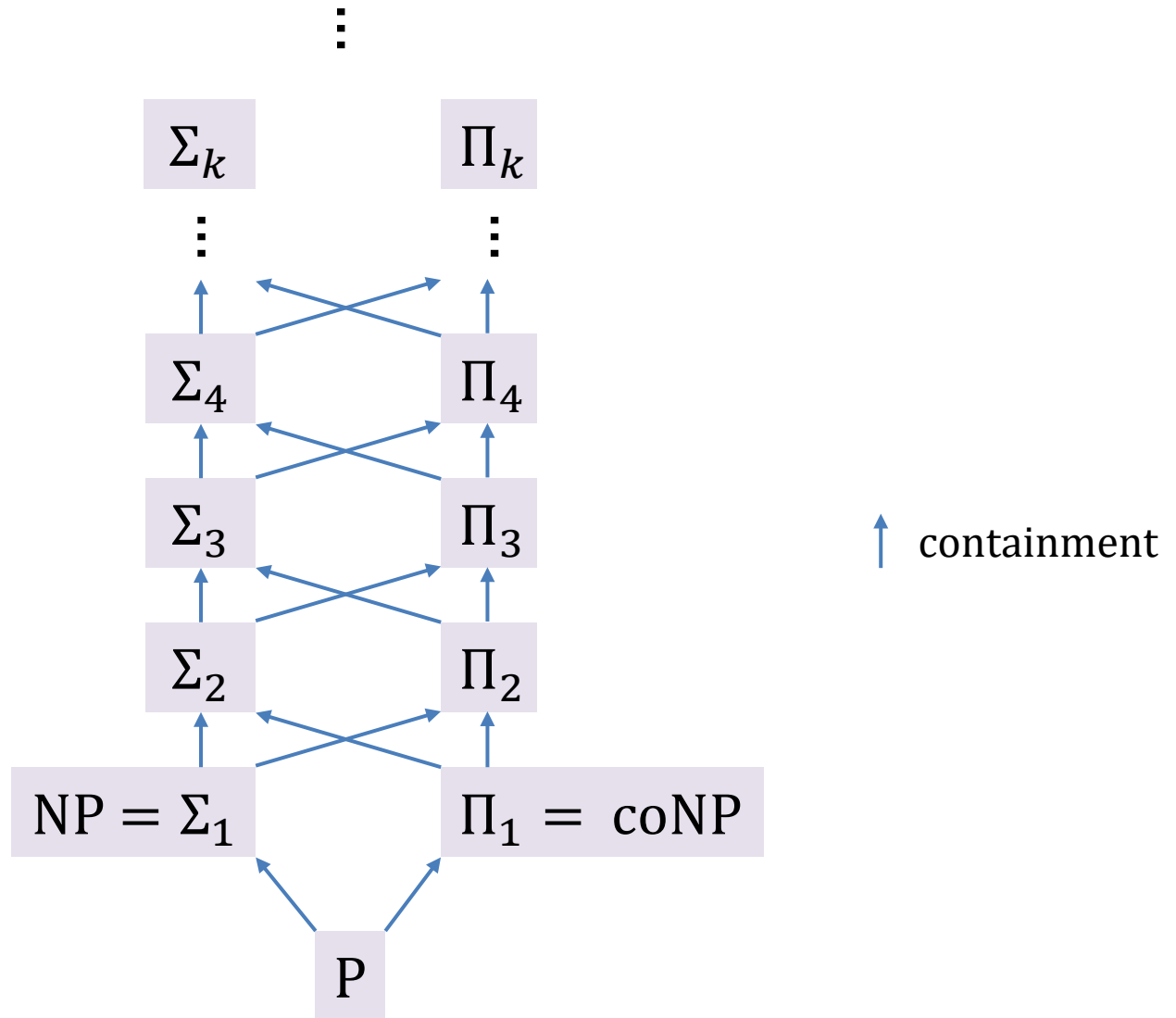


# PSPACE

## Polynomial Hierarchy



# Completeness in the Polynomial-Time Hierarchy

## A Compendium\*

Marcus Schaefer

School of CTI

DePaul University

243 S. Wabash Avenue

Chicago, Illinois 60604, USA

[schaefer@cs.depaul.edu](mailto:schaefer@cs.depaul.edu)

Christopher Umans

Computer Science Department

Caltech

1200 East California Boulevard

Pasadena, CA 91125

[uumans@cs.caltech.edu](mailto:uumans@cs.caltech.edu)

October 5, 2008

### **Abstract**

We present a Garey/Johnson-style list of problems known to be complete for the second and higher levels of the polynomial-time Hierarchy (polynomial hierarchy, or **PH** for short). We also include the best-known hardness of approximation results. The list will be updated as necessary.

