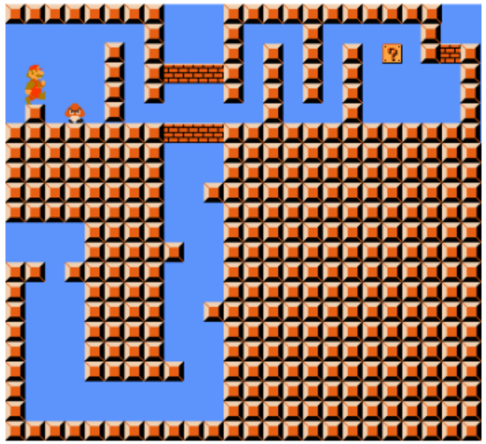
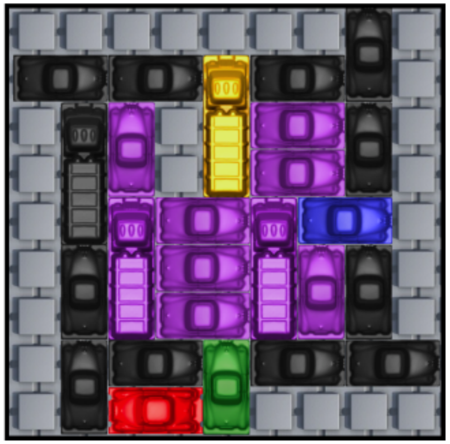
Algorithmic Lower Bounds:
Fun with Hardness Proofs

Super Mario Bros.



Crossover gadget for NP-hardness

Rush Hour



AND gadget for PSPACE-hardness

Minesweeper

OR gadget for NP-hardness

Hardness Made Easy*

Learn **when to give up** the search for efficient algorithms; see **connections** between computational problems; **solve puzzles** to prove theorems, solve **open problems,** and write papers.

Topics: NP, PSPACE, EXPTIME, EXPSPACE, 3SUM, approximation, fixed parameter, games & puzzles, key problems, gadgets, and proof styles.

Spring 2019

6.892 taught by Professor Erik Demaine

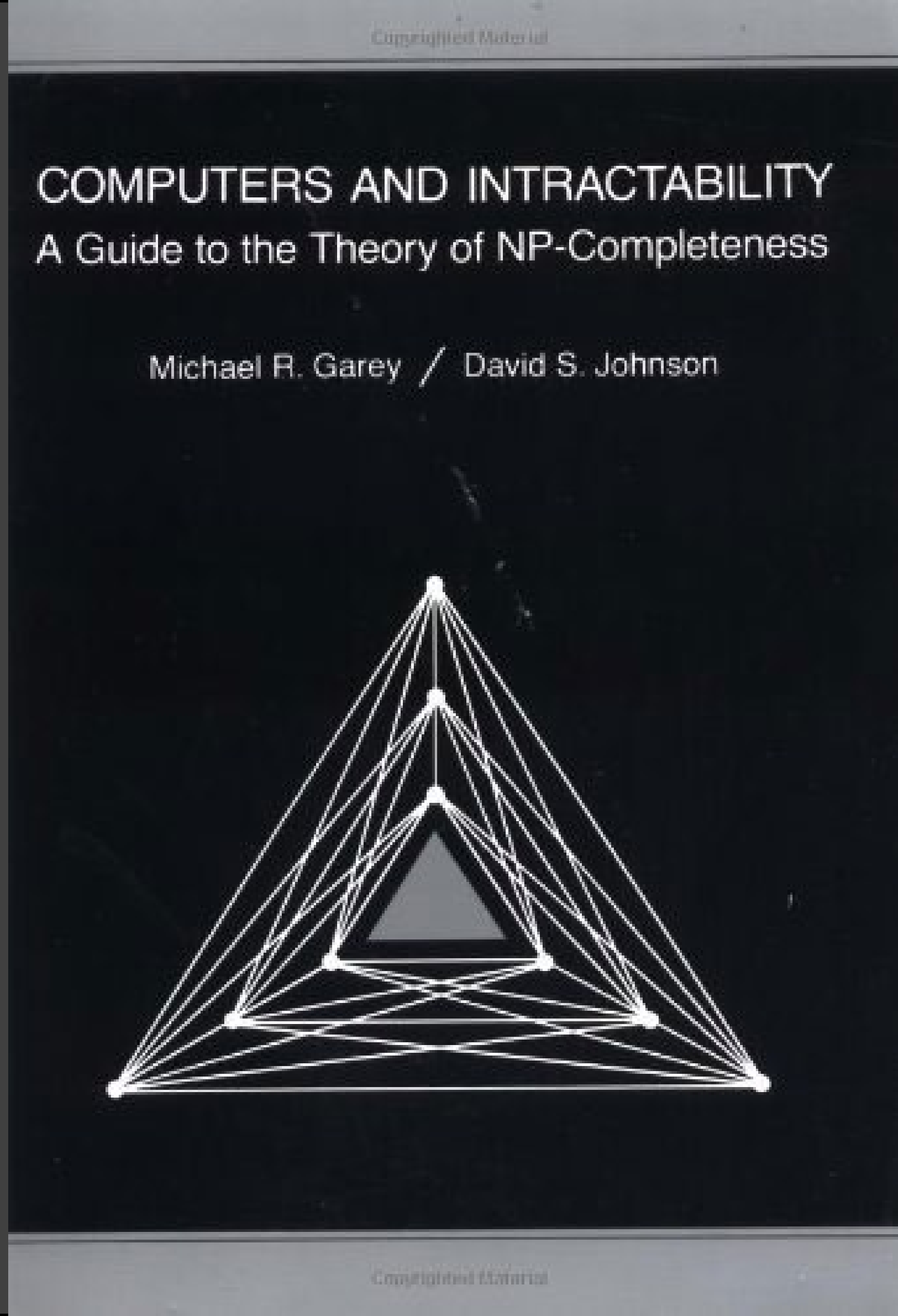
II, AAGS, and Theoretical CS Concentration

