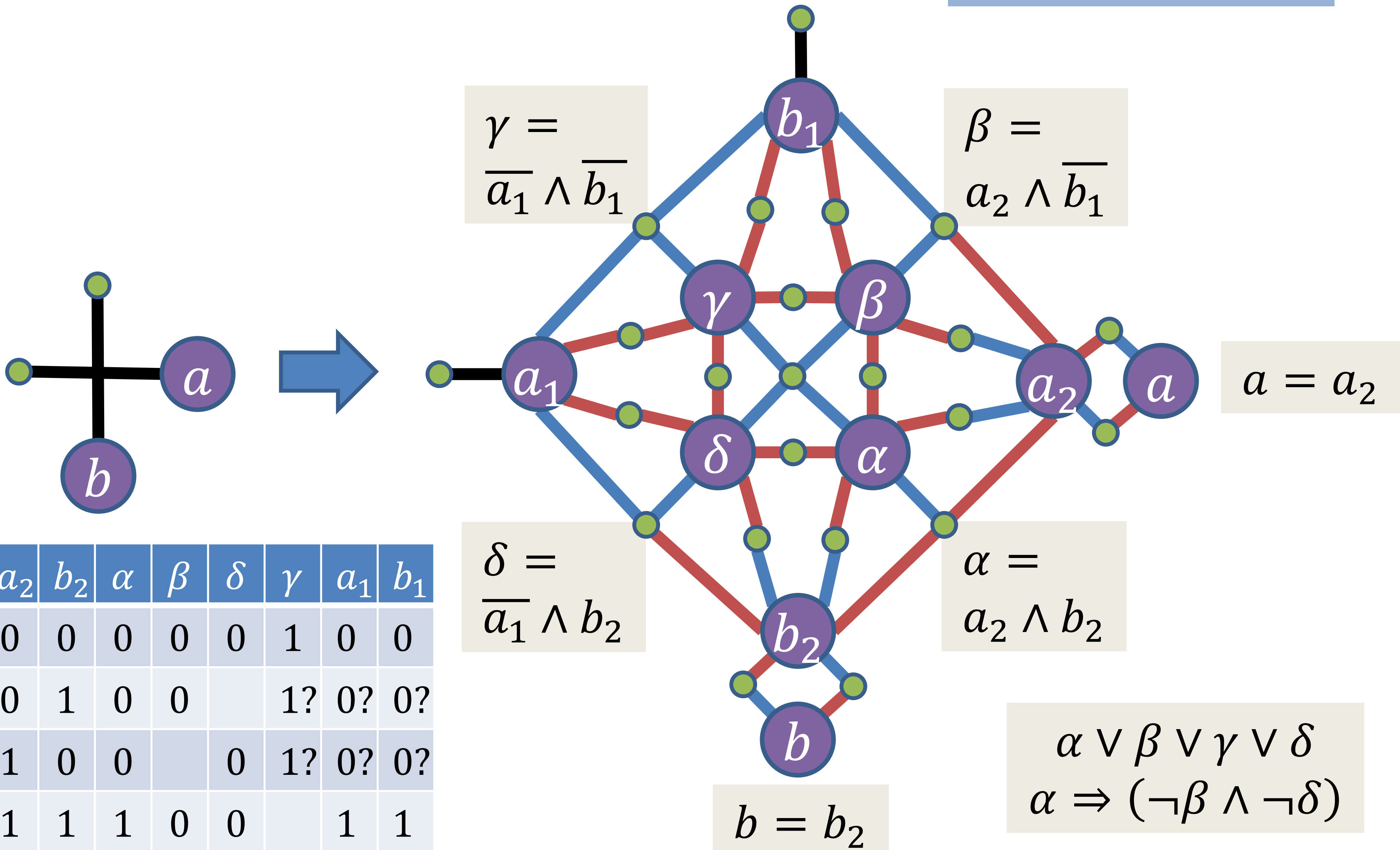
Planar 3SAT is NP-hard

[Lichtenstein 1982]

parsimonious



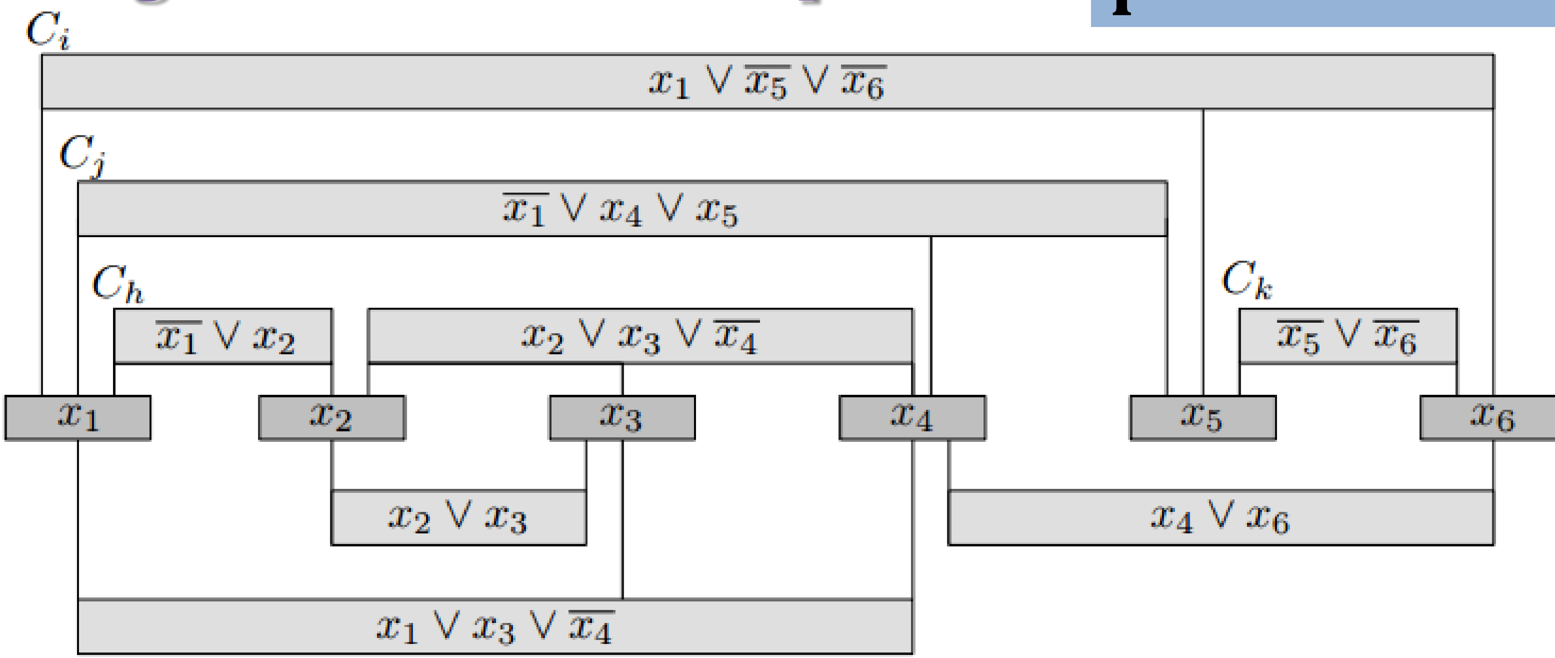
a_2	b_2	α	β	δ	γ	a_1	b_1
0	0	0	0	0	1	0	0
0	1	0	0		1?	0?	0?
1	0	0		0	1?	0?	0?
1	1	1	0	0		1	1

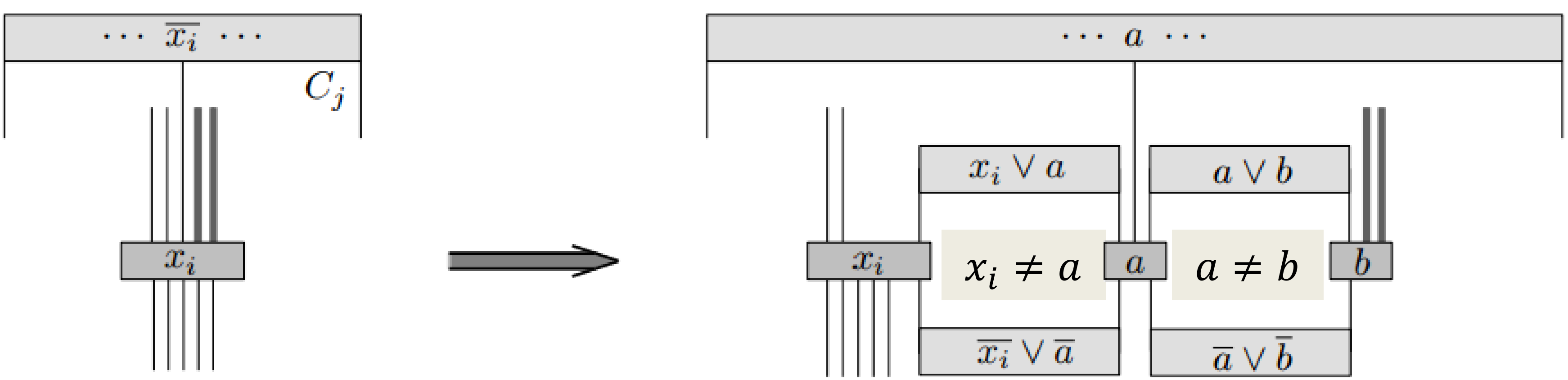
$$\alpha \vee \beta \vee \gamma \vee \delta$$
$$\alpha \Rightarrow (\neg\beta \wedge \neg\delta)$$

Planar Monotone Rectilinear 3SAT

[de Berg & Khosravi 2010]