# A General Theory of Motion Planning Complexity: Characterizing Which Gadgets Make Games Hard

Erik D. Demaine\*

Dylan H. Hendrickson\*

Jayson Lynch\*

<https://arXiv.org/abs/1812.03592>

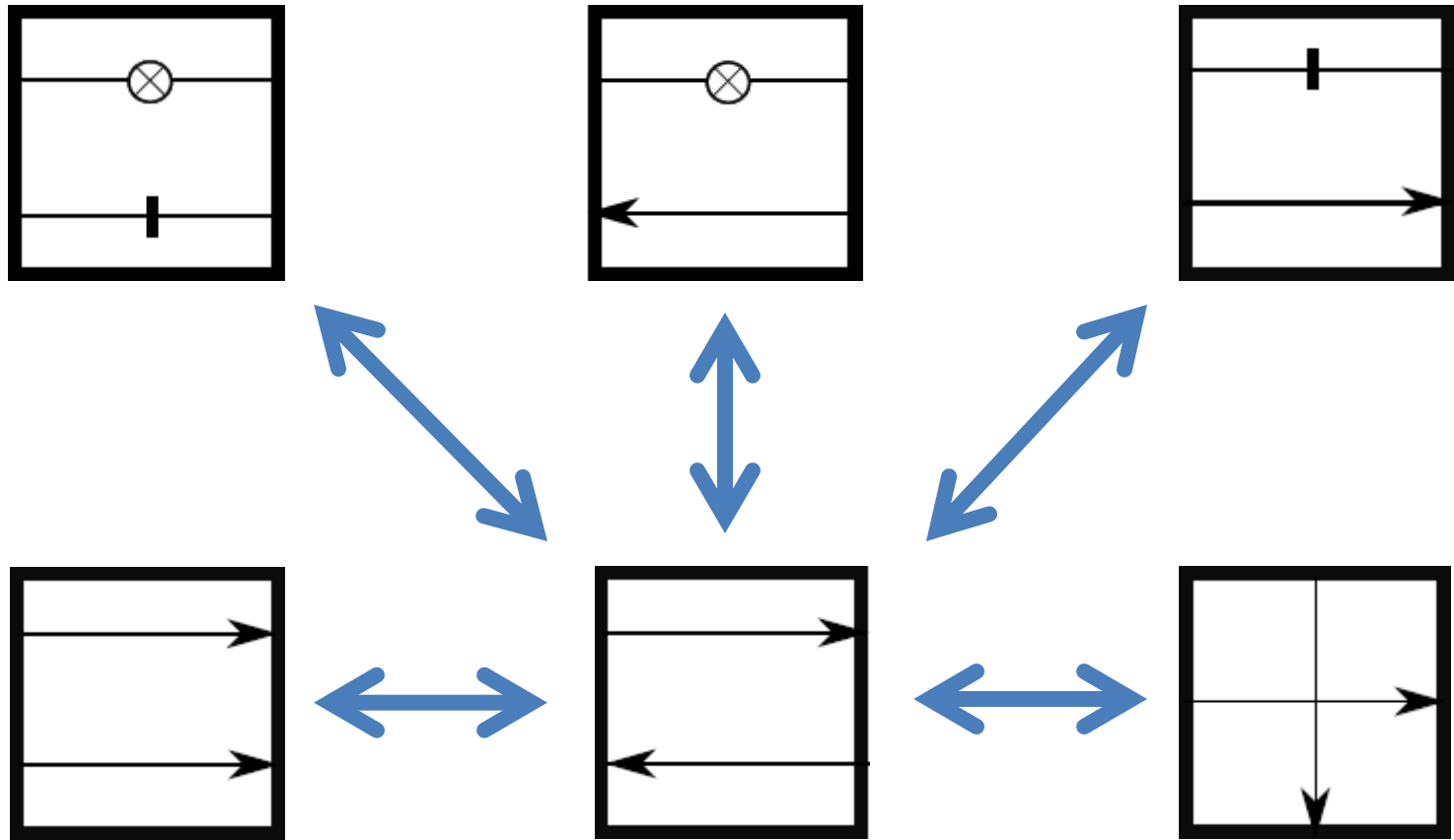
## Abstract

We build a general theory for characterizing the computational complexity of motion planning of robot(s) through a graph of “gadgets”, where each gadget has its own state defining a set of allowed traversals which in turn modify the gadget’s state. We study two families of such