Wednesday 7:00-9:30pm in room 32-082

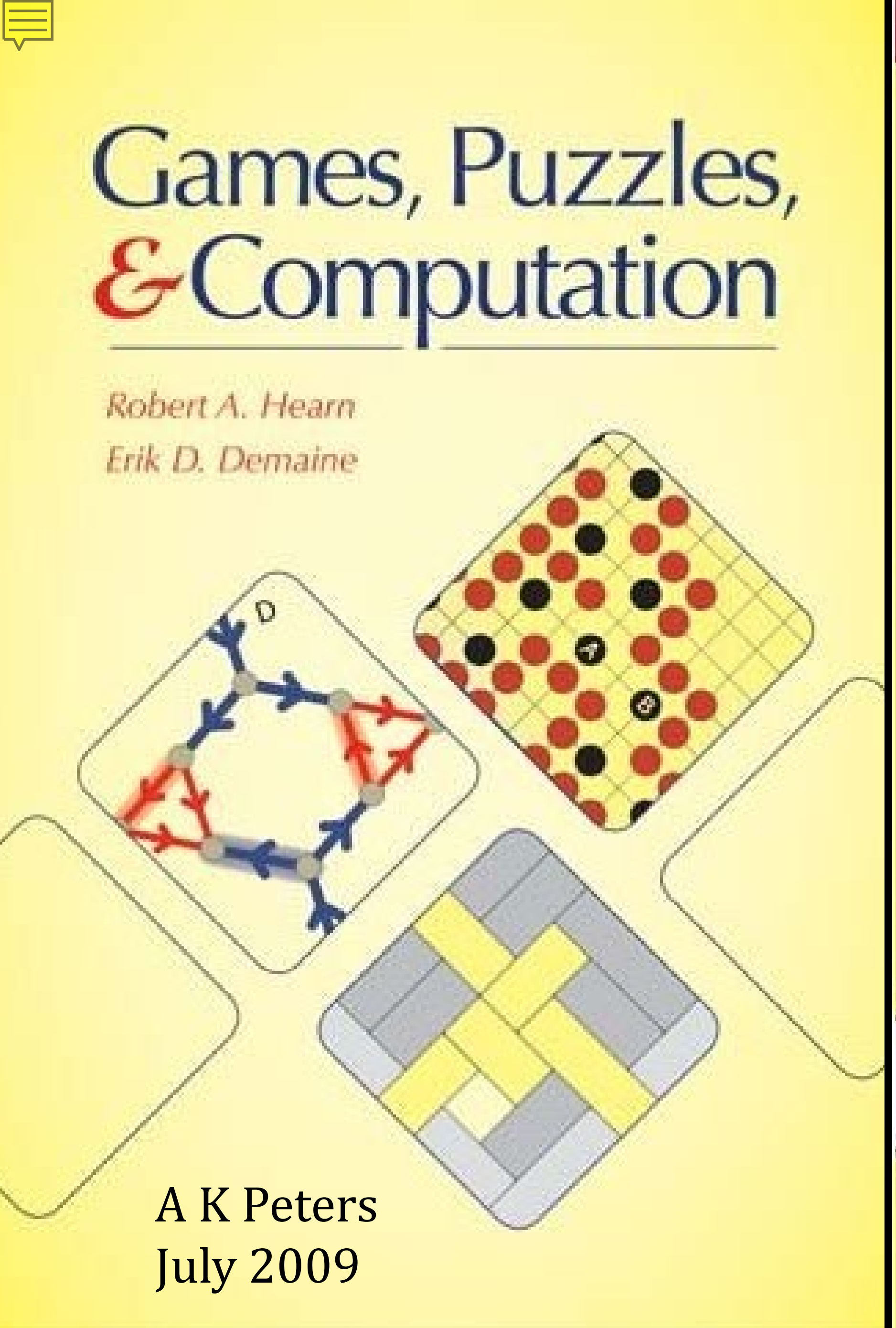
http://courses.csail.mit.edu/6.892/spring19/

sign up for our mailing list to join the class

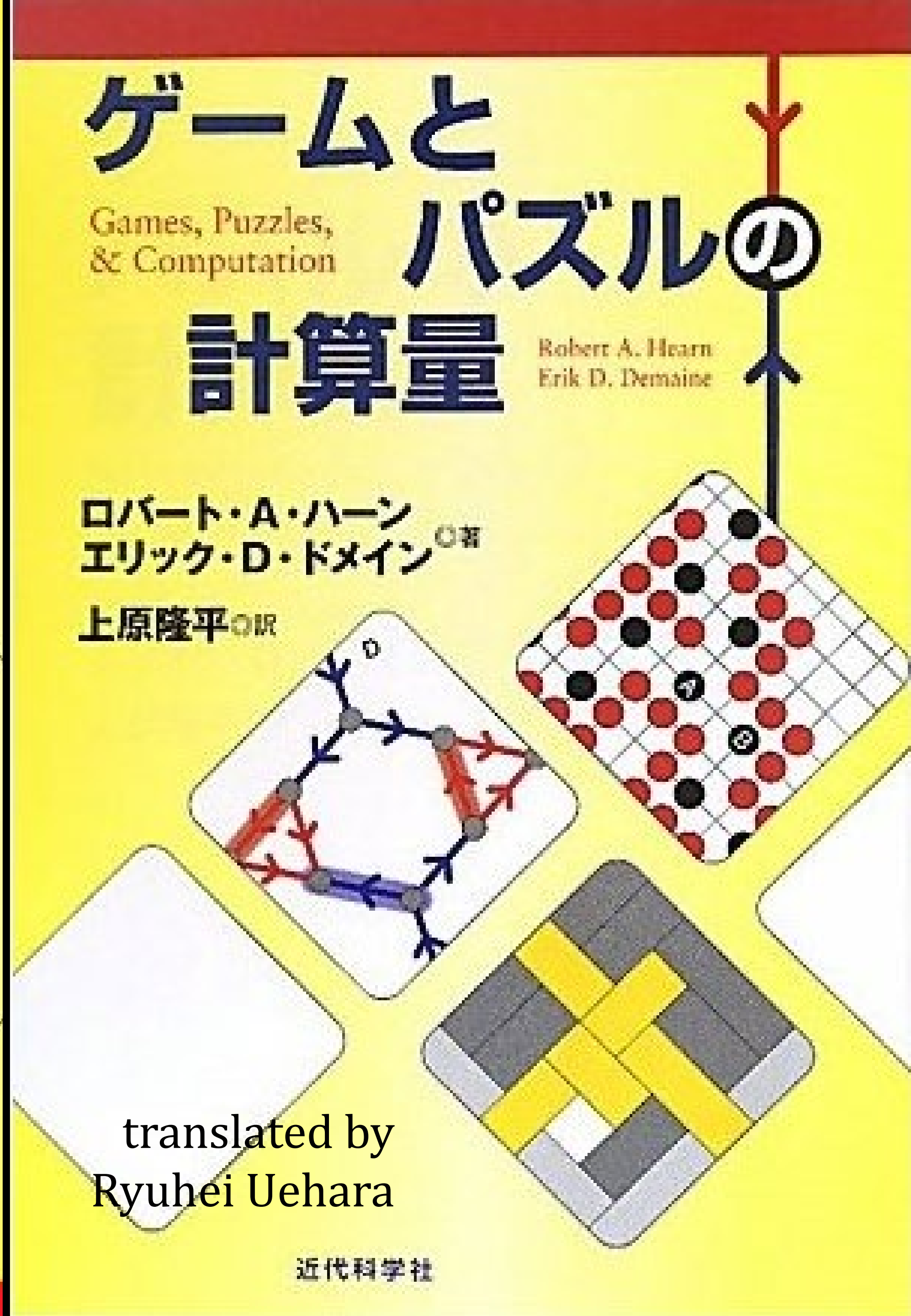
** Easiness not guaranteed. Side effects such as open problems and a heightened sense of complexity may occur. Ask your advisor if 6.892 is right for you!*