parsimonious

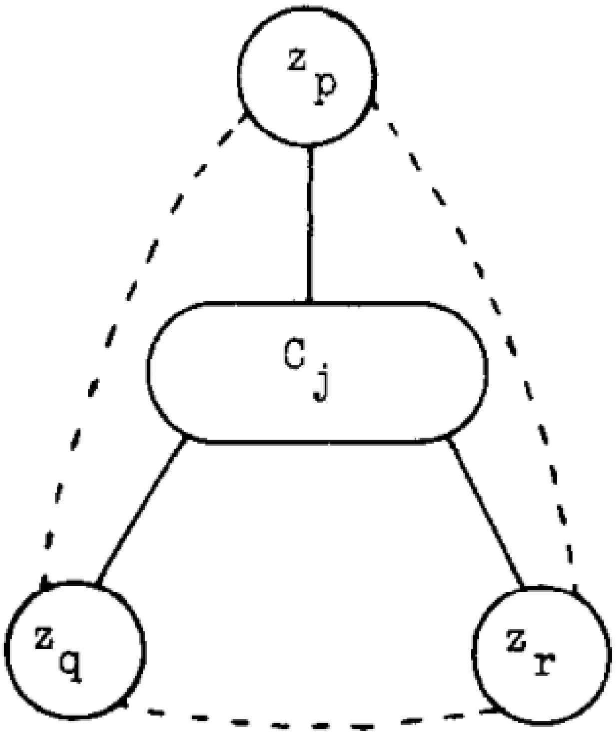


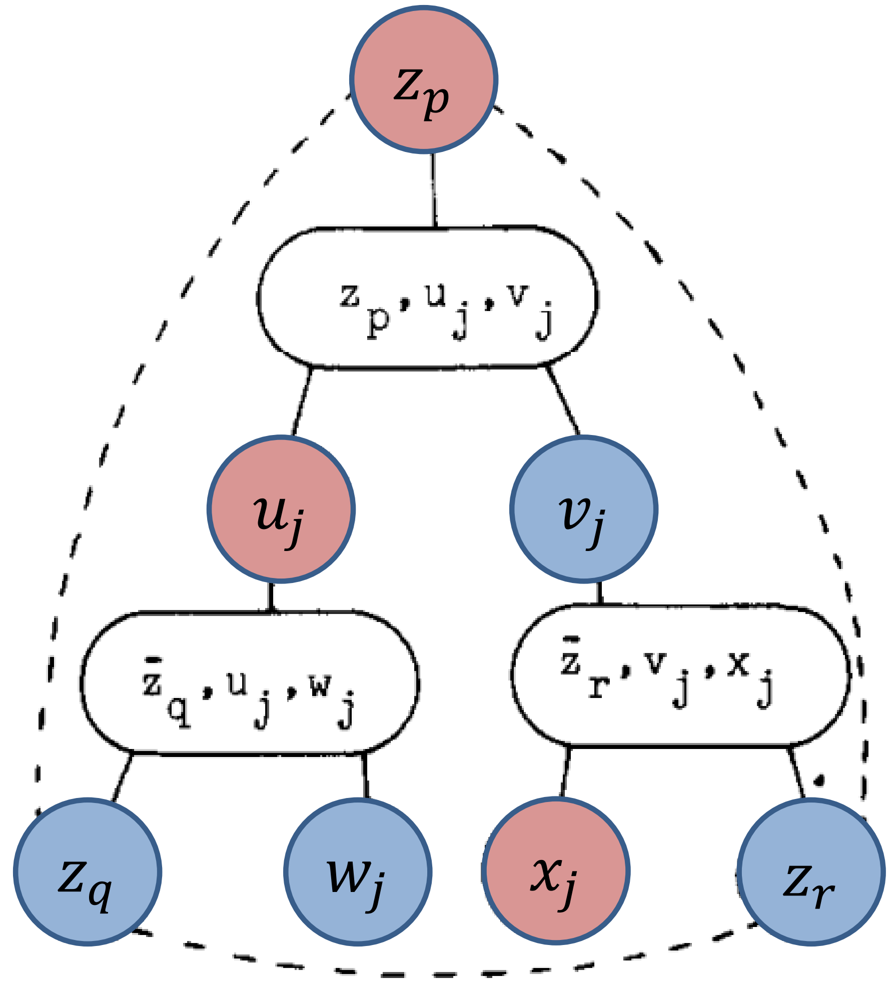


Planar 1-in-3SAT

[Dyer & Freeze 1986]

not parsimonious



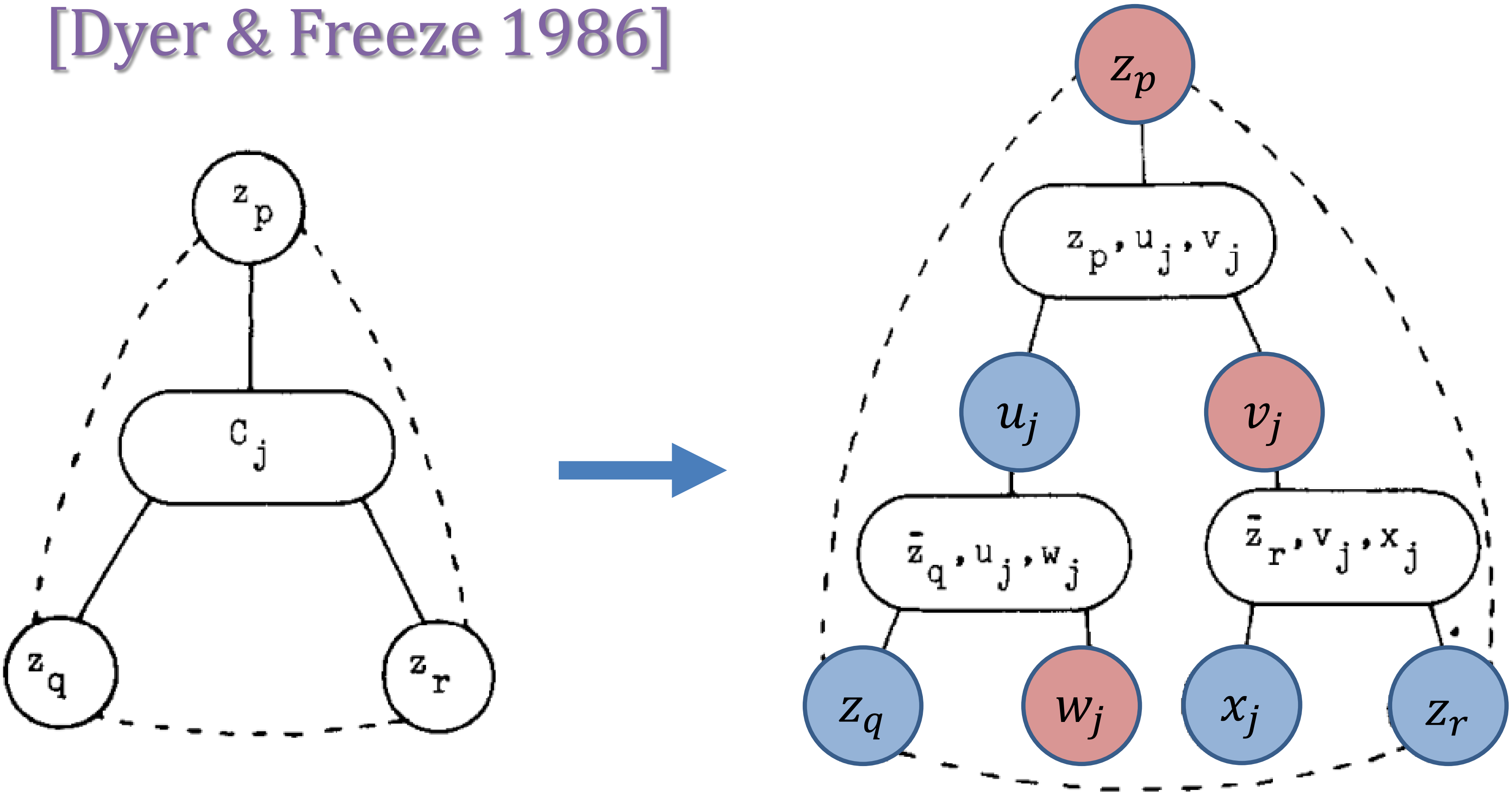


$C_j = \{z_p, z_q, z_r\}$ $\{z_p, u_j, v_j\}, \{\bar{z}_q, u_j, w_j\}, \{\bar{z}_r, v_j, x_j\}$

Planar 1-in-3SAT

[Dyer & Freeze 1986]

not parsimonious

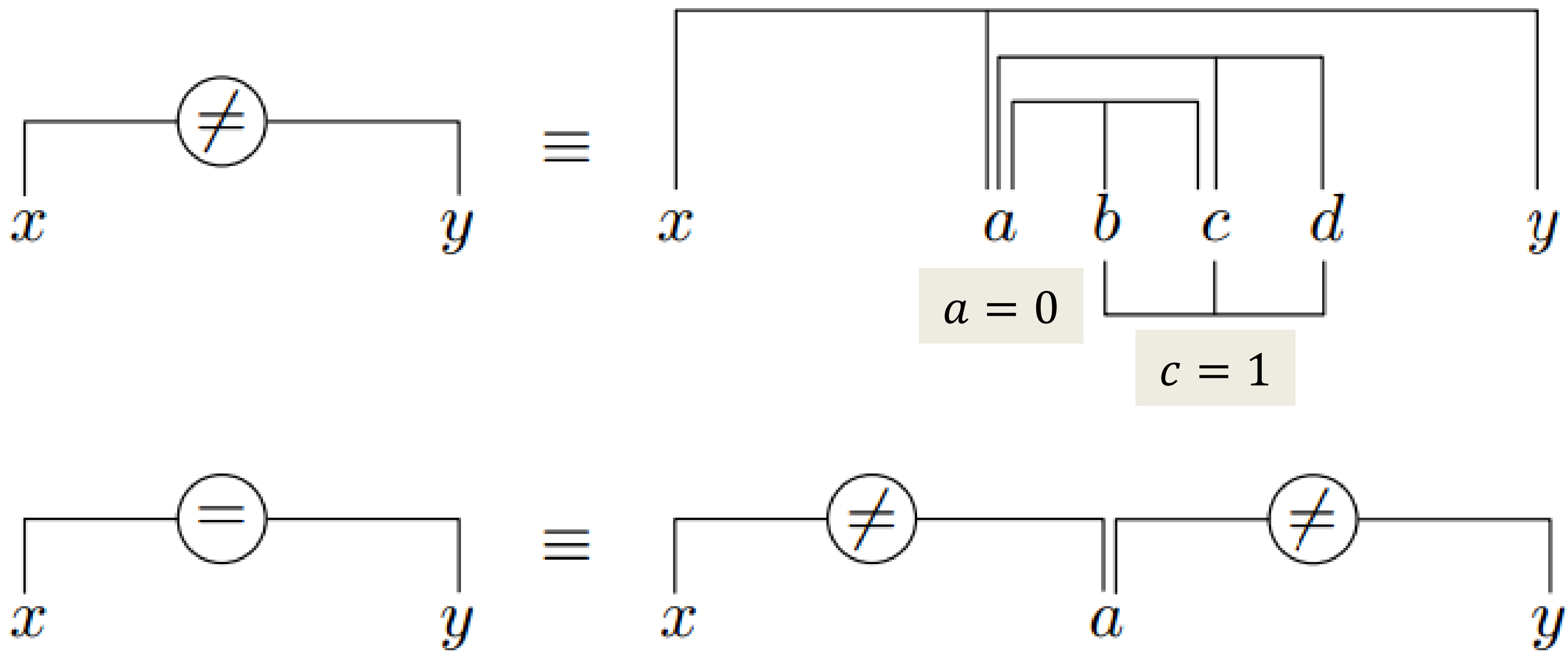


$C_j = \{z_p, z_q, z_r\}$ $\{z_p, u_j, v_j\}, \{\bar{z}_q, u_j, w_j\}, \{\bar{z}_r, v_j, x_j\}$

Planar Positive Rectilinear 1-in-3SAT

[Mulzer & Rote 2008]

