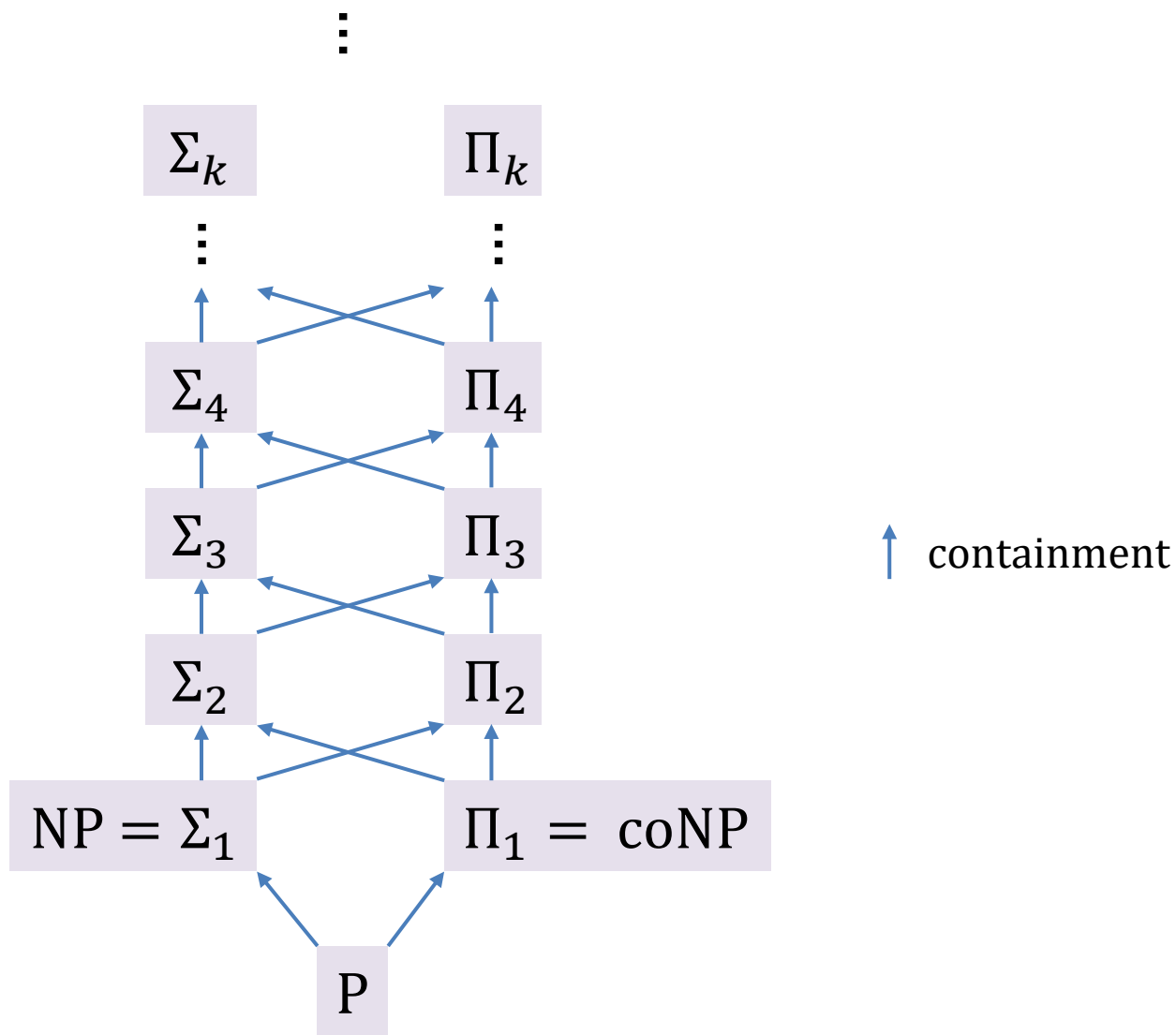




# PSPACE

## Polynomial Hierarchy



full game

PSPACE

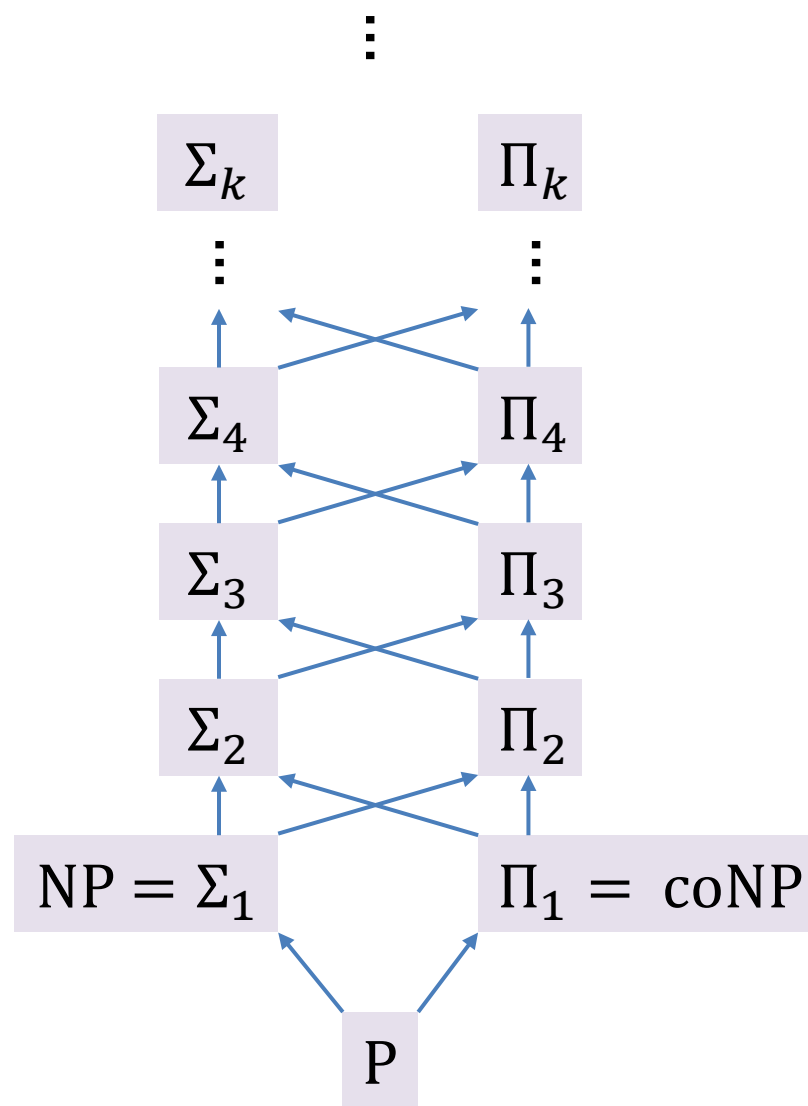
# Polynomial Hierarchy

lose-in-2

mate-in-2

lose-in-1

mate-in-1



2nd player  
lose-in-2

2nd player  
mate-in-1

2nd player  
lose-in-1

2nd player  
mate-in-0



# $\exists \mathbb{R}$ -complete Problems

Other complete problems for the existential theory of the reals include:

- the [art gallery problem](#) of finding the smallest number of points from which all points of a given polygon are visible.<sup>[22]</sup>
- recognition of [unit distance graphs](#), and testing whether the [dimension](#) or Euclidean dimension of a graph is at most a given value.<sup>[9]</sup>
- stretchability of pseudolines (that is, given a family of curves in the plane, determining whether they are [homeomorphic](#) to a [line arrangement](#));<sup>[4][23][24]</sup>
- both weak and strong satisfiability of geometric [quantum logic](#) in any fixed dimension  $>2$ ;<sup>[25]</sup>
- the algorithmic [Steinitz problem](#) (given a [lattice](#), determine whether it is the face lattice of a [convex polytope](#)), even when restricted to 4-dimensional polytopes;<sup>[26][27]</sup>
- realization spaces of arrangements of certain convex bodies<sup>[28]</sup>
- various properties of [Nash equilibria](#) of multi-player games<sup>[29][30][31]</sup>
- embedding a given abstract complex of triangles and quadrilaterals into three-dimensional Euclidean space;<sup>[17]</sup>
- embedding multiple graphs on a shared vertex set into the plane so that all the graphs are drawn without crossings;<sup>[17]</sup>
- recognizing the [visibility graphs](#) of planar point sets;<sup>[17]</sup>