W. H. Freeman
1979

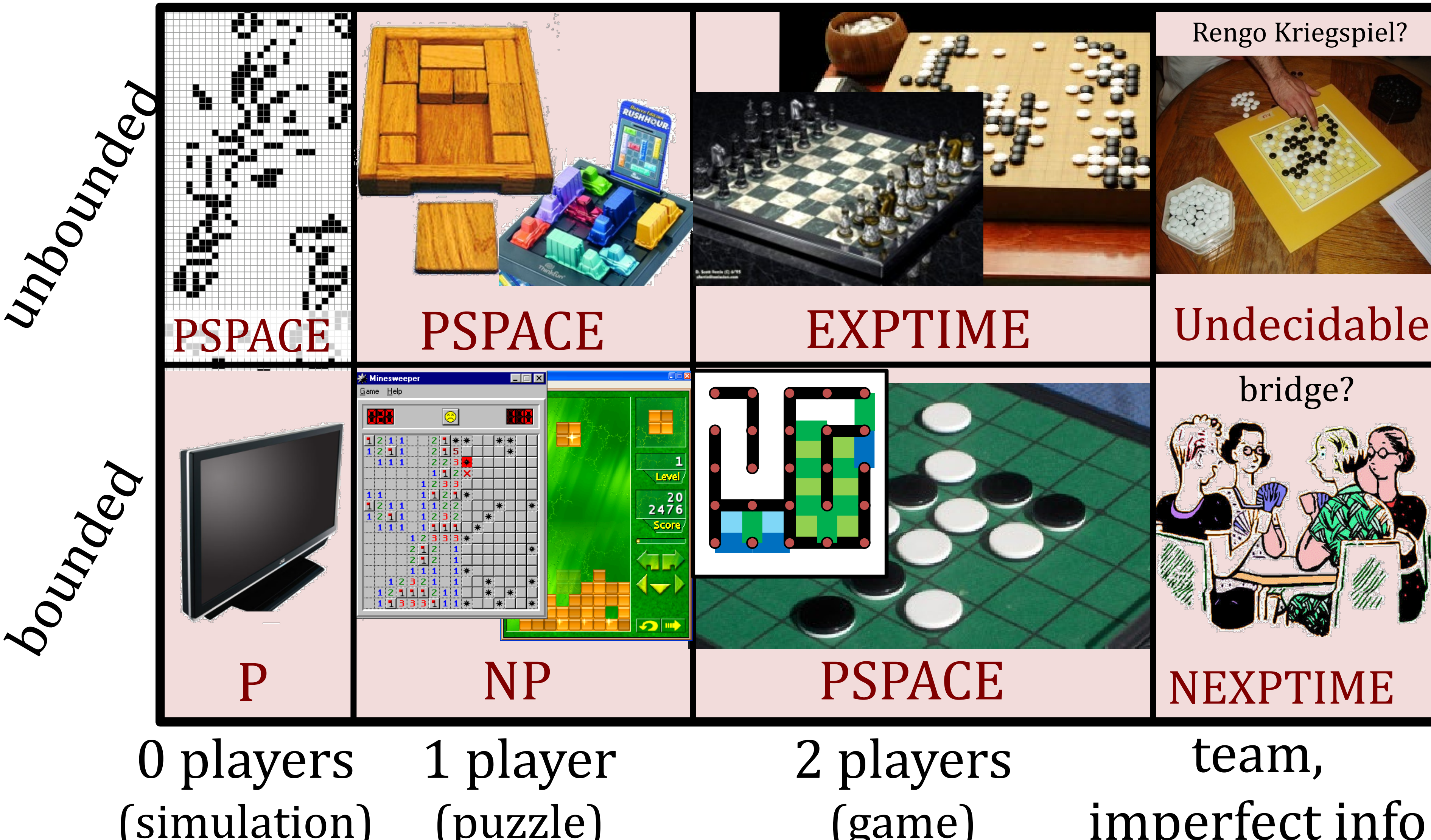


A K Peters
July 2009



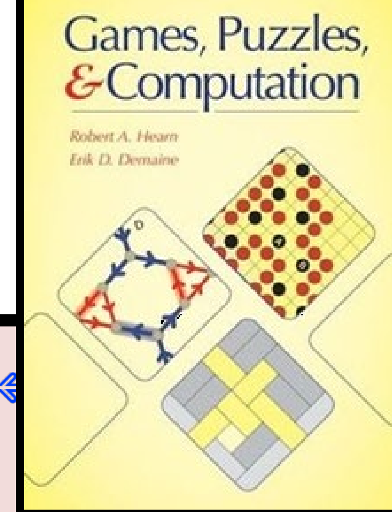
translated by
Ryuhei Uehara

Complexity of Games & Puzzles



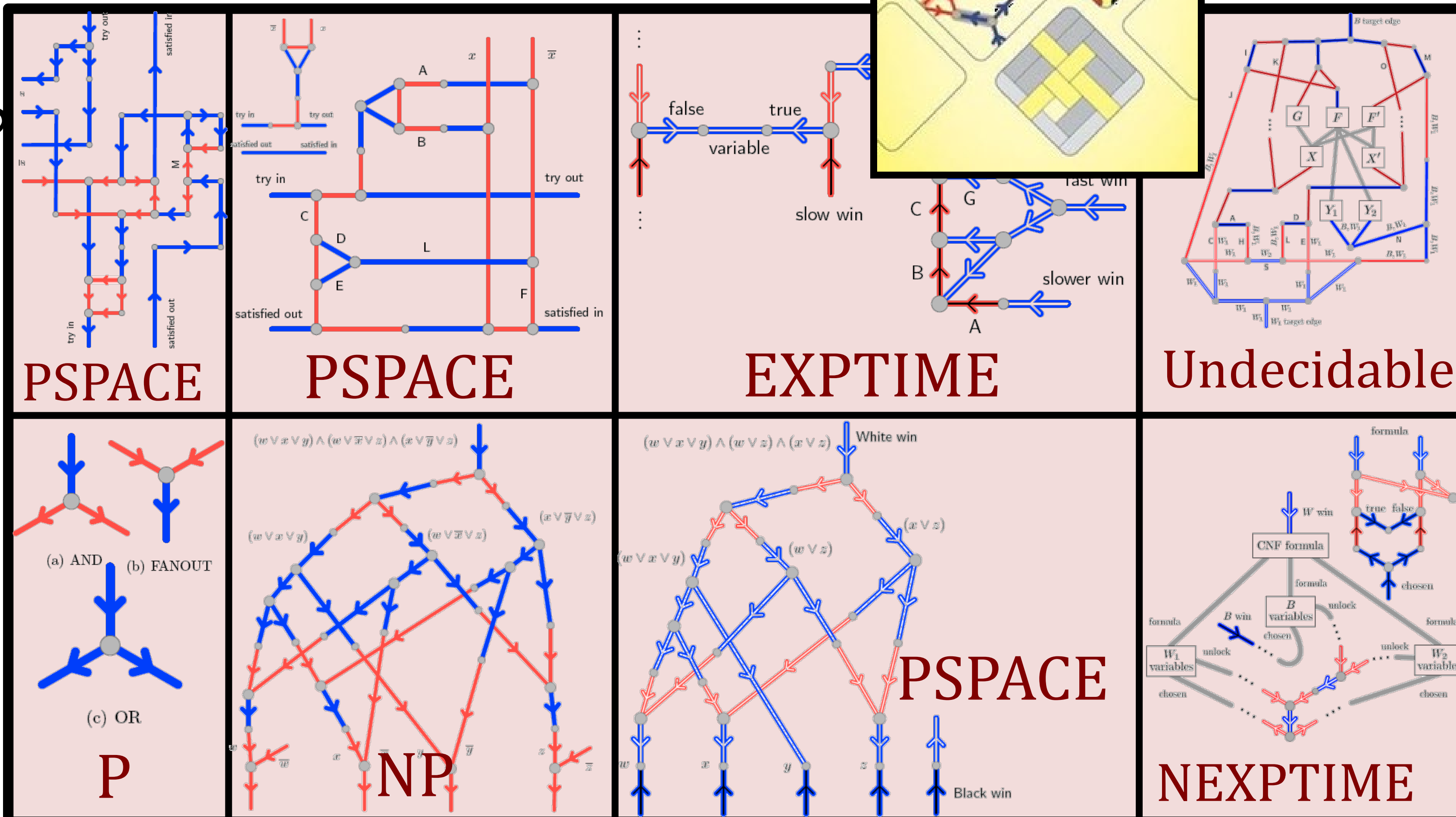
