Polynomial Hierarchy

\[ P = \Sigma_1 \subseteq \Pi_1 = \text{coNP} \subseteq \Sigma_2 \subseteq \Pi_2 \subseteq \ldots \subseteq \Sigma_k \subseteq \Pi_k \subseteq \ldots \subseteq \text{PSPACE} \]

\[ \text{containment} \]
Polynomial Hierarchy

\[ \text{P} = \Sigma_1 \cap \Pi_1 = \text{coNP} \]

\[ \Sigma_2 \cap \Pi_2 \]

\[ \Sigma_3 \cap \Pi_3 \]

\[ \Sigma_4 \cap \Pi_4 \]

\[ \vdots \]

\[ \vdots \]

\[ \text{P} \]

\[ \text{NP} = \Sigma_1 \]

\[ \Pi_1 = \text{coNP} \]

2nd player lose-in-2
2nd player lose-in-1
2nd player lose-in-1
2nd player mate-in-0
2nd player mate-in-1
2nd player mate-in-1
2nd player mate-in-0

mate-in-2
mate-in-2
lose-in-2
lose-in-1
mate-in-1

full game

PSPACE
∃R-complete Problems

Other complete problems for the existential theory of the reals include:

- the art gallery problem of finding the smallest number of points from which all points of a given polygon are visible.\[^{[22]}\]
- recognition of unit distance graphs, and testing whether the dimension or Euclidean dimension of a graph is at most a given value.\[^{[9]}\]
- stretchability of pseudolines (that is, given a family of curves in the plane, determining whether they are homeomorphic to a line arrangement);\[^{[4][23][24]}\]
- both weak and strong satisfiability of geometric quantum logic in any fixed dimension >2;\[^{[25]}\]
- the algorithmic Steinitz problem (given a lattice, determine whether it is the face lattice of a convex polytope), even when restricted to 4-dimensional polytopes;\[^{[26][27]}\]
- realization spaces of arrangements of certain convex bodies\[^{[28]}\]
- various properties of Nash equilibria of multi-player games\[^{[29][30][31]}\]
- embedding a given abstract complex of triangles and quadrilaterals into three-dimensional Euclidean space;\[^{[17]}\]
- embedding multiple graphs on a shared vertex set into the plane so that all the graphs are drawn without crossings;\[^{[17]}\]
- recognizing the visibility graphs of planar point sets;\[^{[17]}\]
- (projective or non-trivial affine) satisfiability of an equation between two terms over the cross product;\[^{[32]}\]
- determining the minimum slope number of a non-crossing drawing of a planar graph.\[^{[33]}\]

Based on this, the complexity class \∃R has been defined as the set of problems having a polynomial-time many-one reduction to the existential theory of the reals.\[^{[4]}\]
A General Theory of Motion Planning Complexity: Characterizing Which Gadgets Make Games Hard

Erik D. Demaine*  Dylan H. Hendrickson*  Jayson Lynch*

https://arXiv.org/abs/1812.03592

Abstract

We build a general theory for characterizing the computational complexity of motion planning of robot(s) through a graph of “gadgets”, where each gadget has its own state defining a set of allowed traversals which in turn modify the gadget’s state. We study two families of such

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2-player bounded motion planning is PSPACE-complete

[Demaine, Hendrickson, Lynch 2018]