Planar 3SAT:
- NP-hard special case of 3SAT
- Variable-clause bipartite graph is planar
  - Edge \((v_i, c_j)\) whenever \(v_i\) or \(\overline{v_i}\) is in \(c_j\)
- Remains planar after connecting variables in a cycle: \(V_1 \rightarrow V_2 \rightarrow \cdots \rightarrow V_n \rightarrow V_1\)
  - OR after connecting variables & clauses in a cycle [Dyer & Frieze 1986]
- Remains planar if we require \(v_i\)'s positive connections separated from negative connections.
  - I.e. split \(v_i\) into \(v_i\) and \(\overline{v_i}\).
- Positive connections negative connections [De Berg & Khosravi - COCOON 2010]
- Remains planar if we require all positive connections on one side of cycle & negative connections on other side \(\Rightarrow\) monotone 3SAT
- Reductions from 3SAT
Planar rectilinear 3SAT: (essentially Lichtenstein 1982) [Knuth & Raghunathan 1992]
- variable = horizontal segment on x axis
- clause = horizontal segment (off x axis) + 3 vertical connections to variables
- no crossings/overlap (other than connections)

Planar monotone rectilinear 3SAT: as above
+ monotone 3SAT: each clause all positive or all negative
+ positive clauses above x axis
+ negative clauses below x axis
[de Berg & Khosravi - COCOON 2010]
- reduction from planar rectilinear 3SAT

Careful:
- if all clauses on one side of variable cycle (above x axis in planar rectilinear 3SAT)
then CP via tree dynamic program
⇒ if clauses also connected in a path
then CP (would force clauses on same side)
(wanted this e.g. for Push-1/Nintendo)
Planar 1-in-3SAT: [Dyer & Frieze 1986]
- NP-hard special case of 1-in-3SAT
- variable-clause bipartite graph is planar
  + remains planar after connecting variables
    in a cycle: \( v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_n \rightarrow v_1 \)
  - or after connecting variables & clauses
    in a cycle

Reduction from Planar 3SAT:
- clause gadget

Planar positive 1-in-3SAT: no negations [Mulzer & Rote - J.ACM 2008]
  + remains planar after connecting variables
    in a cycle: \( v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_n \rightarrow v_1 \)

Rectilinear ...
- variable = horizontal segment on x axis
- clause = horizontal segment (off x axis)
  + 3 vertical connections to variables

Reduction from Planar 3SAT:
- equal & not-equal gadgets
- remove negations
- expand clauses (2 cases: \( u = 0 \) or \( 1 \))
Careful: Planar NAE 3SAT is polynomial! [Moret - SIGACT News 1988]

Reduction to Planar Max Cut: 2-color vertices of planar graph to maximize red-blue edges

\[ \leq \text{P} \] [Orlova & Dorfman 1972] [Hadlock - SICOMP 1975]

(in dual, red-blue edges are non-doubled edges in Chinese Postman problem)

- variable gadget/wire
- NAE clause

Planar X3C: [Dyer & Frieze 1986]

- bipartite graph of elements vs. 3-sets is planar
- reduction from planar 1-in-3SAT

Planar 3DM: [Dyer & Frieze 1986]

- special case where elements are 3-colored & each 3-set is trichromatic
+ remains planar if elements connected in cycle
- reduction from planar 1-in-3SAT
Planar vertex cover: [Lichtenstein 1982]
- given a planar graph
- choose $k$ vertices to hit all edges
- reduction from planar 3SAT
  - variable gadget: even cycle
  - clause gadget: triangle
- maximum degree 3

Planar (directed) Hamiltonian cycle: [Lichtenstein 1982]
- reduction from planar 3SAT
  - visit cycle through variables
  - variable gadget = ladder
  - clause gadget
  - can't jump var. $\Rightarrow$ clause $\Rightarrow$ other var.
  - same reduction claimed for undirected

Shakashaka [Guten 2008; Nikoli 2012-]
- reduction from Planar 3SAT

Flattening fixed-angle chains: [Soss & Toussaint 2000]
- reduction from Partition
- reduction from planar monotone rectilinear 3SAT [Demaine & Eisenstat 2011]