Today: video games & PSPACE. First some NP:

**Metatheorem 1:** \([\text{Viglietta-Fun 2012 & arXiv:1201.4995}]\)

- player traversing planar environment from start
- location traversal & single-use paths \(\Rightarrow\) NP-hard
  - player must visit some locations
  - player can traverse only once

- reduction from Planar Max-deg-3 Hamiltonicity
  - vertex \(\Rightarrow\) location traversal
    - visit each vertex \(\geq\) once
  - edge \(\Rightarrow\) single-use path
    - max. degree 3 \(\Rightarrow\) never revisit vertex

- applications
  - Boulderdash
  - Lode Runner
  - Zelda II \([\text{Aloupis, Demaine, Guo, Viglietta 2019}]\)

**Metatheorem 2:** \([\text{Viglietta-Fun 2012 & arXiv:1201.4995}]\)

- location traversal & tokens + toll roads \(\Rightarrow\) NP-hard
  - can pick (one) up \(\Rightarrow\)
  - need token to traverse

- vertex \(\Rightarrow\) location traversal + token
- edge \(\Rightarrow\) toll road
  - traversing twice \(\Rightarrow\) stranded without token
- application: Pac-Man
Recall from L1:

$\text{PSPACE} = \{\text{problems solvable in polynomial space}\}$
- $\subseteq \text{EXP}$: only exponentially many states
- $\supseteq \text{NP}$: simulate all executions, take running or
- $= \text{NPSPACE}$ [Savitch 1970]

Base $\text{PSPACE}$-complete problems:
- simulate linear-space algorithm (e.g. Turing machine)
- QSAT: (AKA QBF & TQBF)
  - given (fully) quantified Boolean formula, is it true?
  - e.g. $\forall x \exists y : (\overline{x} \lor y) \land (x \lor \overline{y})$ $(x=y)$
  - can assume quantifiers in front (prenex) & alternate $\forall/\exists$ ($\exists$ only $\Rightarrow$ SAT $\Rightarrow$ NP-comp)
- Schaefer-style dichotomy theorem:
  - $\in \text{P} \iff$ Horn, dual-Horn, 2-CNF, or X(N)OR
    (not if satisfied by all true/all false)
  - PSPACE-complete otherwise [Chen-C.Surveys 2009]
- planar Q3SAT [Schaefer - SICOMP 1981] [L7]
- add $\exists$ for new variables at end of quantifiers
- planar 1-in-3 Q3SAT (as in L7)
**Metatheorem 3:** (Viglietta - Fun 2012 & arXiv:1201.4995)

- player traversing planar environment from specified start to specified goal
- door + open pressure plate + close pressure plate traversable \( \Rightarrow \) walk on it only if open \( \Rightarrow \) open specific door \( \Rightarrow \) ditto, close
- reduction from Q3SAT
- clause gadget
- existential quantifier gadget
- universal quantifier gadget
- one plate of each type for each door
- applications:
  - many FPSs e.g. Doom, Quake, Heretic, Hexen, ...
  - many RPGs e.g. Eye of the Beholder
  - many adventure games e.g. SCUMM engine (Maniac Mansion, Monkey Island, Space Quest, ...)
  - Prince of Persia
Metatheorem 4: buttons instead of pressure plates
- pressure plate gadget
- in fact 2 doors per button suffice
  [Bodlaender & van der Zanden - unpublished 2014]
- applications: MANY
  - Sonic the Hedgehog (Sega Genesis)
  - The Lost Vikings (Super NES; PC) "Erik the Swift"
  - Tomb Raider (Sega Saturn & PS1; PC)

Metatheorem 5: [Aloupiis, Demaine, Guo, Viglietta 2014]
- door with traverse, open, close paths \( \Rightarrow \) \( \text{PSPACE-hard} \)
  only if open \( \Rightarrow \) can open, \( \Rightarrow \) must close

- applications:
  - Legend of Zelda: A Link to the Past
    (Ocarina of Time, Majora's Mask, Oracle of Seasons, The Minish Cap, Twilight Princess \( \Rightarrow \) PushPush-1)
  - Donkey Kong Country 1, 2, 3
  - Super Mario Bros. [Demaine, Viglietta, Williams - unpublished, 2014]
  - Lemmings [Viglietta - Fw 2014]