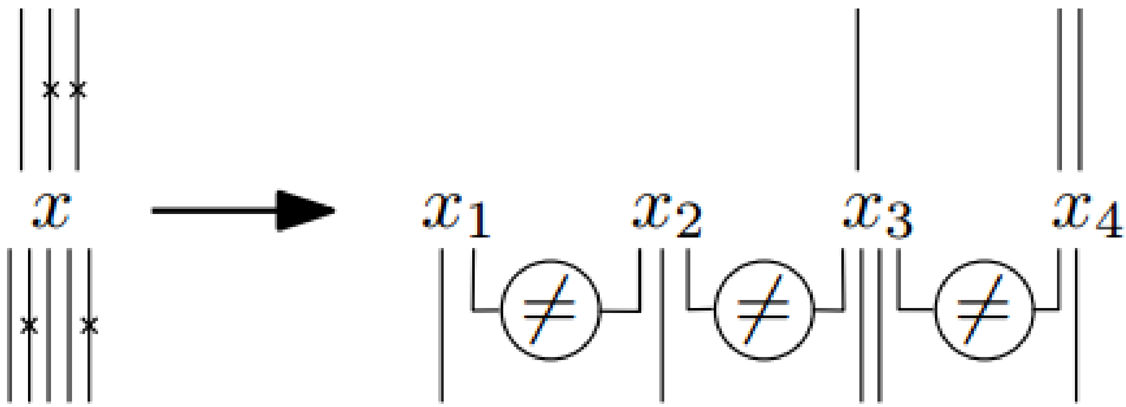
parsimonious

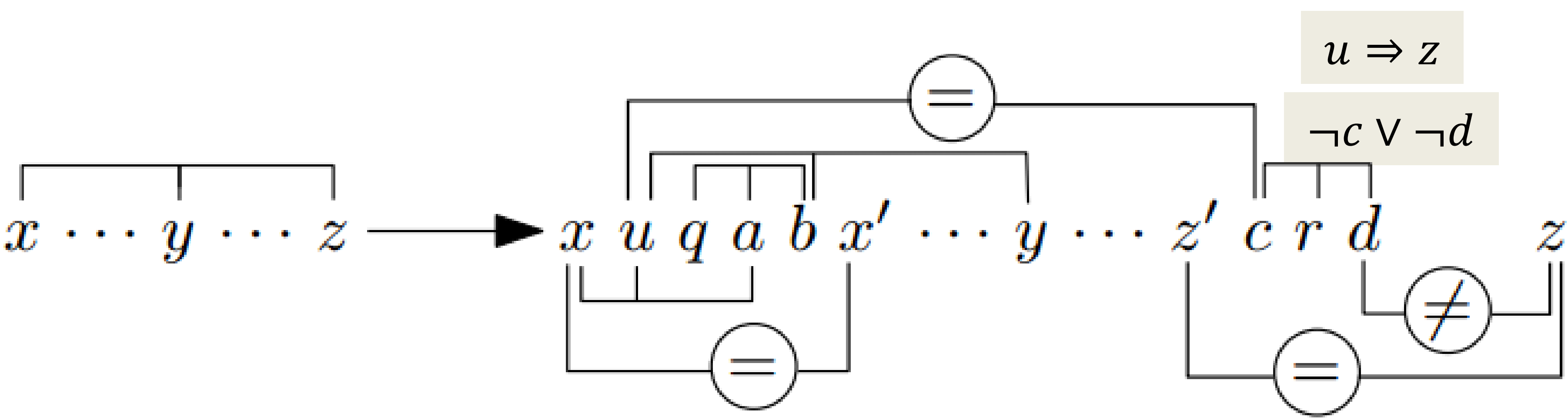


Planar Positive Rectilinear 1-in-3SAT

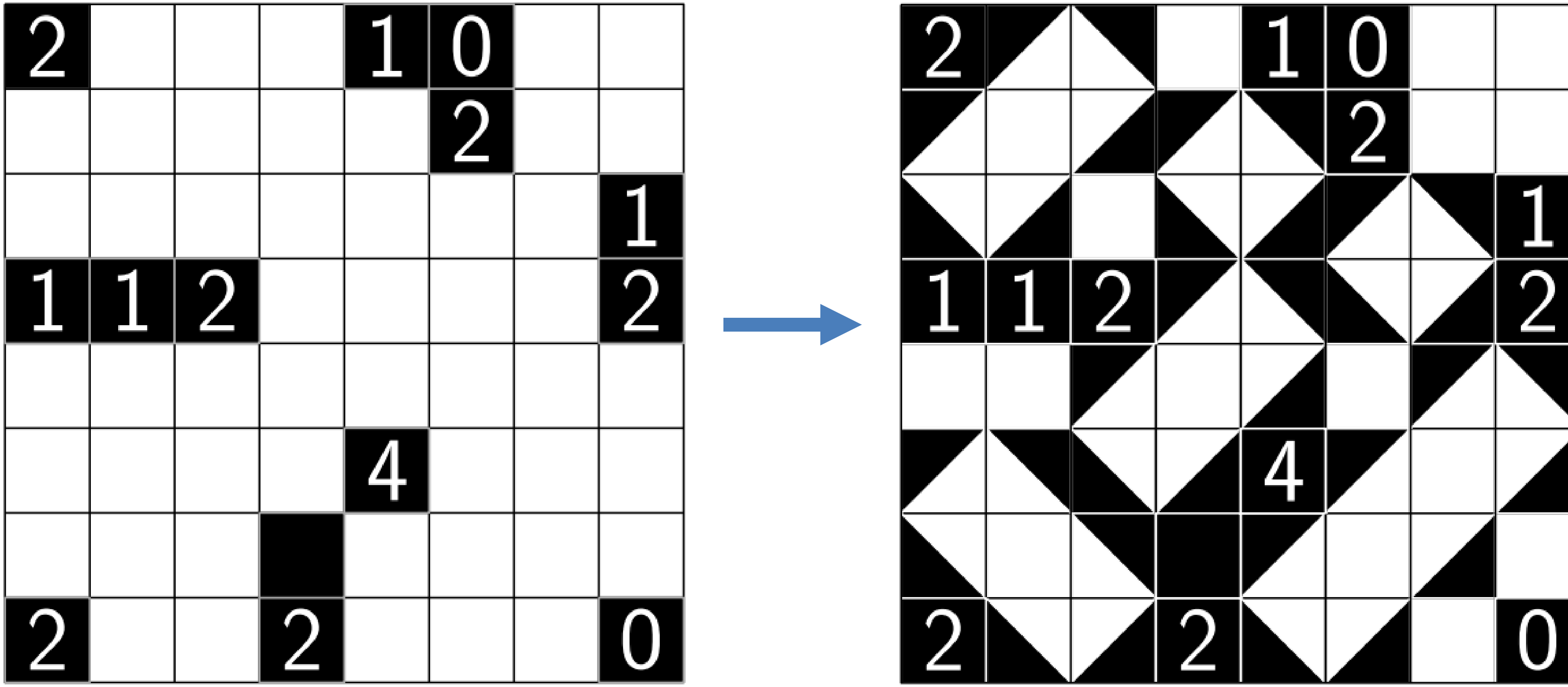
[Mulzer & Rote 2008]

parsimonious





Shakashaka [Guten 2008; Nikoli 2012–]



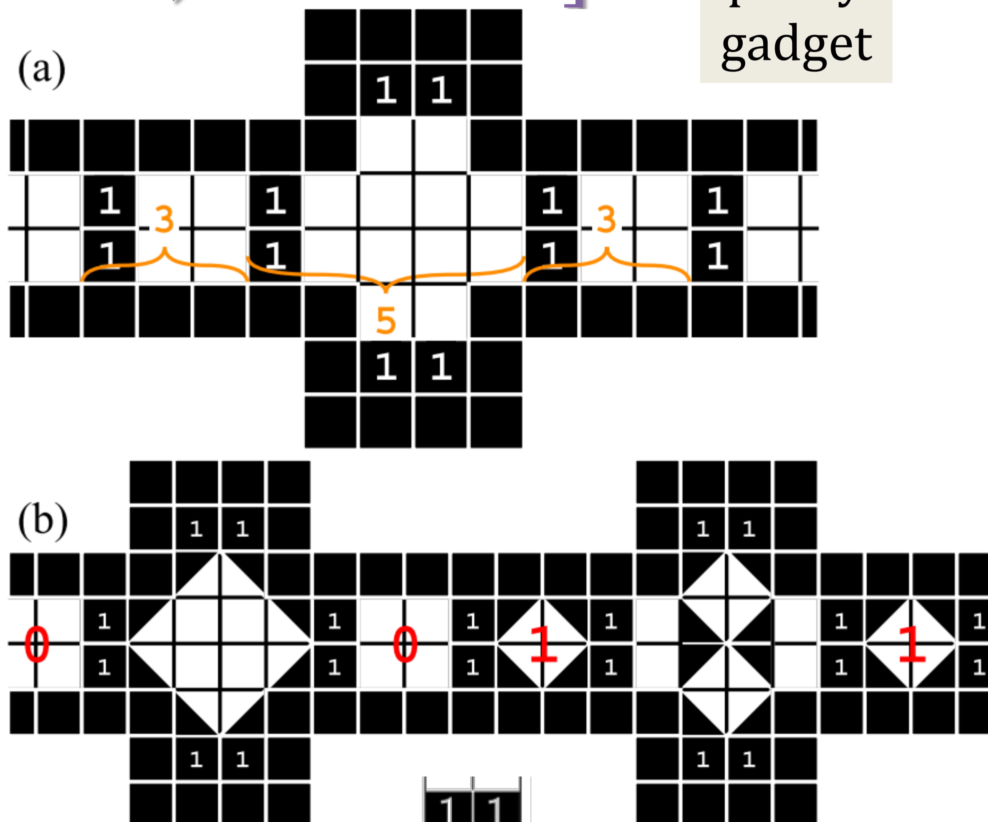
- White squares
- Black squares
  - Labeled 0, 1, 2, 3, 4, or nothing

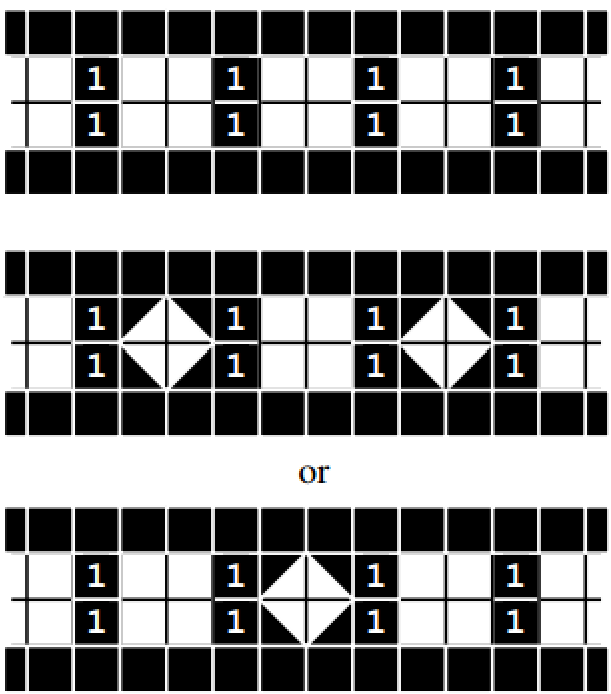
- Half-fill some white squares
- Labels specify number of half-filled neighbors
- Rectangular white regions

Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]