Constraint Logic

[Hearn & Demaine 2009]



unbounded

bounded



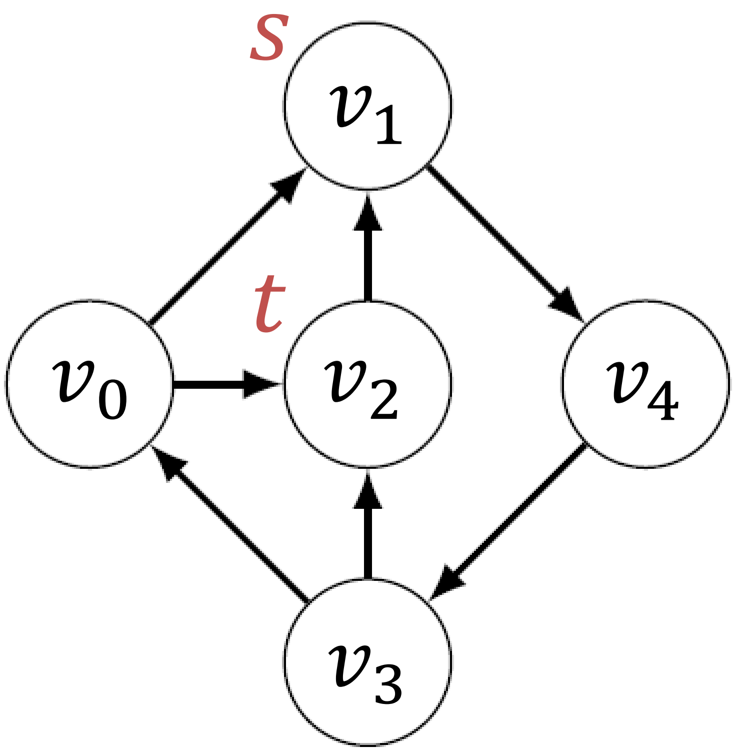
0 players (simulation)

1 player (puzzle)

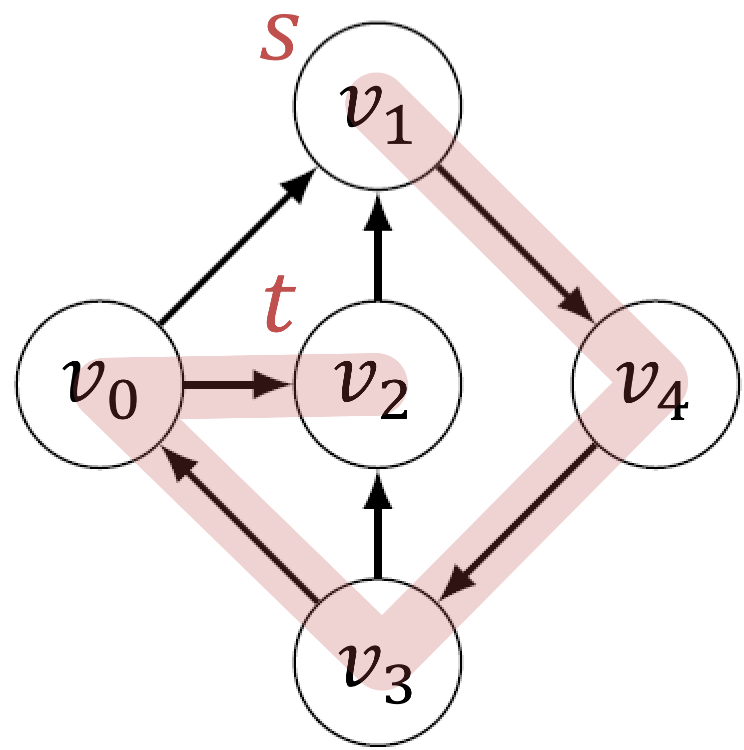
2 players (game)

