[Calinescu, Dumitrescu, Pach 2006]



$$\text{OPT} = |A| + 2 \cdot \text{Set Cover}$$