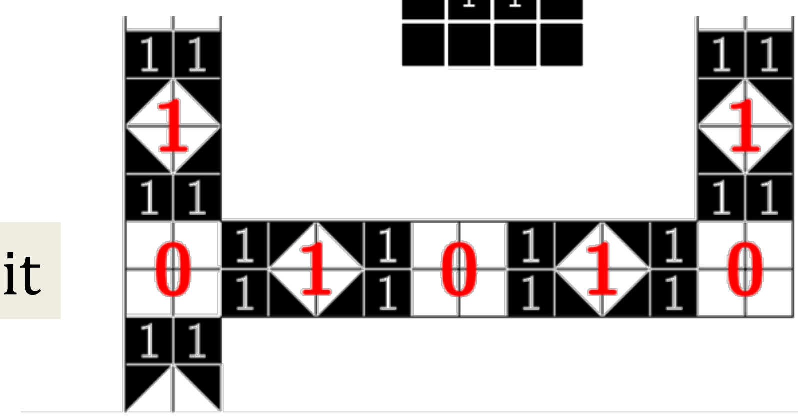
parity gadget





wire

split



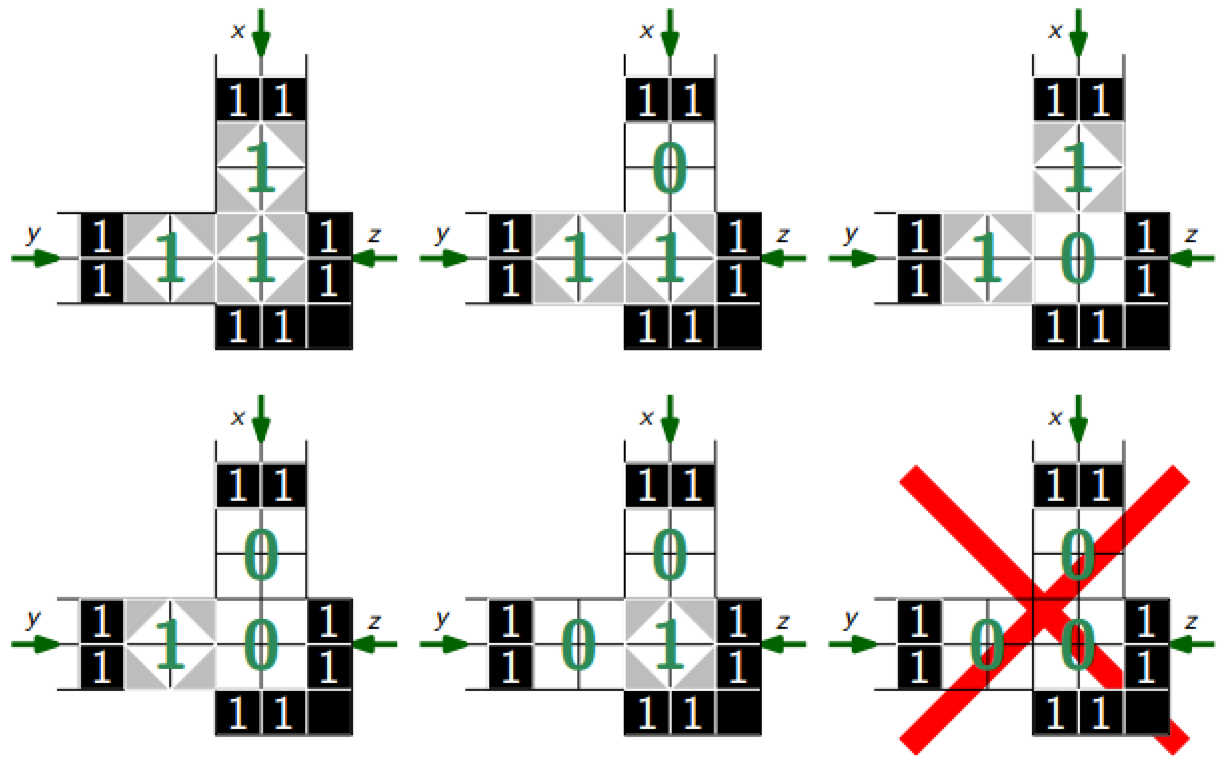
turn

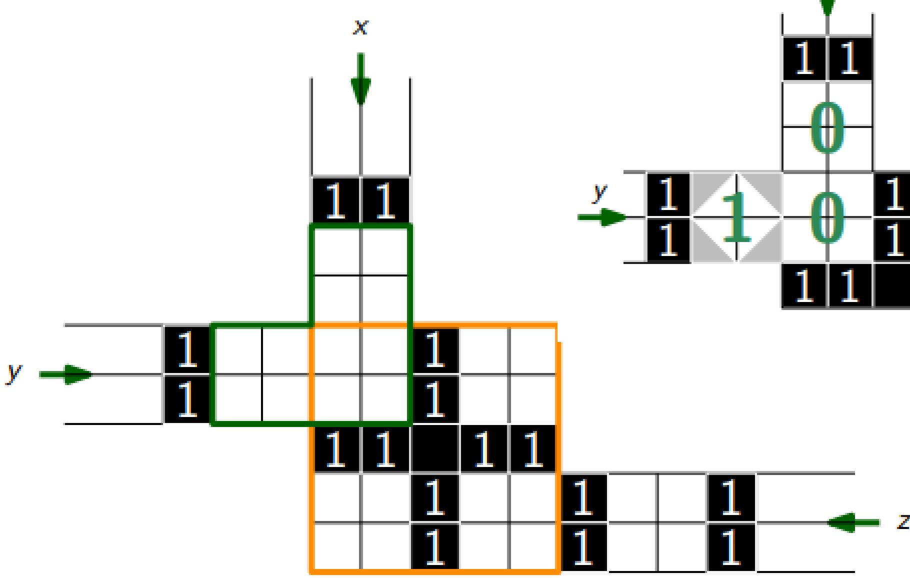
Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]

clause

$x \vee y \vee z$

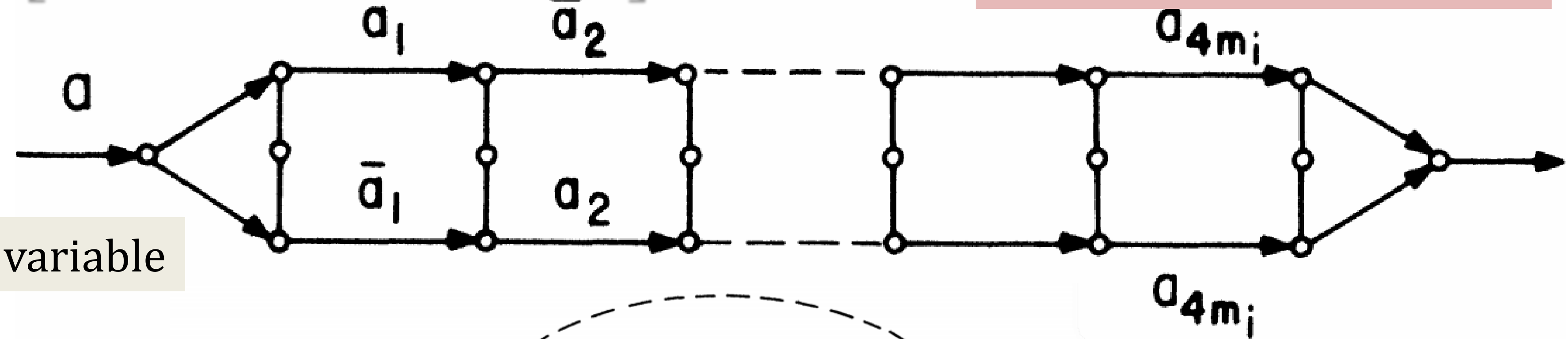




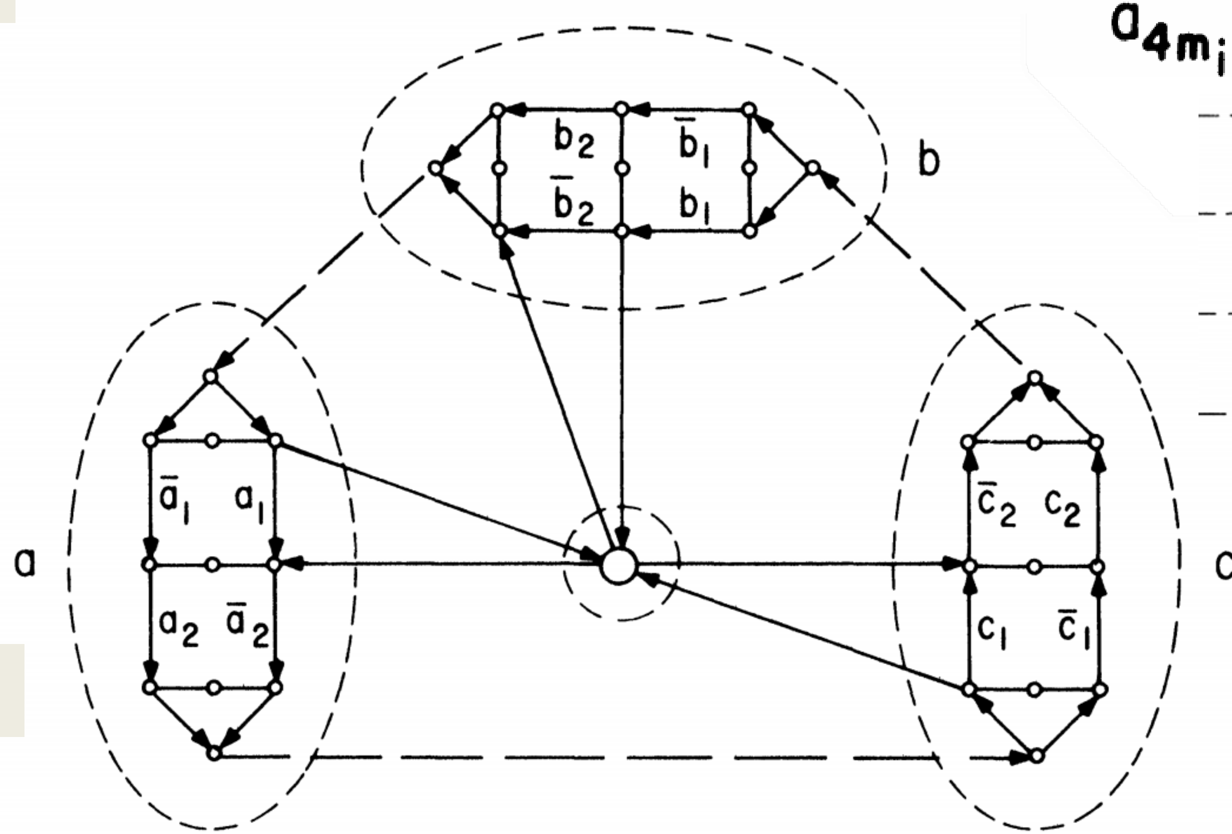
Planar (Directed) Hamiltonian Cycle

[Lichtenstein 1982]

