Recall: (L18)

Bounded 2-player Constraint Logic (ACL)
- each edge is either white or black
- each edge can be reversed only once
- goal: each player has target edge & wins if they reverse it

- \( \text{PSPACE-complete for planar constraint graphs with white AND, SPLIT, OR, CHOICE} \)
  \& VARIABLE vertex
- reduction from impartial game positive CNF SAT
- players take turns setting variables
- positive \( \Rightarrow \) white wants true, black wants false
- black can't win (edge irreversible)
- white wins \( \Leftrightarrow \) formula satisfied
- crossover gadget (only use of \text{CHOICE})
- can make \text{OR} protected using \text{free edge}
  
  \( \text{no constraint at degree-1 end} \)
Amazons:  [Walter Zamkauskas 1988]
- queens on chessboard
- move = queen move + queen shot
  destroy board position at queen-reachable location
- last player to move wins

- PSPACE-complete  [Hearn 2005]
- polynomial # moves: shot consumes board
- reduction from Bounded 2CL
Konane  [Hawaii - ancient Hawaiian Polynesians]  
  (documented by Captain James Cook in 1778)

- **move** = jump your piece over 1 or more opponent pieces in a straight line:
  
  ![Diagram of Konane move]

  → remove captured opponent pieces

- last player to move wins

- **PSPACE-complete**  [Hearn 2005]
  - polynomial # moves: move consumes ≥1 piece
  - reduction from Bounded 2CL
  - conditional gadget for AND, SPLIT, shift :
    - can traverse input 2 → output 2 
      only after input 1 → output 1 (else captured)
    - ignore output 1 ⇒ AND 
    - prime input 2 ⇒ SPLIT
    - both ⇒ parity shift
Cross Purposes: [Michael Albert 2004]
- black stones = 1\times1\times2 towers
- white stones = fallen towers
- move = \[ \begin{array}{c}
\text{black} \\
\Rightarrow
\end{array} \] \[ \begin{array}{c}
\text{white} \\
\end{array} \] (right)
- Vertical player can only move up/down
- Horizontal player can only move left/right
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial \# moves: move consumes black stone
  - reduction from Bounded 2CL
  - H forced to help V after variable settings
  - protected OR (& free edge) to avoid second activation terminating leaving H \% move

Stochastic games: [Papadimitriou – JCSS 1985]
- one player (of 2) plays randomly "nature"
- PSPACE-complete to win with probability \( > \frac{1}{2} \) (via amplification)
- SSAT: \( \exists x_1 : \mathcal{R} x_2 : \exists x_3 : \mathcal{R} x_4 : \cdots : \Pr[\exists F] > \frac{1}{2} \)
- OPEN: real games?
Unbounded formula games: EXPTIME-complete
[Stockmeyer & Chandra - SICOMP 1979]

- start with arbitrary variable assignment
- can set variables to 0 or 1 many times (unlimited)
- all partizan: black & white variables,
  plus possibly “turn variable” \( t = 0 \) if player 2
  \( t = 1 \) if player 1

\[ G_1: \text{move} = \text{set all variables of your color} \]
& set (common) turn variable \( t = 0 \) if player 2
lose if you satisfy (common) 4DNF formula
i.e. move must satisfy common 4CNF formula

\[ G_2: \text{move} = \text{set one variable of your color} \]
(can pass by not changing it)
win if you satisfy your 12DNF formula (2 of them)

\[ G_3: \text{move} = \text{flip one variable of your color} \]
lose if you satisfy your 12DNF formula (2 of them)

\[ G_4: \text{move} = \text{set one variable of your color} \]
(can pass)
win if you satisfy (common) 13DNF formula
\((G_5 = G_6 \text{ but without CNF constraint}) \)

\[ G_6: \text{move} = \text{set one variable of your color} \]
(can pass)
player 1 wins if anyone satisfies (single) CNF formula