team, imperfect info

Hamiltonian (s, t)-Path

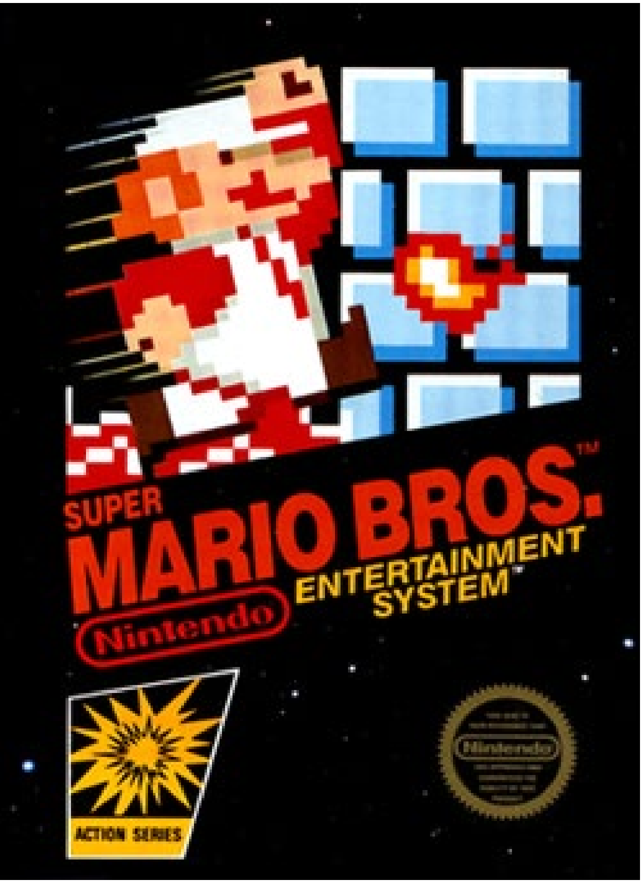


input

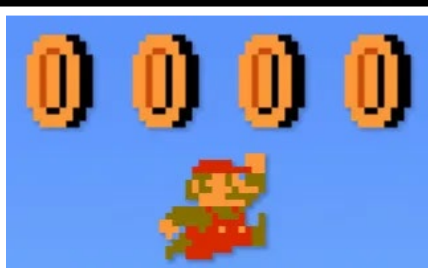


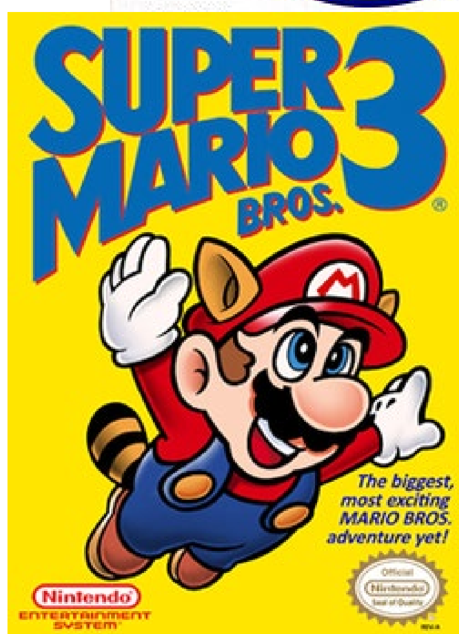
goal

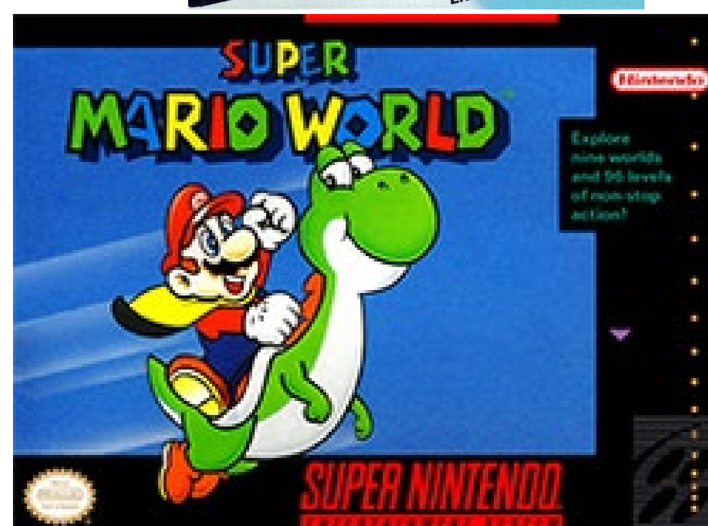
100% Speedrun is NP-hard: Mario



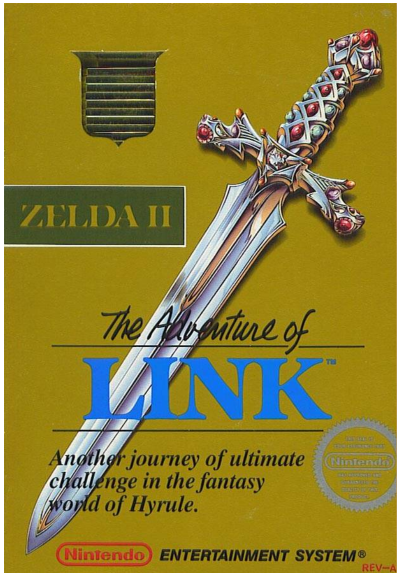
The Lost Levels

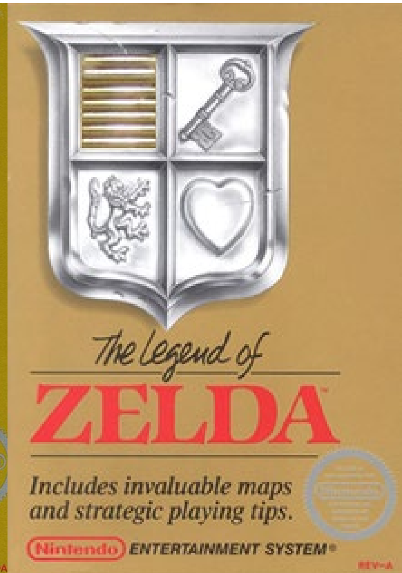






Speedrun is NP-hard: Zelda



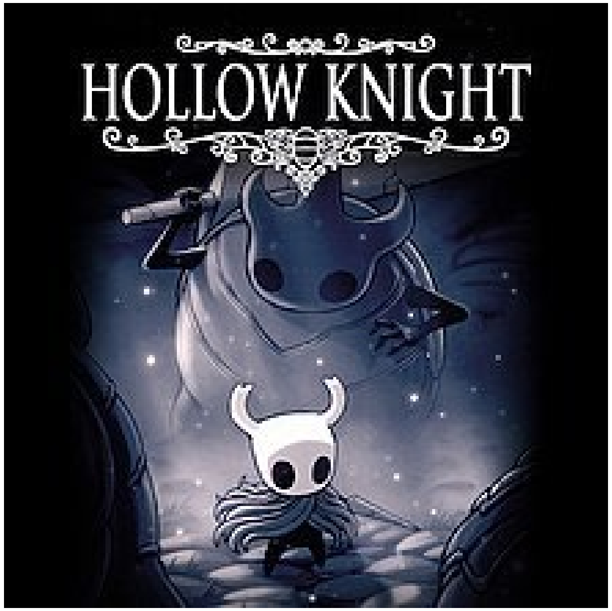


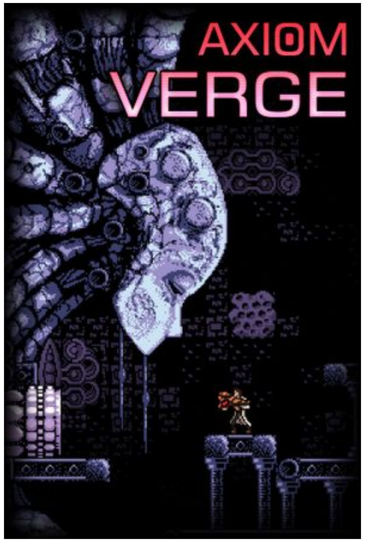
Small Key

Artwork of a Small Key from Ocarina of Time

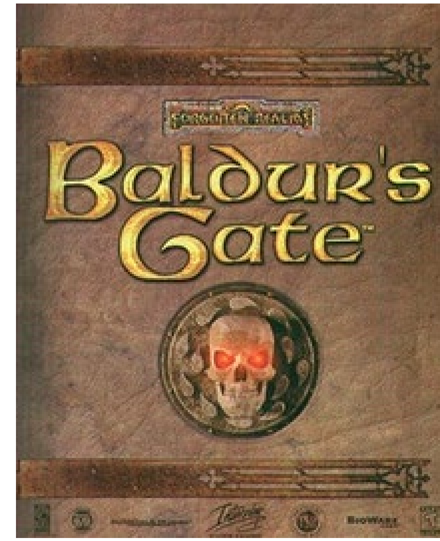
First appearance	*The Legend of Zelda* (1987)
Appearances	*The Legend of Zelda* *Adventure of Link* *A Link to the Past* *Link's Awakening* *Ocarina of Time* *Majora's Mask* *Oracle of Ages* *Oracle of Seasons* *The Wind Waker* *Four Swords* *The Minish Cap* *Four Swords Adventures* *Twilight Princess* *Phantom Hourglass* *Spirit Tracks* *Skyward Sword* *A Link Between Worlds* *Breath of the Wild*
Found	Dungeons (*Breath of the Wild*) Shrines
Use	Open small locks