not parsimonious

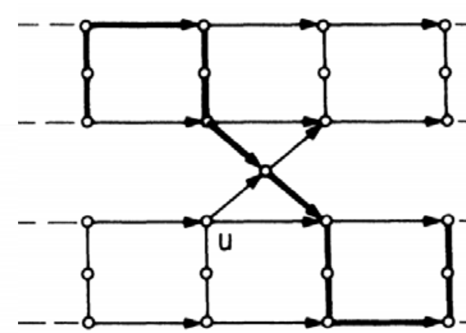


variable



clause

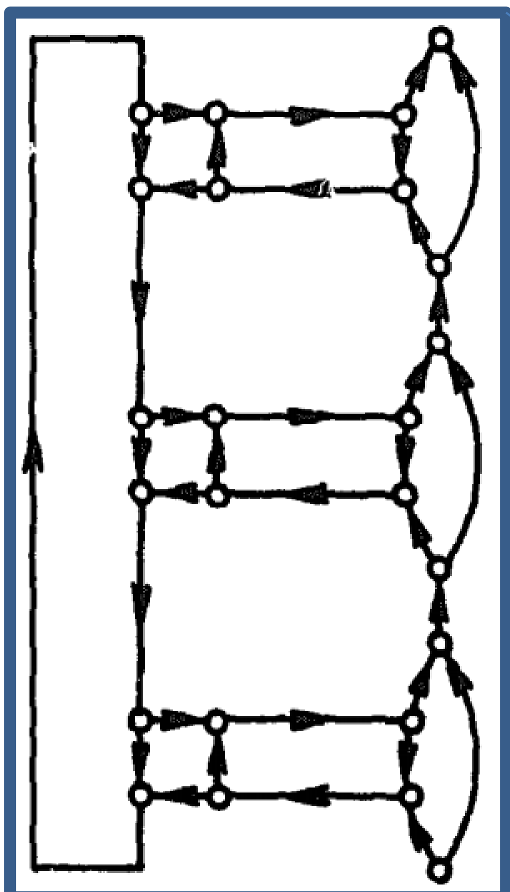
$a \vee \bar{b} \vee c$



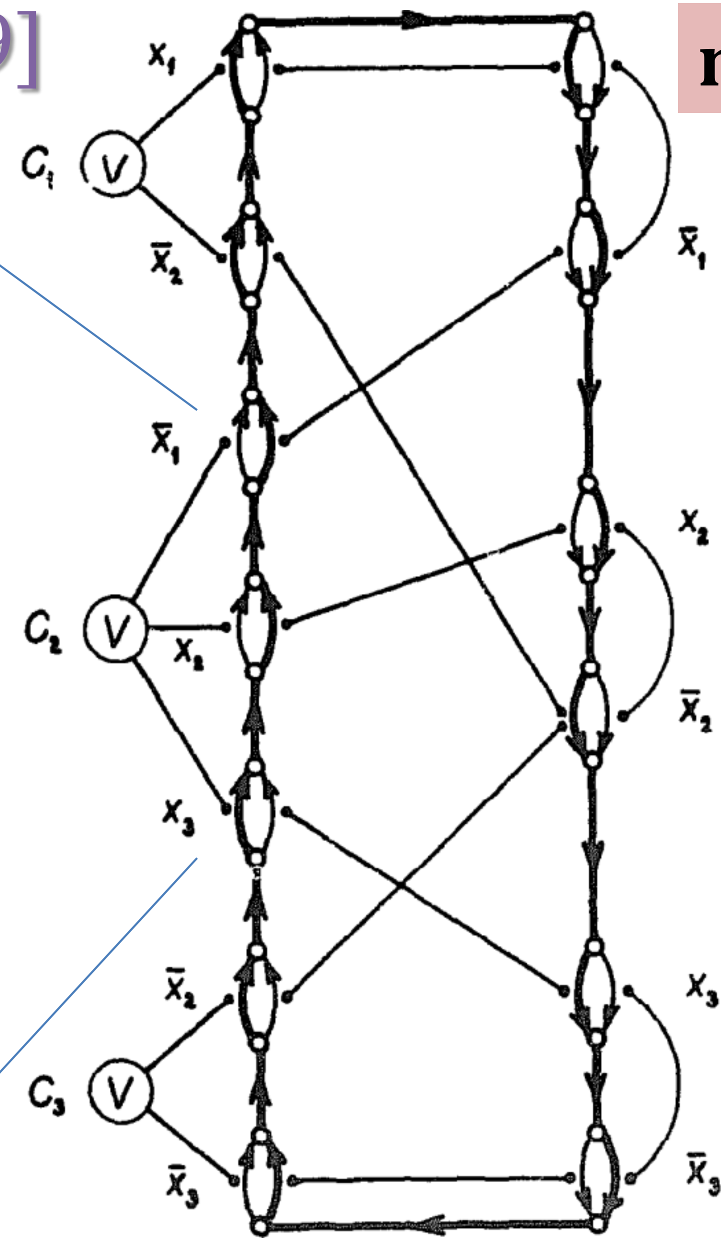
Planar Directed Max-Degree-3

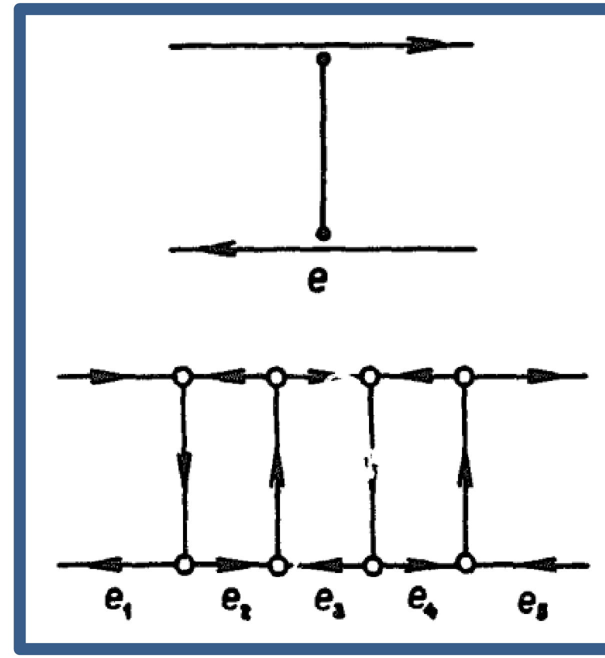
[Plesńik 1979]

not parsimonious



clause gadget



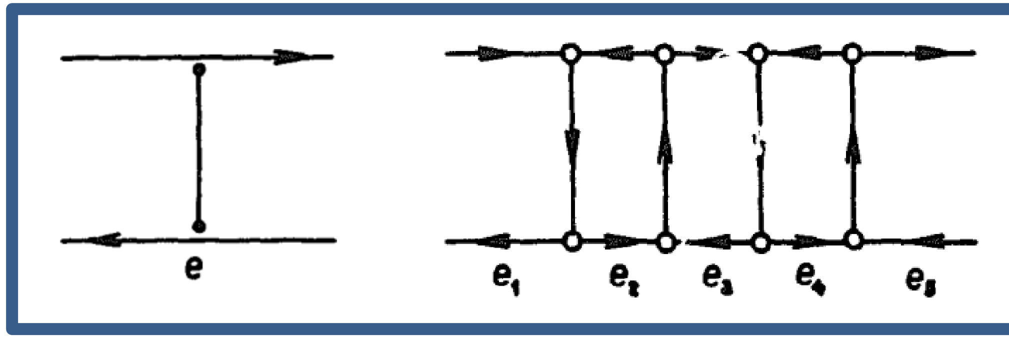


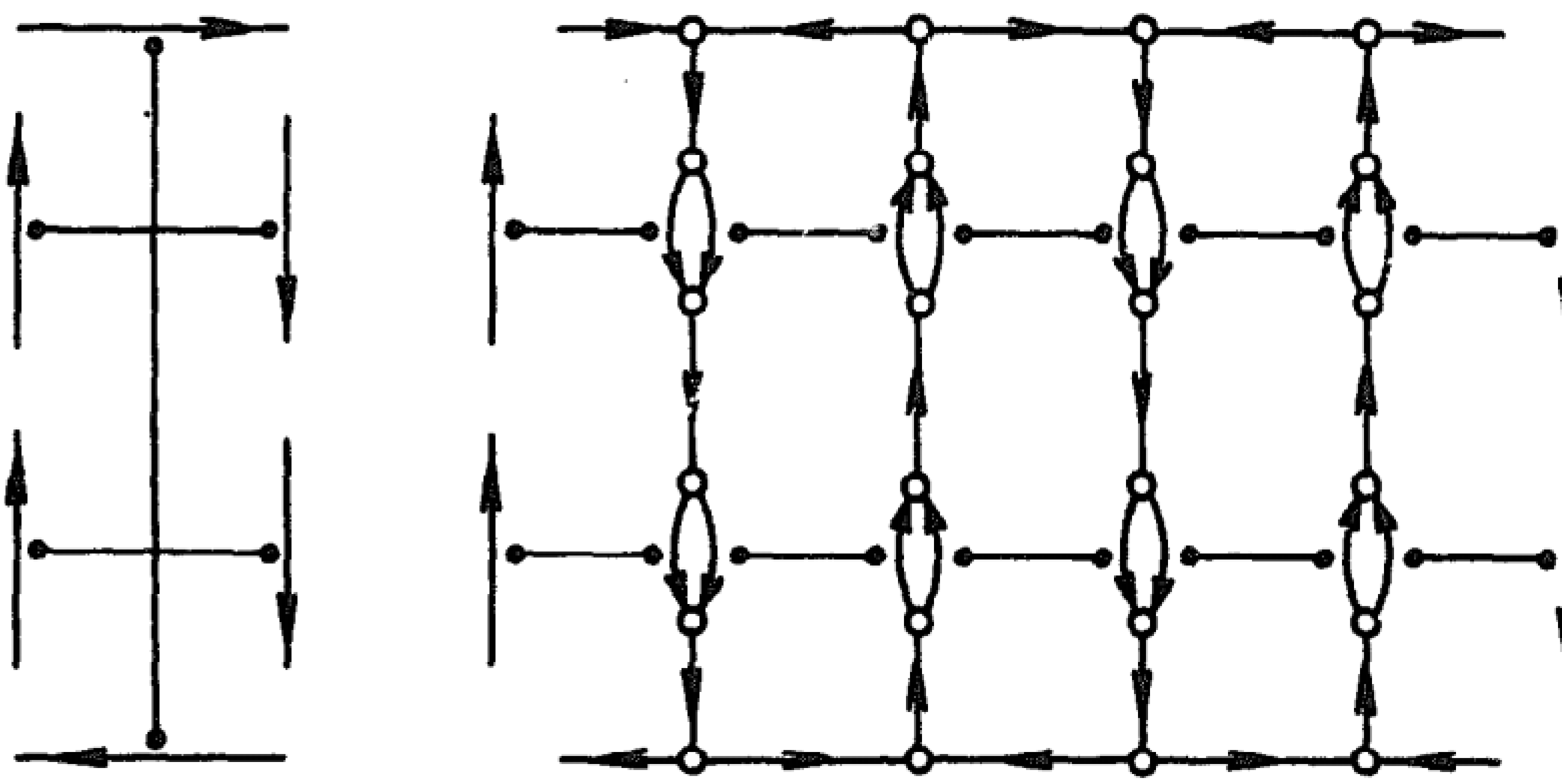
XOR gadget

$$(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})$$

Planar Directed Max-Degree-3

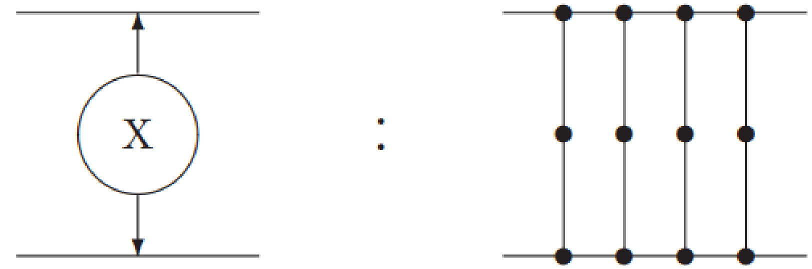
[Plesńik 1979]



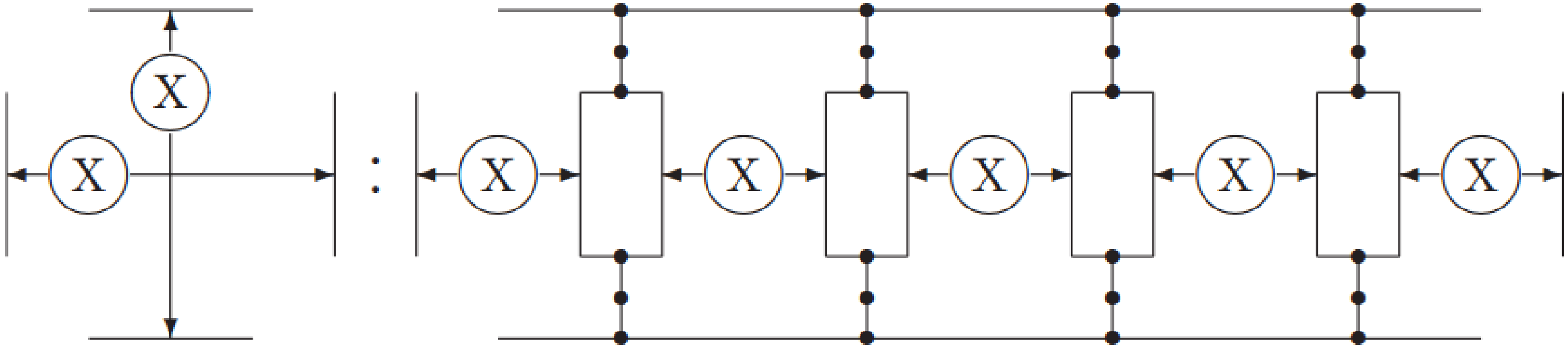


Planar # Ham. Cycles [Sato 2002]