|                                                            | 1-Player Game                                                                                 | 2-Player Game                                                | Team Game                                                                                      |
|------------------------------------------------------------|-----------------------------------------------------------------------------------------------|--------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| Polynomially Bounded (DAG)                                 | <b>NL vs. NP-complete:</b> full characterization [§5]                                         | <b>P vs. PSPACE-complete:</b> full characterization [§6]     | <b>P vs. NEXPTIME:</b> full characterization [§7]                                              |
| Polynomially Unbounded (reversible, deterministic gadgets) | <b>NL vs. PSPACE-complete:</b> full characterization [§2]<br><b>Planar:</b> equivalent [§2.3] | <b>P vs. EXPTIME-complete:</b> partial characterization [§3] | <b>P vs. RE-complete (<math>\Rightarrow</math> Undecidable):</b> partial characterization [§4] |

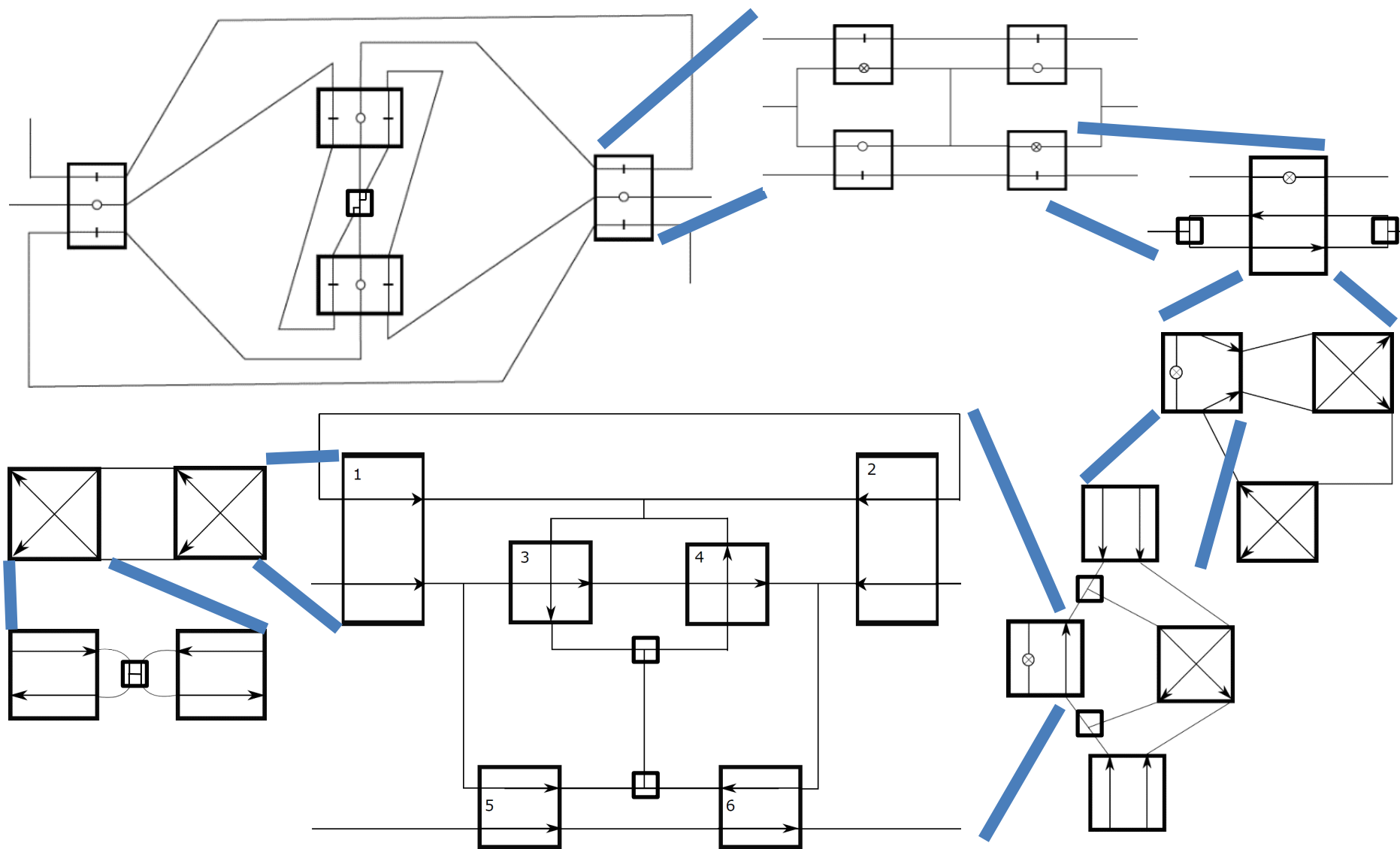
# Everything Simulates Everything

[Demaine, Groszof, Lynch, Rudoy 2018]



# Crossover Gadget

[Demaine, Grosz, Lynch, Rudoy 2018]

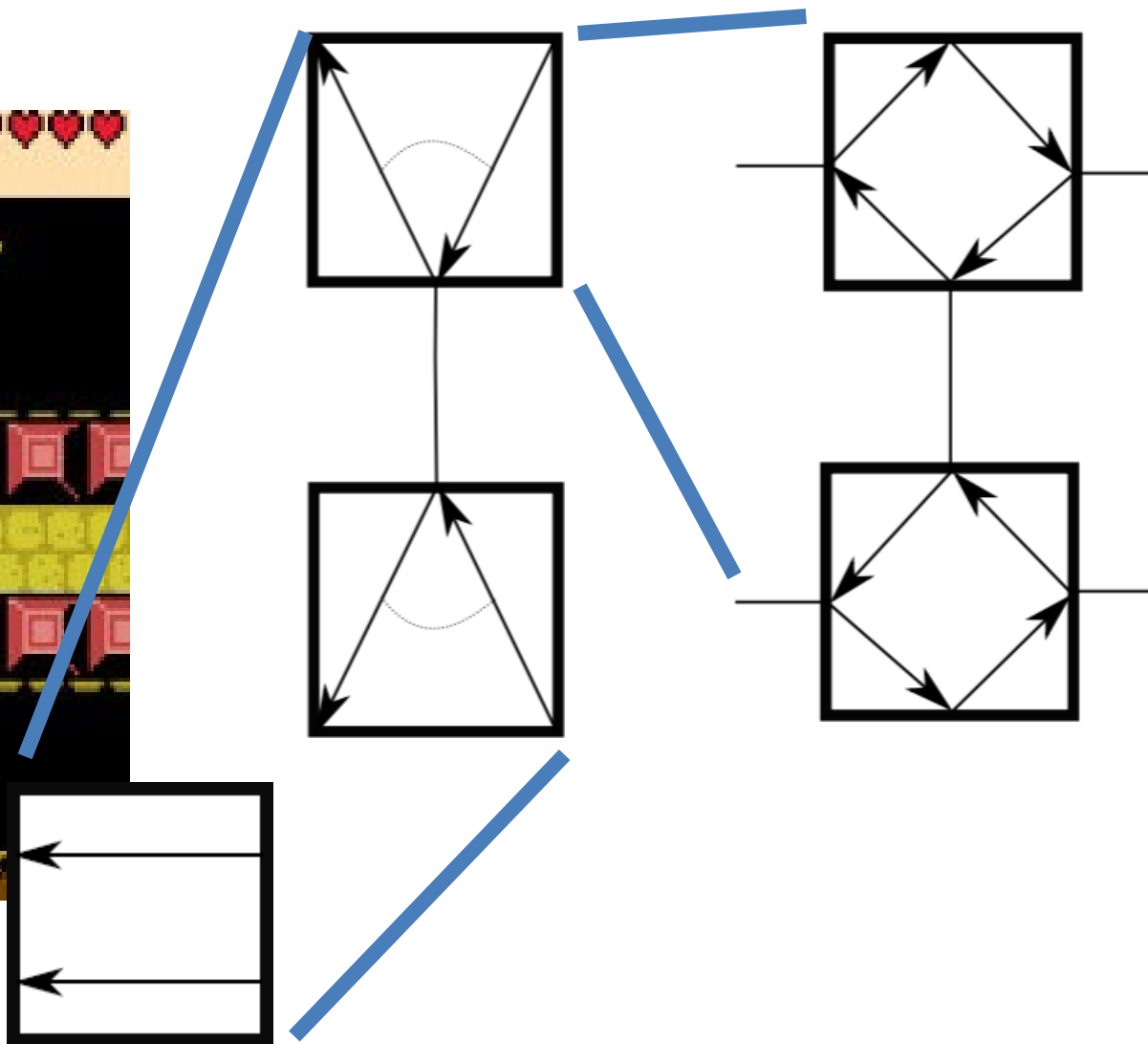


# 4-Spinners are PSPACE-hard

[Demaine, Grosz, Lynch, Rudoy 2018]

