Packing Squares into a Square is Strongly NP-complete

\[ 3B + t \]
\[ B + a_i \]
\[ 3B + t \]
\[ (B + t) \frac{n}{3} \]

[Leung, Tam, Wong, Young, Chin 1990]
Edge-Unfolding Polyhedra

[Biedl, Demaine, Demaine, Lubiw, Overmars, O’Rourke, Robbins, Whitesides 1998]

[Bern, Demaine, Demaine, Eppstein, Kuo, Mantler, Snoeyink 1998]
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete
[Abel & Demaine 2011]
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]
Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

[Abel & Demaine 2011]
Snake Cube (Cubra)

Open: History? (c. 1990?)
Snake Cube is NP-complete
[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl 2012]

- Reduction from 3-Partition

\( a_i \) gadget:

\[
\begin{array}{c}
\text{huge} \\
8 a_i \\
\text{huge}
\end{array}
\]
Snake Cube is NP-complete
[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl 2012]

- Zigzag is universal
  - $2 \times 2 \times 2$ refinement makes any Hamiltonian shape
  - $4 \times 4 \times 4$ refinement makes any shape

- **Parity issue:** Path alternates cell parity each step

- **Claim:** Can start and end at any faces of cells of opposite parity
Snake Cube is NP-complete
[Abel, Demaine, Demaine, Eisenstatat, Lynch, Scharlel 2012]

- Reduce target box $\rightarrow$ target shape
- Reduce target cube $\rightarrow$ target box
- $\Rightarrow$ NP-hard to fold snake cube into target cube
Disk packing

Viet Elser’s disk packing puzzle

Robert Lang
Disk packing is NP-hard

[Demaine, Fekete, Lang 2010]
Disk packing is NP-hard

[Demaine, Fekete, Lang 2010]
Clickomania / Same Game
[Schuessler ~2000?]

- **Move** = Remove any connected group of size > 1

**Goal:** Remove everything
Clickomania Complexity

[Biedl, Demaine, Demaine, Fleischer, Jacobsen, Munro 2000]

- **Polynomial** for one row/column via CFG
- **NP-hard** for
  - 2 columns & 5 colors
  - 5 columns & 3 colors
- **Open:**
  - 2 rows
  - 2 colors
inverse of below

\[ B \cdot t \]

\[ B \cdot a_i \]

\[ B = \frac{4}{3} n \]

[Biedl, Demaine, Demaine, Fleischer, Jacobsen, Munro 2000]
In Honor of your Intellectual Contribution to the Art of Tetris,

FOR PROVING NP-COMPLETENESS IN MAXIMIZATION OF LINES,
TETRISES, PIECES PLAYED, OR MINIMIZATION OF SQUARE HEIGHT,

we masters of the Harvard Tetris Society hereby confer the title of

TETRIS MASTER

upon

Erik D. Demaine

on the sixteenth day of the twelfth month in the year 17 Anno Tetri (2002)
Claim 5: When terminate, we do so on the left.

if not:
  no RG ...

first LG goes into R-terminus:
  XLS.

Red LG goes in here

Other:
  XLS.

... in R-terminus?

problems:
  if multiple R-termini, then have a LG-sink.
  have a LGLS, LG-sink.
  & partial LS sink
  LS could be speak up & come early.

idea:
  count each piece using sinks = trouble.

end send W3 "1"s
  without LG
  If yes "1", in a pile,
  Sam is screwed
  if q scales by 2

starts RG, RG

fill: LG, LS, LG

end: RG needs RG

claims 2-4

(fillery go in right sides with right alignments)

Initial Board

\[ \approx t \text{ notches} \]

(target sum)

\[ n/3 \text{ buckets} \]

(one per sum)

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

(it is possible to actually get here)
For each input $a_i$: 
Failure to Launch  [Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

“unprimed” buckets

(a)  (b)  (c)  (d)  (e)  

(f)  (g)  (h)  (i)  (j)  (k)
Forced Moves
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale Pieces
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale Pieces
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale

Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale

Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kusters, Liben-Nowell 2003]
Finale
Pieces
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Finale Pieces

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kusters, Liben-Nowell 2003]
Hardness of Approximation
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]
Tetris Open Problems
[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]

- Complexity of Tetris with
  - Initially empty board?
  - $O(1)$ columns?
  - $O(1)$ rows?
  - Restricted piece sets (e.g. \[\square\])?
  - No last-minute slides?

- Is two-player Tetris PSPACE-complete?
- What can we say about online (regular) Tetris?
1-planar Graph  [Ringel 1985]

- Each edge has at most one crossing

[David Eppstein]
1-planarity is NP-complete

[Grigoriev & Bodlaender 2007]

$K_6$

double wheel

uncrossable edge
1-planarity is NP-complete

[Grigoriev & Bodlaender 2007]

\[ A = \{2, 3, 3, 3, 4, 5\} \]
GeoLoop & Ivan’s Hinge

[Kenneth Stevens 1993]

[Piano-Hinged Dissections
Time to Fold!
Greg N. Frederickson]

[Jan Essebaggers & Ivan Moscovich 1993]
GeoLoop & Ivan’s Hinge

[Abel, Demaine, Demaine, Horiyama, Uehara 2014]

NP-complete

universal & polynomial
2.2. Lower Bounds

Figure 2.5. Ruler folding reduction. Here $x_1 + x_3 + x_4 = x_2 + x_5 + x_6$.

[Hopcroft, Joseph, Whitesides 1985]
(Simple) Map Folding

[Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000]

Figure 14.4. Folding a $2 \times 4$ map via a sequence of three all-layers simple folds.