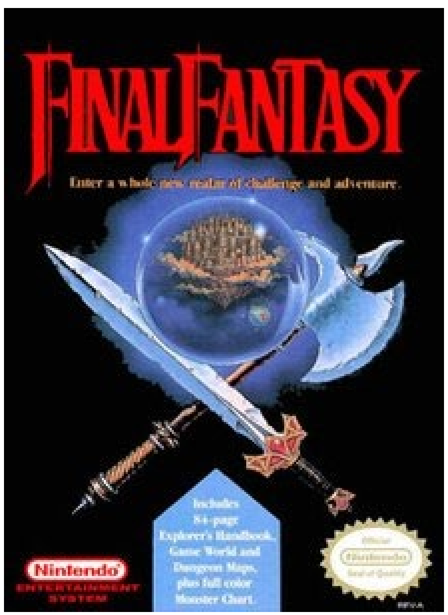
Speedrun is NP-hard: Metroidvania

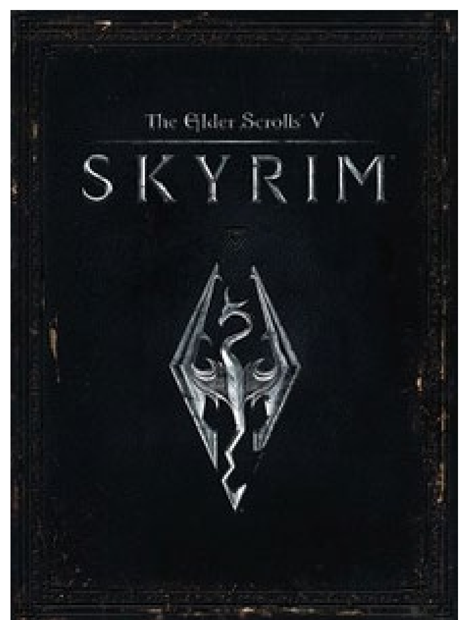




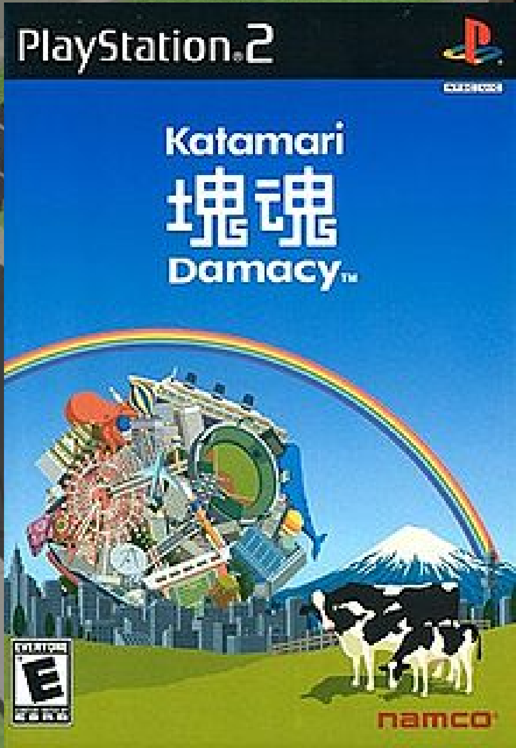