Peek: stack of plates with holes: 1 fixed plate;
\((G_4)\) black & white plates have 2 states, in & out
- move = manipulate one plate (can pass)
- win if hole all the way through
Membership in EXPTIME = ASPACE \[\text{alternating } \forall \exists \text{ guesses}\]
- build set of “mate in k” states for \(k=0, 1, \ldots, c^n\)
- moves \(\leq\) states \(\uparrow\)

Unbounded graph games: EXPTIME-complete
[Stockmeyer & Chandra - SICOMP 1979]

HAM:
- given simple undirected graph
- each edge black or white \& in or out
- move = toggle in/out of an edge of your color
- player 1 wins if in edges form
  a Hamiltonian cycle (after any move)
- reduction from G6

Block:
- given 3 graphs on the same vertex set
- each player has tokens of their color
  on some of the vertices (\(\leq 1\) token per vertex)
- move = move 1 token of your color along
  a path in one of the 3 graphs
  such that target \& intermediate vertices
  have no tokens
- player i wins if they get a token to a vertex \(v_i\)
- reduction from \(G_3\)
  - variable \& clause gadget
Real games that are EXPTIME-complete: ⇒ $\exists P$!

- Checkers [Robson - SICOMP 1981]
  - reduction from $G_3$ where about to lose after every turn
  - initially players adjust kings between T/F
  - then player mounts an attack: move A or B forcing opponent to follow path, fork as desired
  - if all attack vars. set & no defense vars. set i.e. DNF clause satisfied then get $x$ free moves
  - with $x$ free moves can trigger outer spiral ⇒ huge material advantage
  - then can form picket lines > size(interior) ⇒ win [Fraenkel, Garey, Johnson, Schaefer, Yesha - FOCS 1978]

- Chess [Fraenkel & Lichtenstein - JCTA 1981]
  - reduction from $G_3$

- Go with Japanese ko rule [Robson - IFIP 1983]
(Unbounded) 2CL:
- each edge is either white or black
- goal: each player has target edge & wins if they reverse it
- EXPTIME-complete even for planar graphs
  - reduction from G6
  - players flip variables
  - if formula satisfied: white (Player 1) will lock all variables & run formula
  - lock = reverse true or false edge
  - black must respond A (then B, C, D) to prevent white from fast win via F
    \[ \Rightarrow \text{black immobilized during locks} \]
- black’s slow win is 1 move longer than formula satisfaction \( \Rightarrow \) white can’t flip its variables after any locking (no time)
- white slower win prevents black from flipping A early, e.g. instead of flipping a variable
- formula uses path equalizer so all satisfying assignments take same time
- NCL crossover
No-repeat rule: \[\text{lose if ever repeat a past game configuration}\]
\[\Rightarrow G_1, G_2, G_3 \text{ become EXPSPACE-complete as do Chess & Checkers}\]
- OPEN: is Go with superko (no-repeat) \[\text{EXPSPACE-complete}\] (as in USA & China)?

Conditional no-repeat rule: \[\text{[Robson - MFCS 1984]}\]
- two special variables \(x\) & \(y\)
- lose if ever repeat a past game configuration
  & at most 1 of \(x\) & \(y\) have changed since
\[\Rightarrow G_1 \text{ becomes 2EXPTIME-complete}\]

Private-information games: \[\text{[Reif - JCSS 1984]}\]
you can see some but not all of opponent's state
\[\Rightarrow G_1 \text{ 5DNF, } G_2 \text{ DNF become 2EXPTIME-complete}\]
\[\Rightarrow \text{version of Peek with half of winning holes visible to each player}\]

Blind games: \[\text{[Reif - JCSS 1984]}\]
player 1's entire state is hidden from player 2
\[\Rightarrow G_2 \text{ DNF becomes EXPSPACE-complete}\]
\[\Rightarrow \text{version of Peek above}\]
- OPEN: Constraint Logic in all these settings