- (projective or non-trivial affine) satisfiability of an equation between two terms over the [cross product](#);<sup>[32]</sup>
- determining the minimum [slope number](#) of a non-crossing drawing of a [planar graph](#).<sup>[33]</sup>

Based on this, the [complexity class](#)  $\exists \mathbb{R}$  has been defined as the set of problems having a polynomial-time many-one reduction to the existential theory of the reals.<sup>[4]</sup>

[https://en.wikipedia.org/wiki/Existential\\_theory\\_of\\_the\\_reals#Complete\\_problems](https://en.wikipedia.org/wiki/Existential_theory_of_the_reals#Complete_problems)

# A General Theory of Motion Planning Complexity: Characterizing Which Gadgets Make Games Hard

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## Abstract

We build a general theory for characterizing the computational complexity of motion planning of robot(s) through a graph of “gadgets”, where each gadget has its own state defining a set of allowed traversals which in turn modify the gadget’s state. We study two families of such

	1-Player Game	2-Player Game	Team Game
Polynomially Bounded (DAG)	<b>NL vs. NP-complete:</b> full characterization [§5]	<b>P vs. PSPACE-complete:</b> full characterization [§6]	<b>P vs. NEXPTIME:</b> full characterization [§7]
Polynomially Unbounded (reversible, deterministic gadgets)	<b>NL vs. PSPACE-complete:</b> full characterization [§2] <b>Planar:</b> equivalent [§2.3]	<b>P vs. EXPTIME-complete:</b> partial characterization [§3]	<b>P vs. RE-complete (<math>\Rightarrow</math> Undecidable):</b> partial characterization [§4]



**2-player  
bounded  
motion  
planning is  
PSPACE-  
complete**