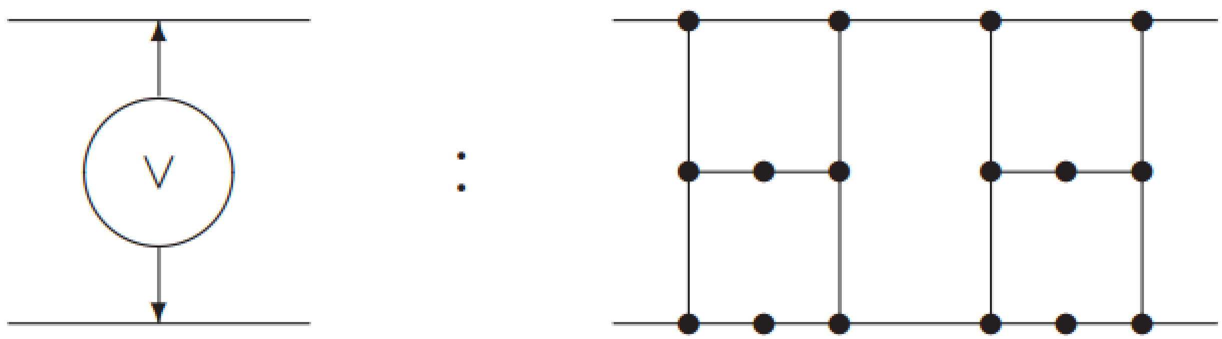
XOR



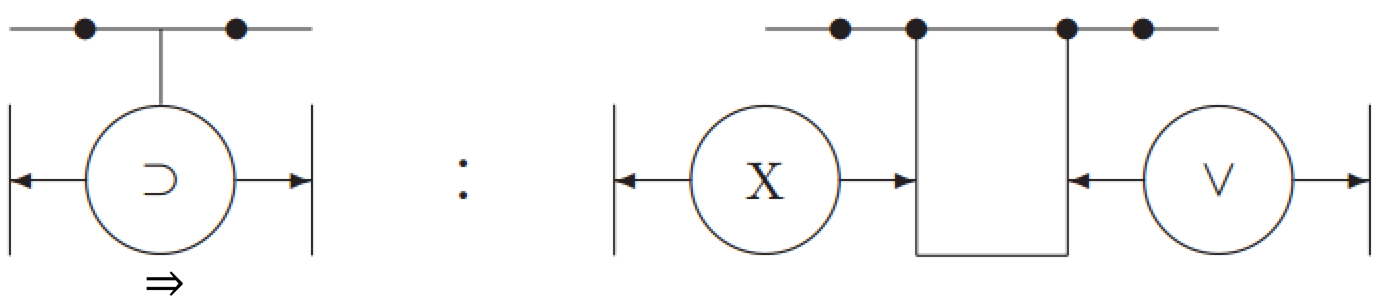
crossing XORs



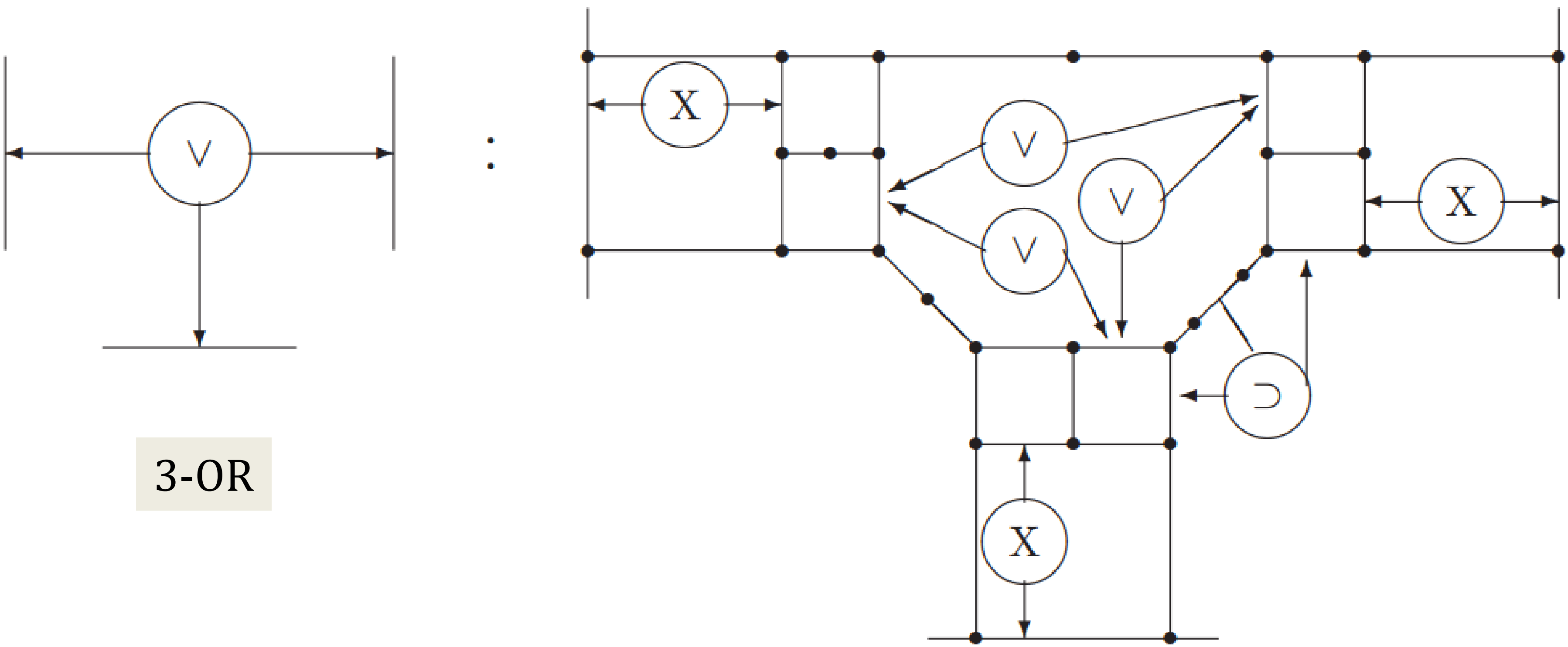
OR



imply

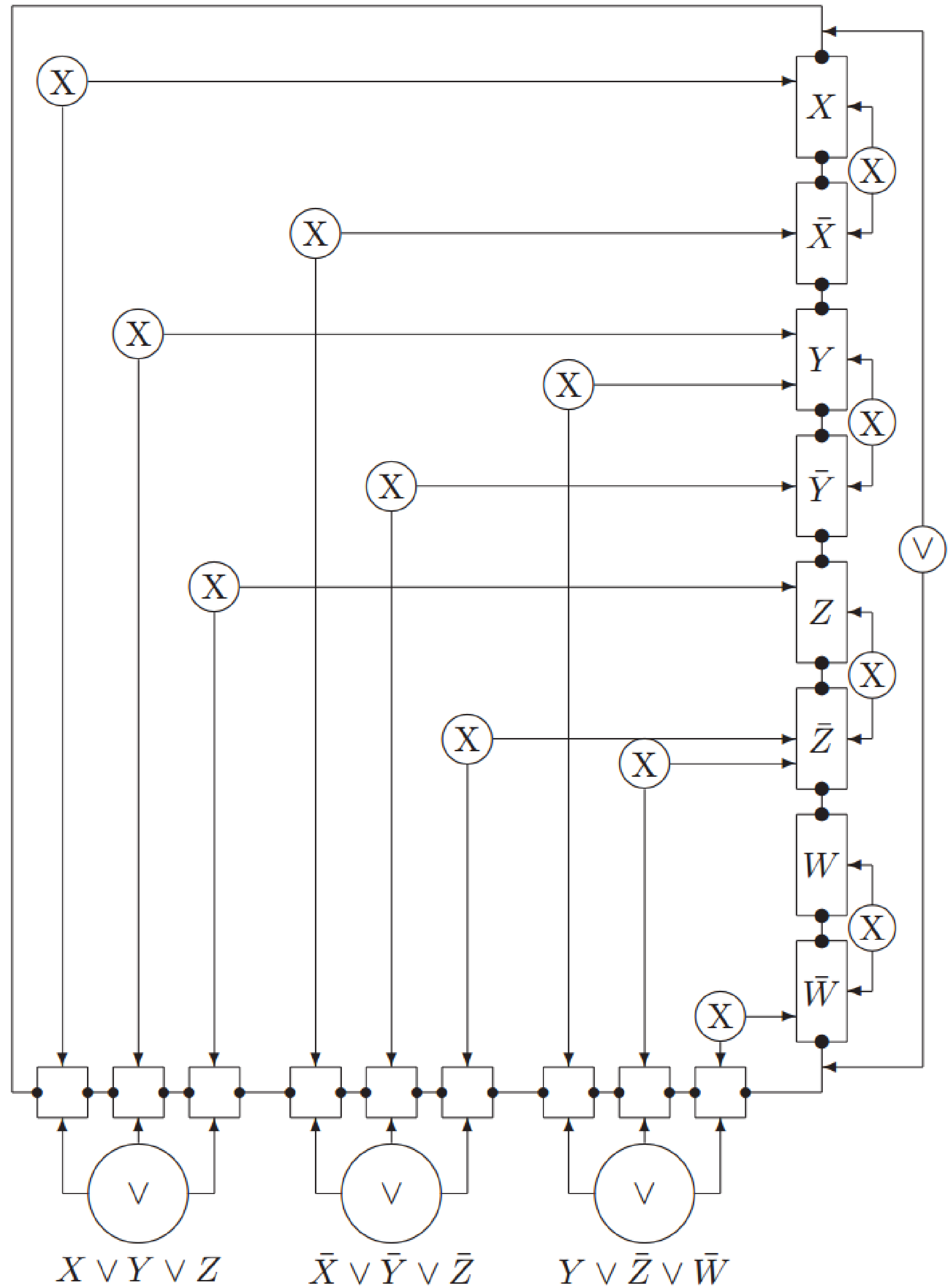


Planar # Ham. Cycles [Sato 2002]

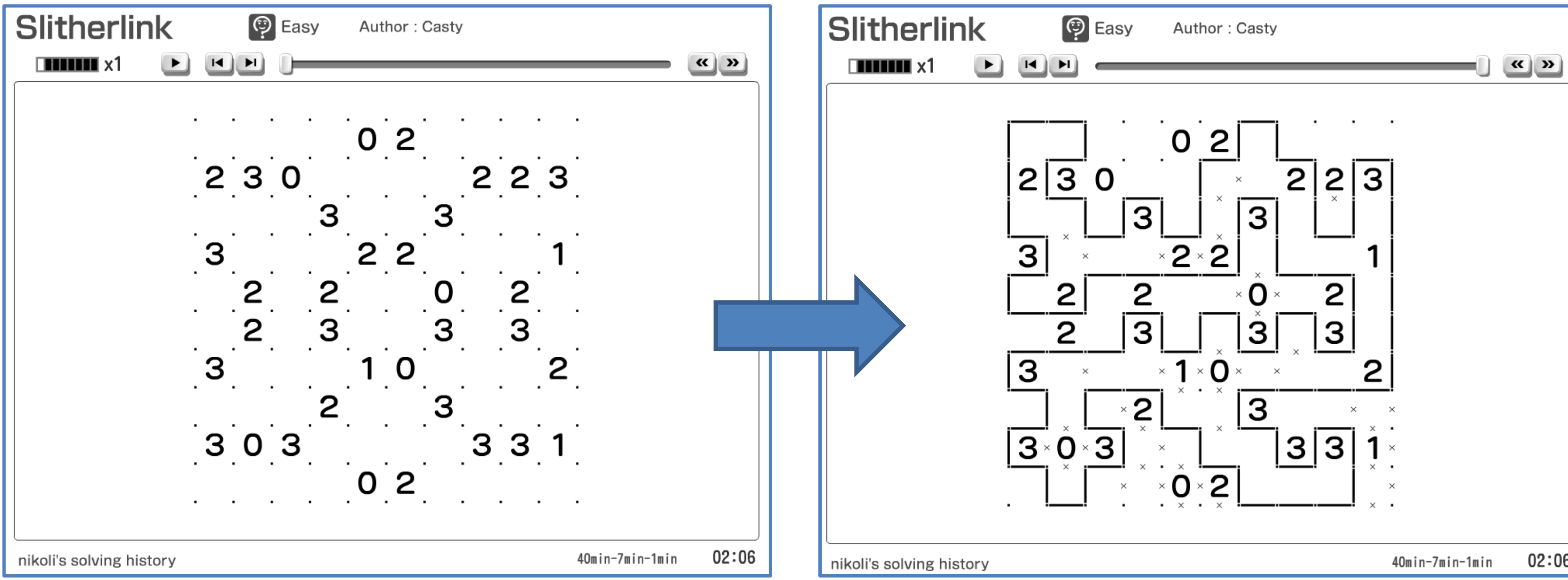


3-OR

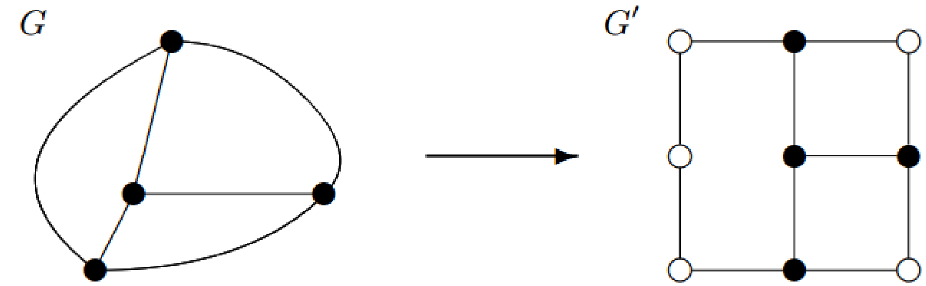
Planar # Ham. Cycles [Sato 2002]