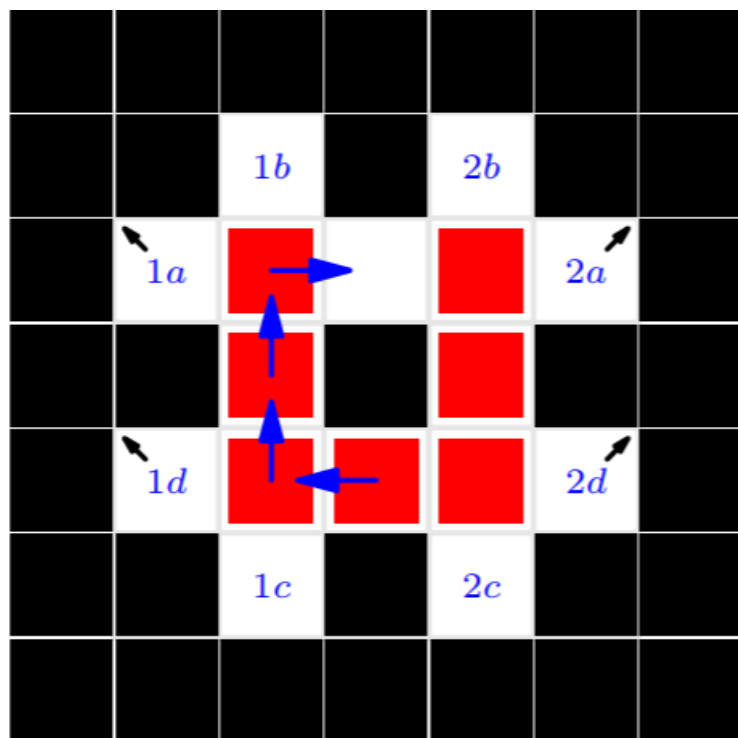


The Legend of Zelda:  
Oracle of Seasons

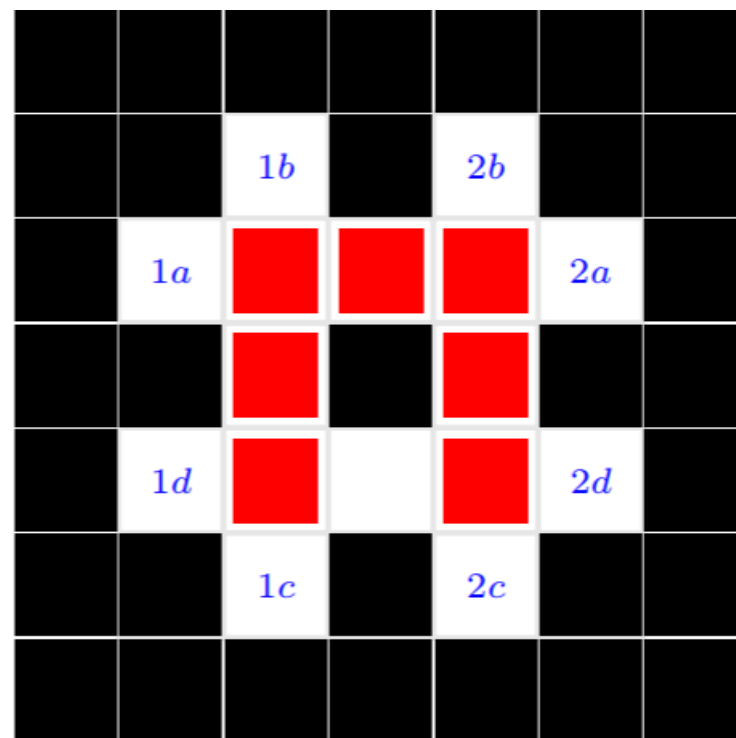


# 3D PushPull-1F is PSPACE-hard

[Demaine, Groszof, Lynch 2017]



(a) 2-Toggle in state *A*. The arrows indicate the transition to state *B*.

