Today: video games & PSPACE. First some NP:

**Metatheorem 1:** [Viglietta-Fun 2012 & arXiv:1201.4995]
- player traversing planar environment from start
- location traversal & single-use paths $\Rightarrow$ NP-hard
  - player must visit some locations
  - player can traverse only once
- reduction from Planar Max-deg-3 Hamiltonicity
- vertex $\Rightarrow$ location traversal
  - $\Rightarrow$ visit each vertex $\geq$ once
- edge $\Rightarrow$ single-use path
  - max. degree 3 $\Rightarrow$ never revisit vertex
- applications
  - Boulder dash
  - Lode Runner
  - Zelda II [Aloupis, Demaine, Guo, Viglietta 2014]

**Metatheorem 2:** [Viglietta-Fun 2012 & arXiv:1201.4995]
- location traversal & tokens + toll roads $\Rightarrow$ NP-hard
  - can pick (one) up $\Rightarrow$ need token to traverse
- vertex $\Rightarrow$ location traversal + token
- edge $\Rightarrow$ toll road
- traversing twice $\Rightarrow$ stranded without token
- application: Pac-Man
Recall from L1:

\[ \text{PSPACE} = \{ \text{problems solvable in polynomial space} \} \]  
- \( \subseteq \text{EXP} \): only exponentially many states  
- \( \supseteq \text{NP} \): simulate all executions, take running or  
- = \text{NPSPACE} \] [Savitch 1970]

Base PSPACE-complete problems:
- simulate linear-space algorithm (e.g. Turing machine)  
- \( \text{QSAT} \): (AKA QBF & TQBF) given (fully) quantified Boolean formula, is it true?  
  - e.g. \( \forall x \exists y : (\bar{x} \lor y) \land (x \lor \bar{y}) \) \( (x = y) \)  
  - can assume quantifiers in front (prenex) & alternate \( \forall / \exists \) (\( \exists \) only \( \Rightarrow \) SAT \( \Rightarrow \) NP-comp)  
- Schaefer-style dichotomy theorem:
  - \( \in \text{P} \) \( \Leftrightarrow \) Horn, dual-Horn, 2-CNF, or \( X(N)OR \)  
    (not if satisfied by all true/all false)  
- PSPACE-complete otherwise [Chen & C. Surveys 2009]  
- planar Q3SAT [Schaefer - SIAM J. 1981] [L7]  
  - add \( \exists \) for new variables at end of quantifiers  
- planar 1-in-3 Q3SAT (as in L7)
**Metatheorem 3:** [Viglietta - Fun 2012 & arXiv:1201.4995]
- player traversing planar environment from specified start to specified goal
- door + open pressure plate + close pressure plate $\xrightarrow{\text{traversable}} \xrightarrow{\text{walk on it}} \Rightarrow \text{PSPACE-hard}$
  only if open $\Rightarrow$ open specific door $\Rightarrow$ ditto, close & exactly 1 open & 1 close plate per door
- reduction from Q3SAT
- clause gadget
- existential quantifier gadget
- universal quantifier gadget
- one plate of each type for each door
- applications:
  - many FPSs e.g. Doom, Quake, Heretic, Hexen, ...
  - many RPGs e.g. Eye of the Beholder
  - many adventure games e.g. SCUMM engine (Maniac Mansion, Monkey Island, Space Quest, ...)
  - Prince of Persia
Metatheorem 4: buttons instead of pressure plates
\( \triangleright \) optional: can press or not
\( \triangleright \) activates 3 doors at once
- pressure plate gadget
- in fact 2 doors per button suffice
  [Bodlaender & van der Zanden - unpublished 2014]
- applications: MANY
  - Sonic the Hedgehog (Sega Genesis)
  - The Lost Vikings (Super NES; PC) “Erik the Swift”
  - Tomb Raider (Sega Saturn & PS1; PC)

Metatheorem 5: [Aloupiis, Demaine, Guo, Viglietta 2014]
- door with traverse, open, close paths \( \Rightarrow \) PSPACE-hard
  only if opened \( \text{can open} \) \( \triangleright \) must close
- applications:
  - Legend of Zelda: A Link to the Past
    (Ocarina of Time, Majora’s Mask, Oracle of Seasons, The Minish Cap, Twilight Princess \( \equiv \) PushPush-1)
  - Donkey Kong Country 1, 2, 3
  - Super Mario Bros. [Demaine, Viglietta, Williams - unpublished, 2014]
  - Lemmings [Viglietta - FW 2014]
    (tested in DOS version = Amiga version = unofficial editor)