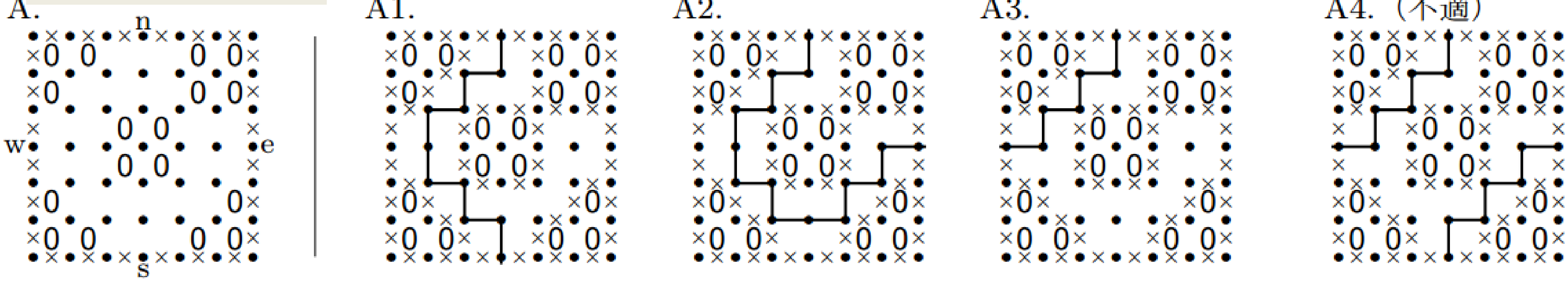
Slitherlink [Nikoli 1989]

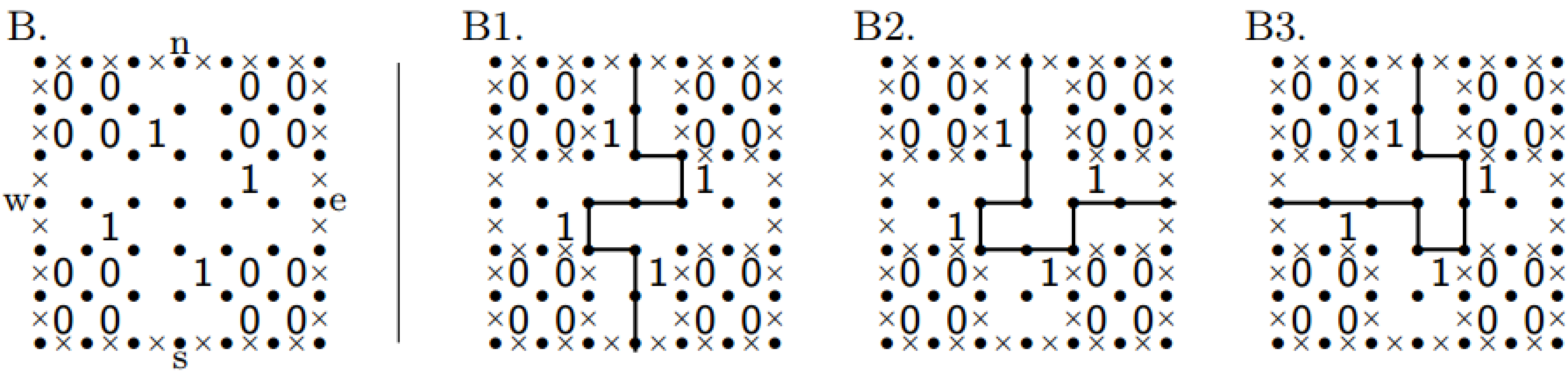


Slitherlink is NP-complete

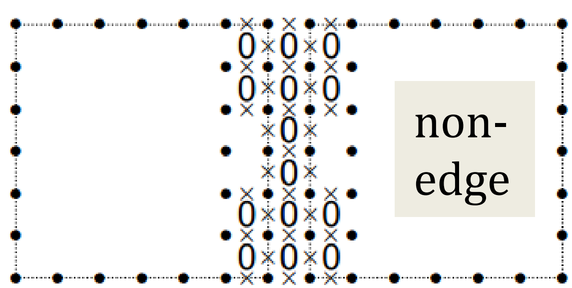


optional vertex

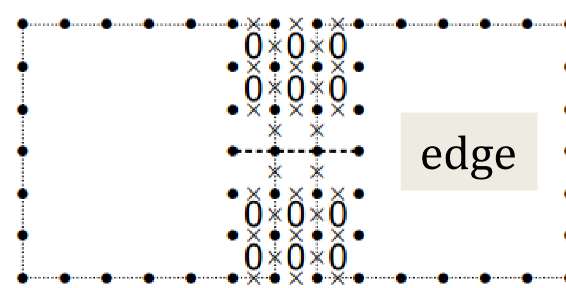




required vertex



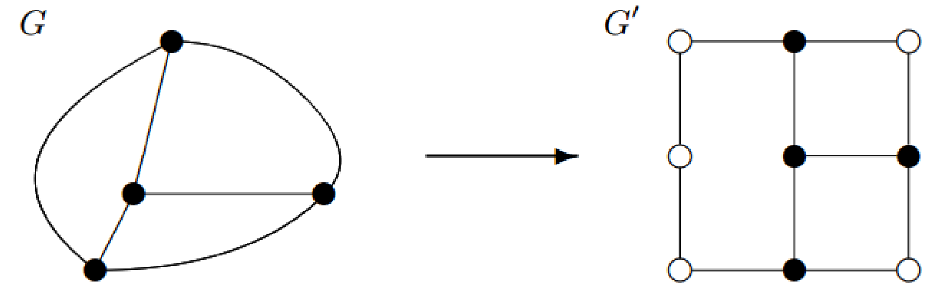
non-edge

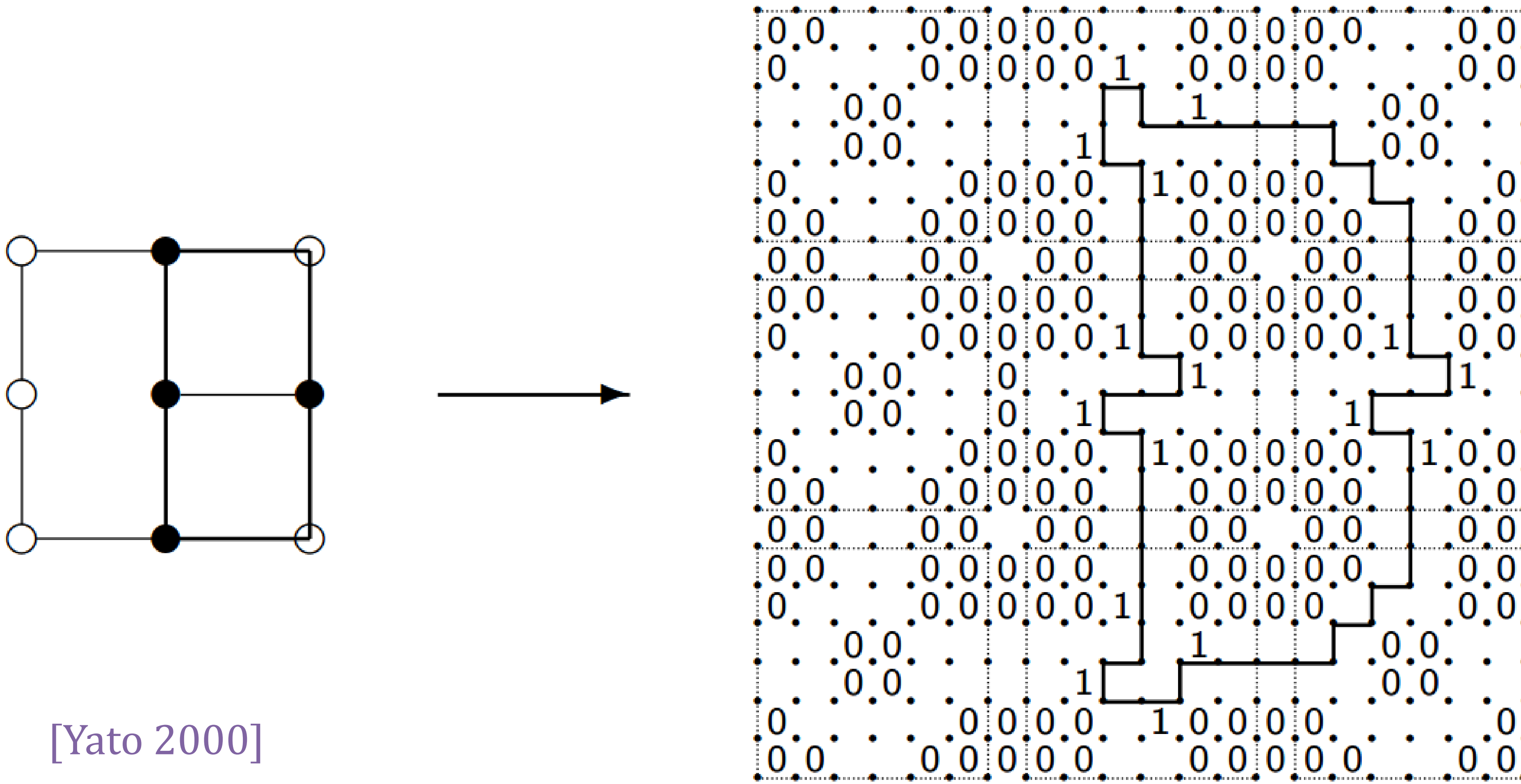


edge

[Yato 2000]