Speedrun is NP-hard: RPG



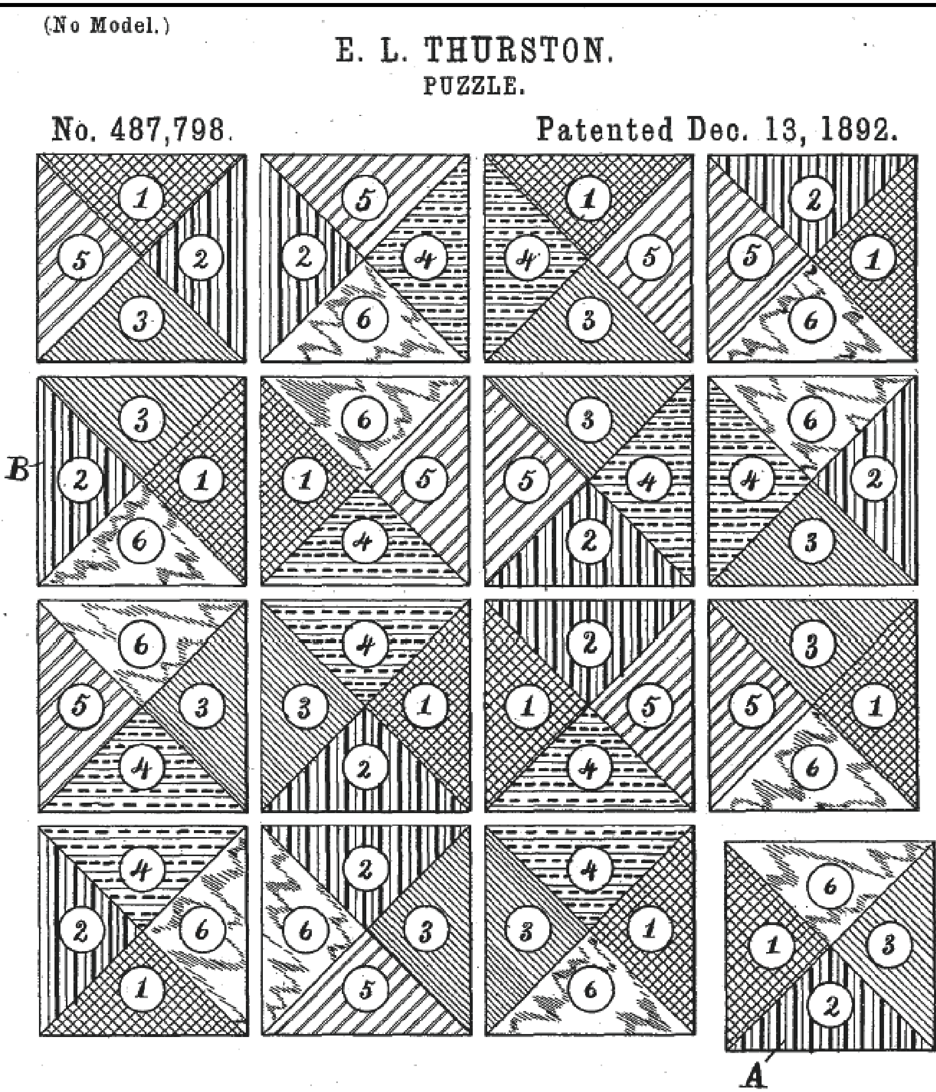


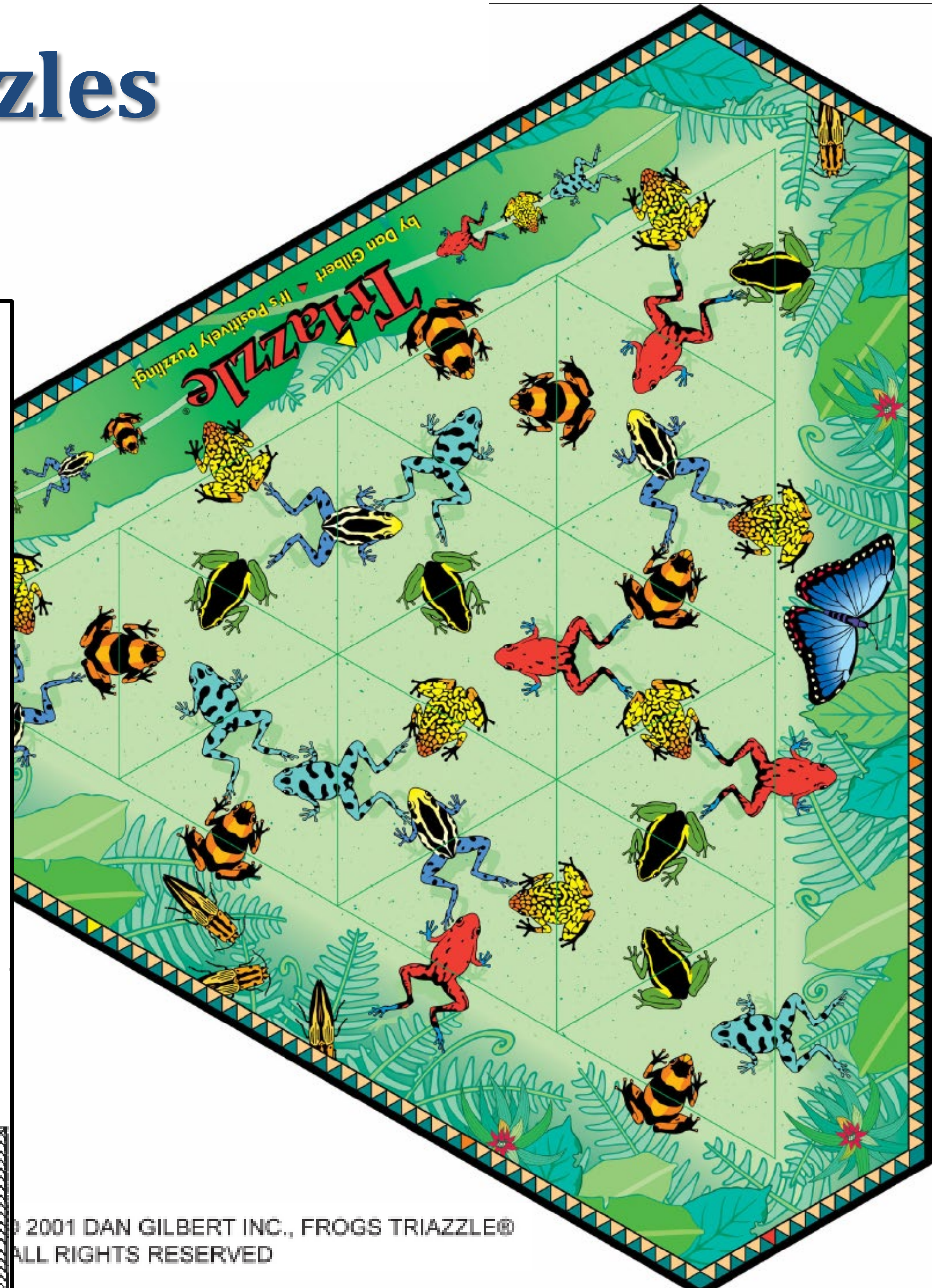


Playing is NP-hard: Katamari



Edge-Matching Puzzles
[1890s—]

