Today: video games & PSPACE. First some NP:

- player traversing planar environment from start
- location traversal & single-use paths $\Rightarrow$ NP-hard
  - player must visit some locations
  - player can traverse only once
- reduction from Planar Max-deg-3 Hamiltonicity
  - vertex $\Rightarrow$ location traversal
    $\Rightarrow$ visit each vertex $\geq$ once
  - edge $\Rightarrow$ single-use path
    - max. degree $3 \Rightarrow$ never revisit vertex
- applications
  - Boulderdash
  - Lode Runner
  - Zelda II [Aloupis, Demaine, Guo, Viglietta 2014]

- location traversal & tokens + toll roads $\Rightarrow$ NP-hard
  - can pick (one) up
  - need token to traverse
- vertex $\Rightarrow$ location traversal + token
- edge $\Rightarrow$ toll road
  - traversing twice $\Rightarrow$ stranded without token
- application: Pac-Man
Recall from L1:

\[ \text{PSPACE} = \{ \text{problems solvable in polynomial space} \} \]
- \( \subseteq \text{EXP} \): only exponentially many states
- \( \supseteq \text{NP} \): simulate all executions, take running or
- \( = \text{NPSPACE} \) [Savitch 1970]

Base \text{PSPACE-complete problems:}
- simulate linear-space algorithm
  (e.g. Turing machine)
- \text{QSAT}: (AKA \text{QBF} & \text{TQBF})
  given (fully) quantified Boolean formula, is it true?
  - e.g. \( \forall x \exists y : (\overline{x} \lor y) \land (x \lor \overline{y}) \quad (x \equiv y) \)
  - can assume quantifiers in front (prenex) & alternate \( \forall / \exists \) (\( \exists \) only \( \Rightarrow \) \text{SAT} \( \Rightarrow \) \text{NP-comp})
- Schaefer-style dichotomy theorem:
  - \( \in \text{P} \iff \text{Horn, dual-Horn, 2-CNF, or X(N)OR} \)
    (not if satisfied by all true/all false)
  - \text{PSPACE-complete otherwise} [Chen-C.Surveys 2009]
- planar \text{Q3SAT} [Schaefer-SICOMP 1981] [L7]
  - add \( \exists \) for new variables at end of quantifiers
- planar 1-in-3 \text{Q3SAT} (as in L7)
**Metatheorem 3** [Viglietta-Fun 2012 & arXiv:1201.4995]

- player traversing planar environment from specified start to specified goal
- door + open pressure plate + close pressure plate
  - traversable \( \Leftrightarrow \) walk on it
  - only if open \( \Rightarrow \) open specific door \( \Rightarrow \) ditto, close
  & exactly 1 open & 1 close plate per door
- reduction from Q3SAT
- clause gadget
- existential quantifier gadget
- universal quantifier gadget
- one plate of each type for each door

- applications:
  - many FPSs e.g. Doom, Quake, Heretic, Hexen, ...
  - many RPGs e.g. Eye of the Beholder
  - many adventure games e.g. SCUMM engine (Maniac Mansion, Monkey Island, Space Quest, ...)
  - Prince of Persia
Metatheorem 4: buttons instead of pressure plates

- optional: can press or not
- activates 3 doors at once

- pressure plate gadget

- in fact 2 doors per button suffice

[Bodlaender & van der Zanden - CIAC 2015]

- applications: MANY
  - Sonic the Hedgehog (Sega Genesis)
  - The Lost Vikings (Super NES; PC) “Erik the Swift”
  - Tomb Raider (Sega Saturn & PS1; PC)

Metatheorem 5: [Aloupiis, Demaine, Guo, Viglietta 2014]

- door with traverse, open, close paths \( \Rightarrow \) PSPACE-hard
  - only if opened can open \( \Rightarrow \) must close

- applications:
  - Legend of Zelda: A Link to the Past
    (Ocarina of Time, Majora’s Mask, Oracle of Seasons, The Minish Cap, Twilight Princess \( \cong \) PushPush-1)
  - Donkey Kong Country 1, 2, 3
  - Super Mario Bros. [Demaine, Viglietta, Williams – unpublished, 2014]
  - Lemmings [Viglietta – FWW 2014]
    (tested in DOS version = Amiga version = unoff. editor)