Slitherlink is NP-complete

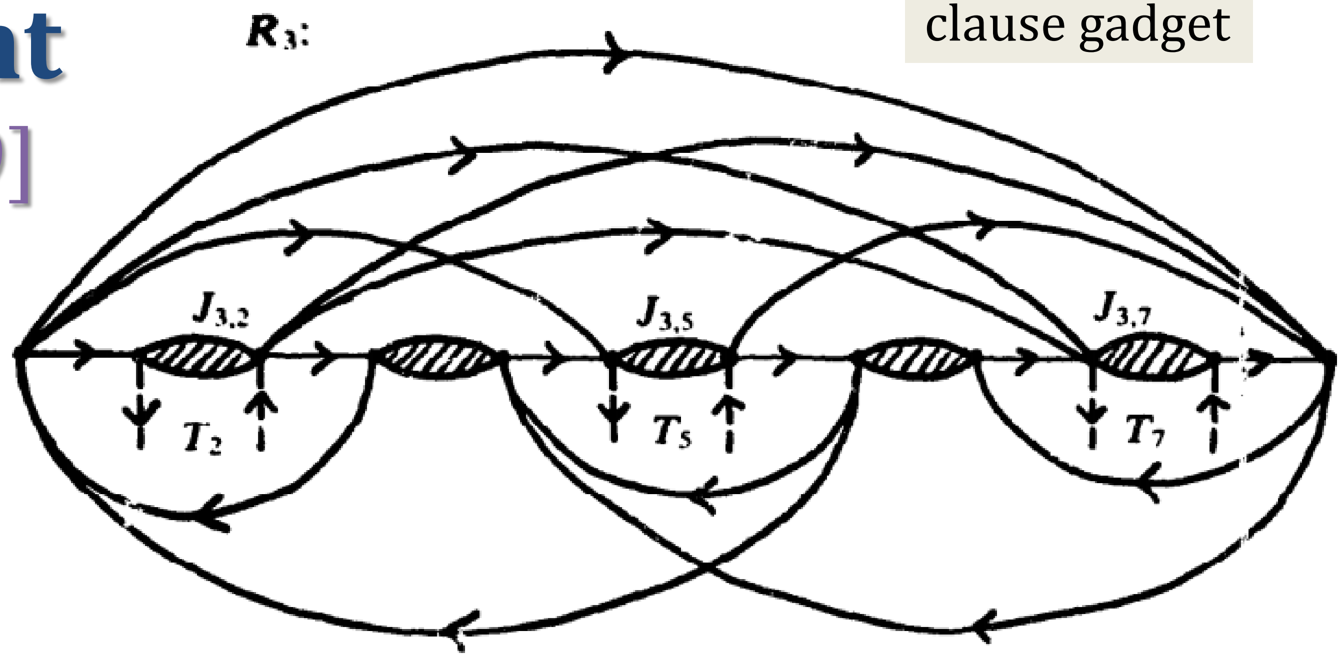


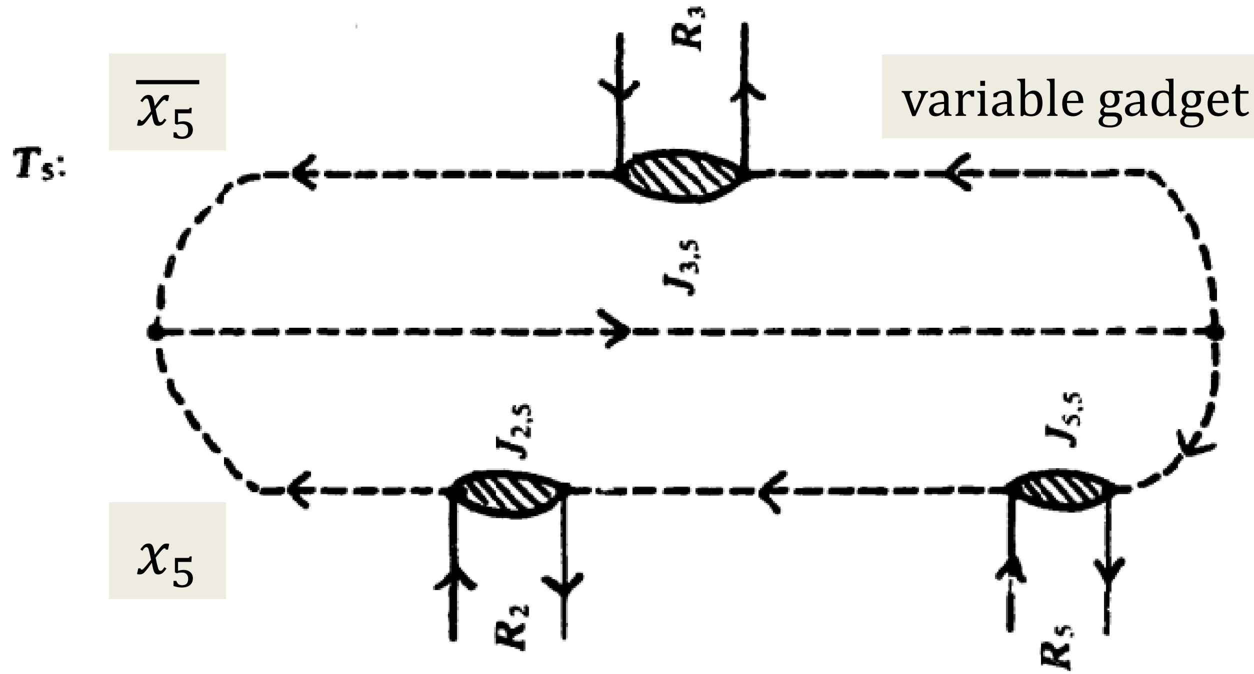


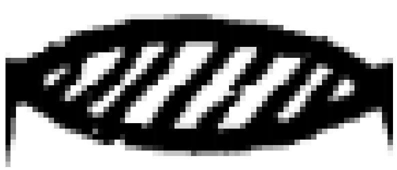
[Yato 2000]

Permanent

[Valiant 1979]

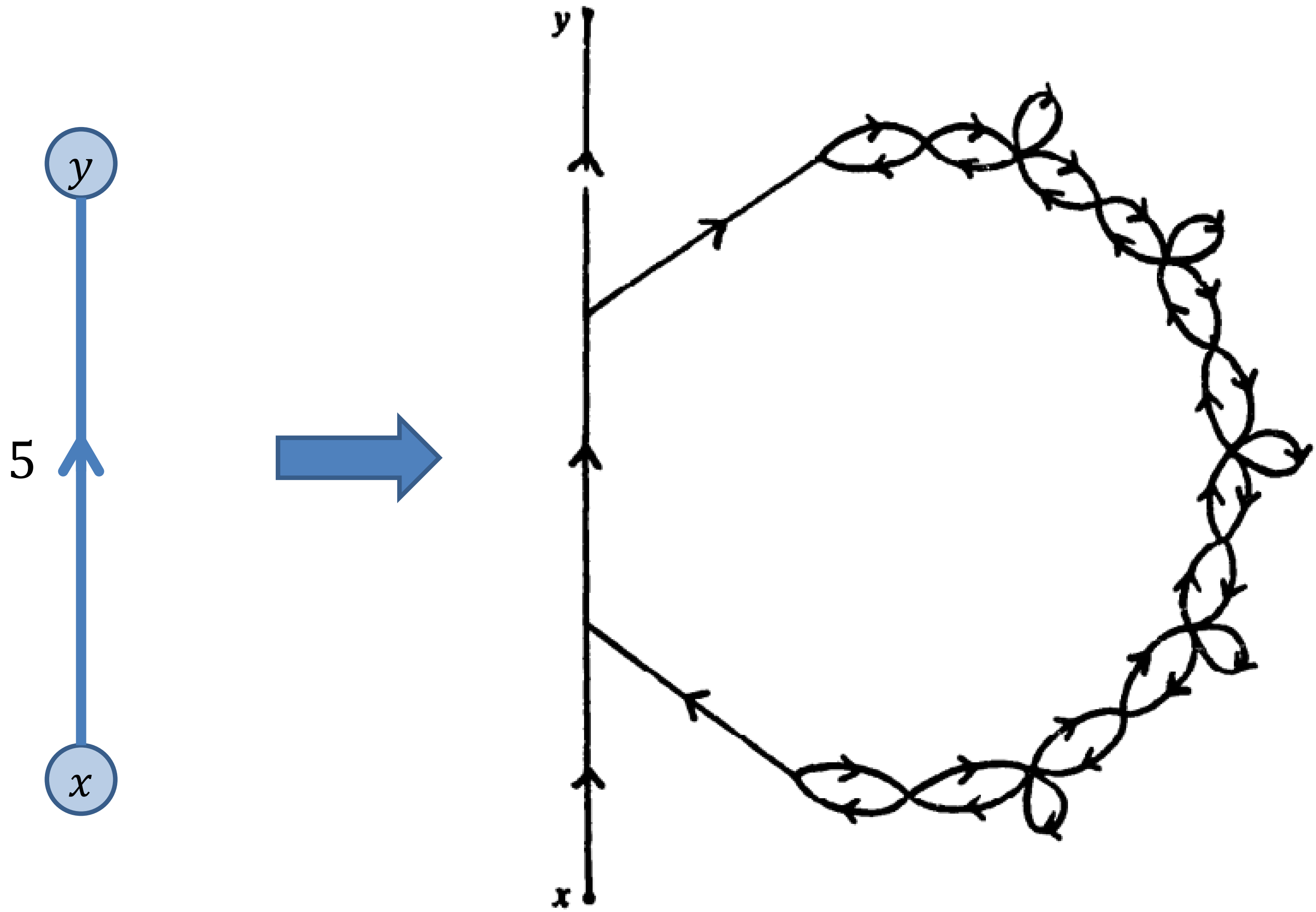






$$\begin{pmatrix} 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 \end{pmatrix}.$$

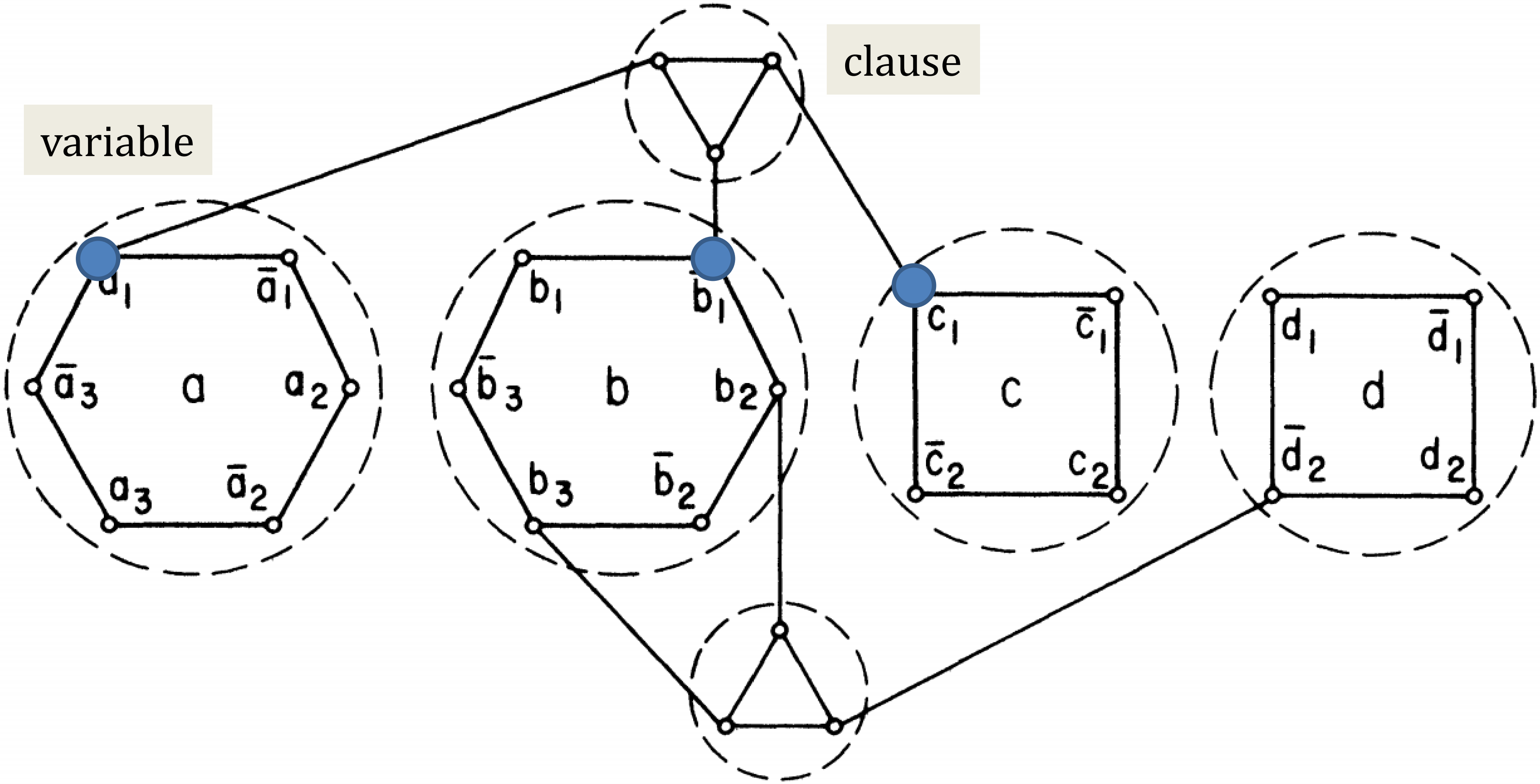
0/1-Permanent
[Valiant 1979]



Planar Vertex Cover

[Lichtenstein 1982]

not parsimonious



Example : $B = (a + \bar{b} + c)(b + b + \bar{d})$