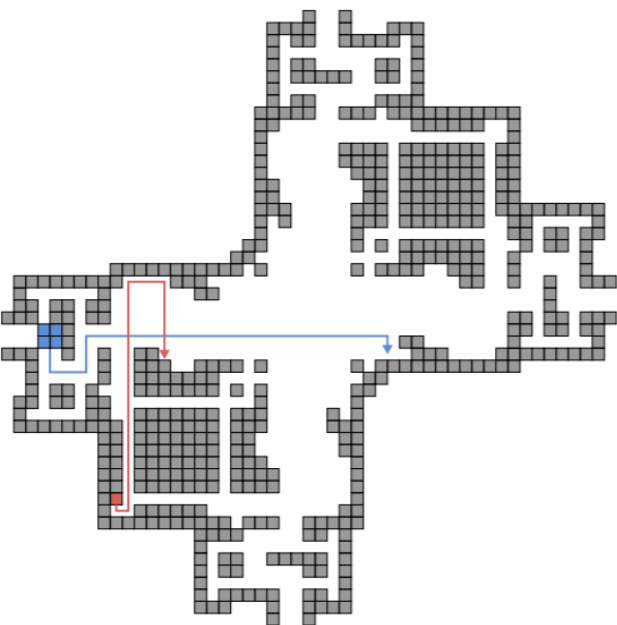


(b) 2-Toggle in state *B*.

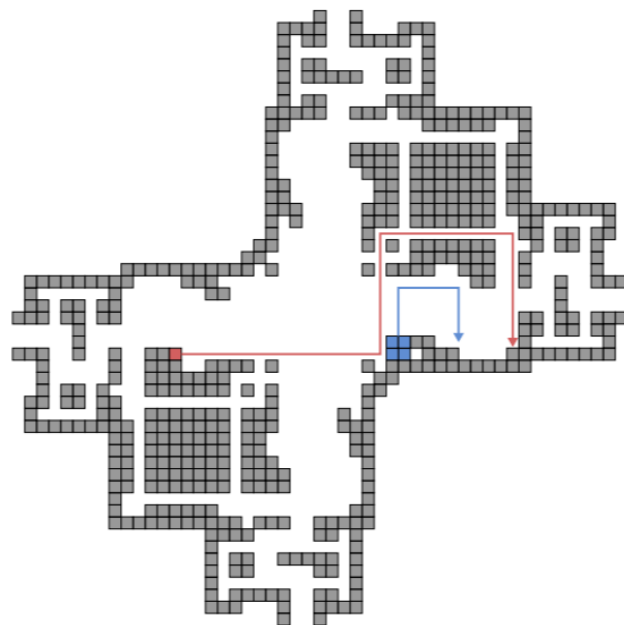


# Tilt Assembly is PSPACE-hard

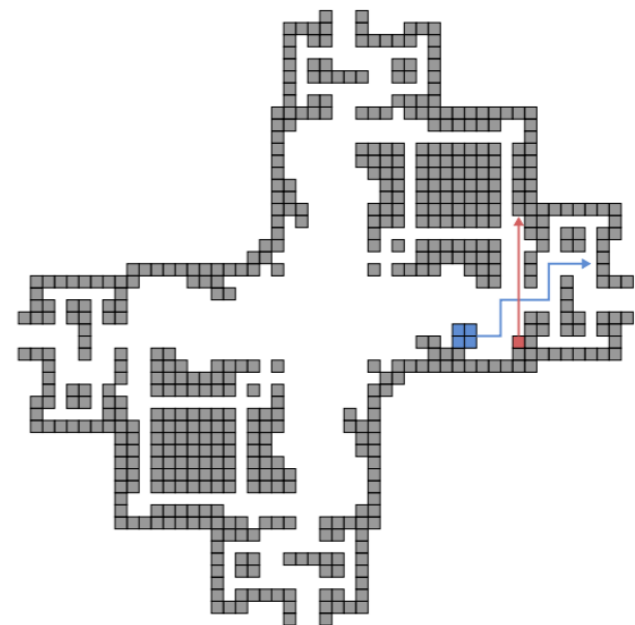
[Balanza-Martinez, Caballero,  
Cantu, Garcia, Luchsinger,  
Reyes, Schweller, Wylie 2019]



(a)  $\langle S, E, N, E, S \rangle$



(b)  $\langle E, N, E, S \rangle$



(c)  $\langle E, N, E, N, E \rangle$

Crossing 2-toggle