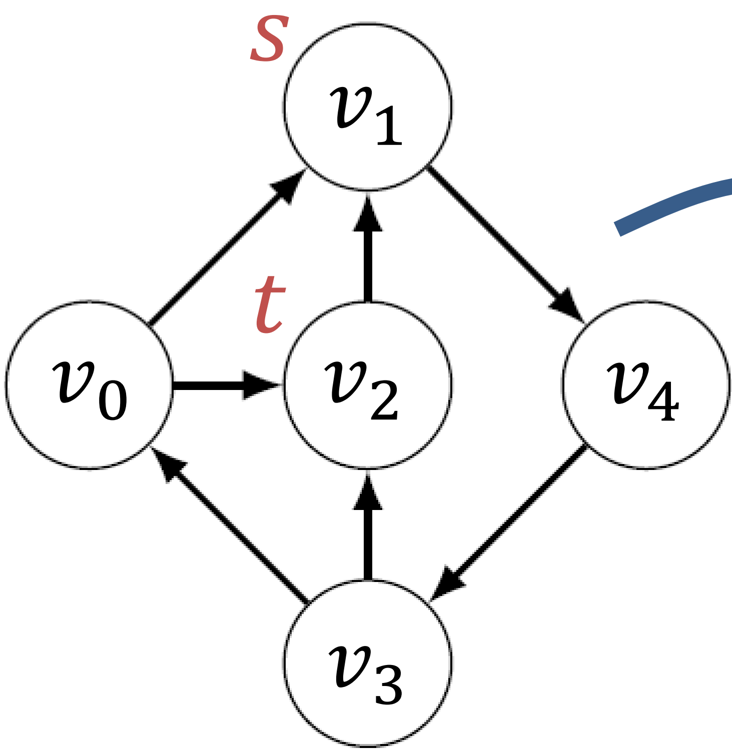


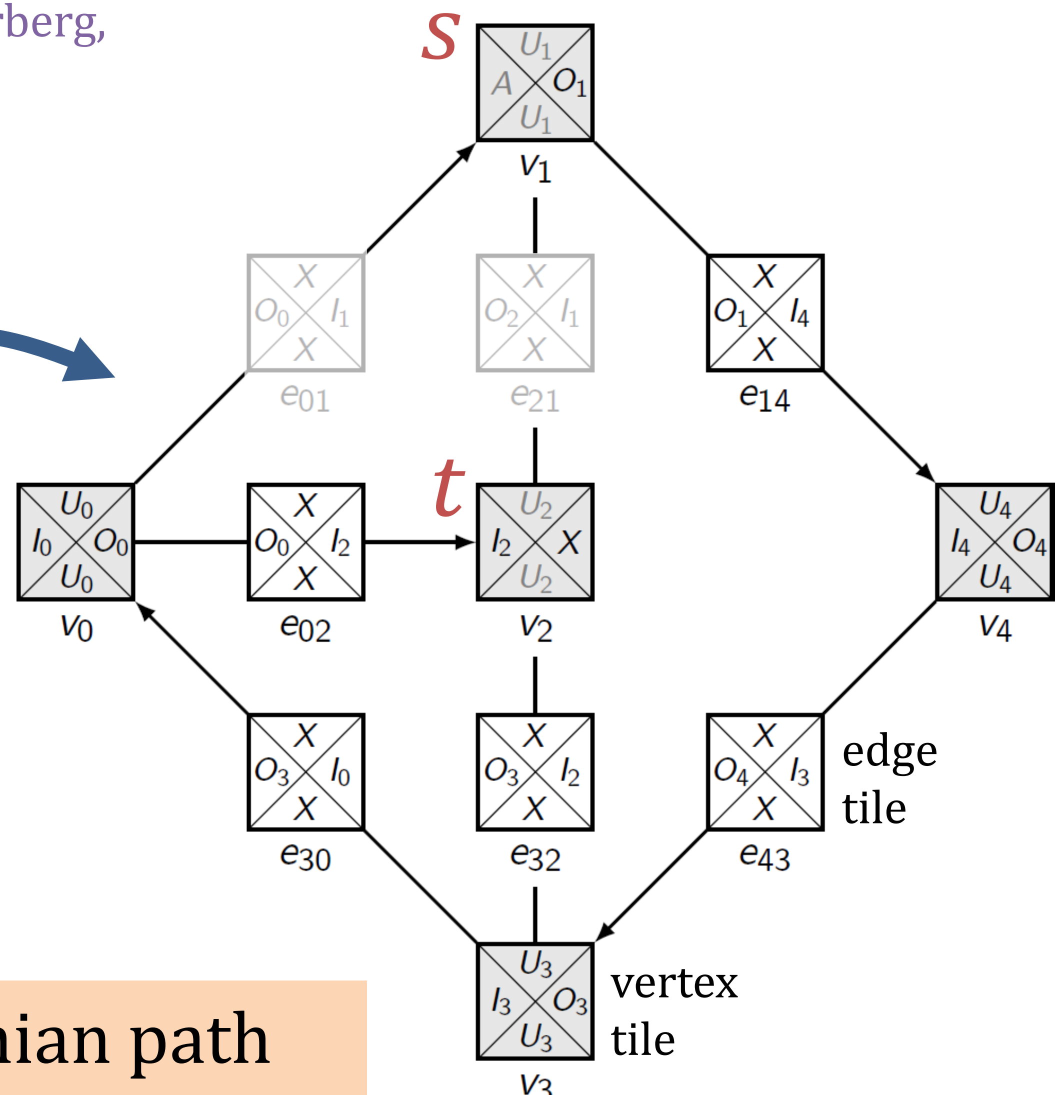



[Christopher Monckton, 2007]
US$2,000,000 prize (expired 2010)
ETERNITY II
WWW.ETERNITYII.COM
x256
photo by Matěj Baťha

NP-hardness of $1 \times n$ Edge Matching

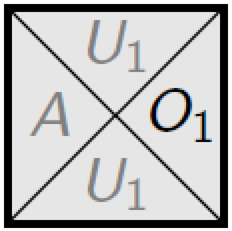
Bosboom, Demaine, Demaine, Hesterberg,
Manurangsi, Yodpinyanee 2017





Reduction from Hamiltonian path
with specified start s & end vertex t

NP-hardness of $1 \times n$ Edge Matching

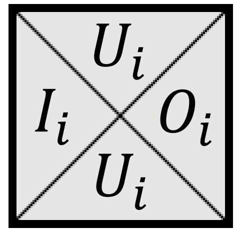


out

$s = v_1$

start vertex tile

in

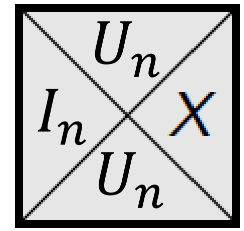


out

v_i

vertex tile

in

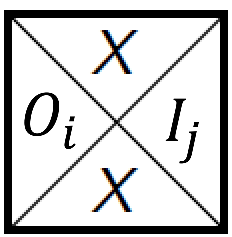


discard

$t = v_n$

end vertex tile

out



in

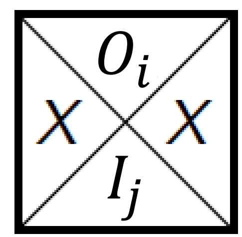
(v_i, v_j)

edge tile

in path

or

discard



discard

(v_i, v_j)

edge tile

not in path

NP-hardness

Bosboom, Demaine, Demaine, Hesterberg,
Manurangsi, Yodpinyanee 2017